RECENT INTERESTING AND USEFUL ENHANCEMENTS OF POLYPHASE FILTER BANKS

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Best Paper Award at Conference

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What the Customer Wants



What ever you told the Customer, this is what he/she Heard:

> More Bandwidth More Dynamic Range More Formats More Options More Bells More Whistles More More More More

What the Customer Expects to Pay



When the Customer Wants it.



Size the Customer Wants.



When a sampled data filter reduces bandwidth, it makes sense that the system should also reduce sample rate.
It would be interesting if the filter could do both; reduce BW and reduce sample rate



Re-Sampling!

Does That Mean We Didn't Do it Right the First time?

Common Down Sampling Filters



Dual LTI Filters Have Same Transfer Function



Dual Graphs:

Replace Nodes with Summing Junctions, Replace Summing Junctions with Nodes, Reverse Direction of Arrows. Dual graph is Seen as transposing State transition matrix

Dual LTV Filters Perform Opposite Function



M-to-1 Down Sampler

1-to-M Up Sampler

Motivation For Using Multirate Filters



Processing Task: Obtain Digital Samples of Complex Envelope Residing at Frequency f_C



See!



DSP Insertion in Communication Systems

Instinctive First Response: Copy Legacy Analog Prototype

- We should avoid this approach!!
- If we don't, we emulate an analog design!
- That is not the reason we invoke and apply DSP!
- DSP is inserted to improve performance and reduce cost!
- Analog prototype systems incorporate design compromises appropriate for the time they were made!
- We don't want to perpetuate those compromises!
- We have access to tools and resources not available to past designers!



Signal Conditioning for DSP Receiver





Fundamental Operations Select Frequency, Limit Bandwidth, Select Sample Rate



Spectral Description Fundamental Operation CHANNEL OF INTEREST INPUT ANALOG FILTER RESPONSE ~ -fs/2 0 fs/2 TRANSLATED SPECTRUM 0 -fs/2 fs/2 OUTPUT DIGITAL FILTER RESPONSES FILTERED SPECTRUM -fs/2 0 fs/2 SPECTRAL REPLICATES AT DOWN-SAMPLED RATE

-fs/M 0

fs/M

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Signal and Filter are at Different Frequencies Which One to Move??



Down Sample Complex Digital IF



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Fundamental Operation Modified



Equivalency Theorem

I learned this from Irwin Jacobs in 1970 when I was a grad student at UCSD Communication Systems Principles of Communication Engineering Wozencraft and Jacobs

$$r(n) = s(n)e^{-j\theta_0 n} * h(k)$$

$$= \sum_k s(n-k)e^{-j\theta_0(n-k)}h(k)$$

$$= e^{-j\theta_0 n}\sum_k s(n-k)h(k)e^{j\theta_0 k}$$

$$= e^{-j\theta_0 n} \{s(n) * h(n)e^{j\theta_0 n}\}$$

Signal Flow Description of Equivalency Theorem



Reorder Translate and Resample



SPECTRAL DESCRIPTION

REORDERED FUNDAMENTAL OPERATION



Successive Transformations to turn Sampled Data Version of Edwin Armstrong's Heterodyne Receiver to Tuned Radio Frequency (TRF) Receiver to Aliased TRF Receiver.



Now Problem Of computing Output samples Destined to be Be discarded

Coefficient Assignment of Polyphase Partition

For M-to-1 resample start at Index r and Increment by M For 3-to-1 resample start at index r and increment by 3



Non-Zero Coefficient

This mapping from 1-D to 2-D is used by Cooley-Tukey FFT. Polyphase Filters and CT-FFT are kissing cousins!

Polyphase Partition of Low Pass Filter 1-Path to M-Path Transformation





$$H(Z) = \sum_{r=0}^{M-1} Z^{-r} \sum_{n=0}^{N-1} h(r + nM) Z^{-nM}$$

M-Path Partition Supports M-to-1 Down Sample Also Supports Rational Ratio M-to-Q and M-to-Q/P Down Sample!



Polyphase Partition of Band Pass Filter 1-Path to M-Path Transformation

Modulation Theorem of Z-Transform

$$G(Z) = \sum_{n=0}^{N-1} h(n) e^{j\theta_k n} Z^{-n} = \sum_{n=0}^{N-1} h(n) (e^{-j\theta_k} Z)^{-n} = H(e^{-j\theta_k} Z)$$

$$G(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r+nM) e^{j\theta_k(r+nM)} Z^{-(r+nM)}$$

$$G(Z) = \sum_{r=0}^{M-1} e^{j\theta_k r} Z^{-r} \sum_{n=0}^{N-1} h(r+nM) e^{j\theta_k nM} Z^{-nM}$$

Polyphase Band Pass Filter and M-to-1 Resampler



Noble Identity: Commute M-units of Delay followed by M-to-1 Down Sample



M-Units of Delay at Input Rate Same as 1-Unit of Delay at Output Rate





Apply Noble Identity to Polyphase Partition



We Reduce Sample Rate M-to-1 Prior to Reducing Bandwidth

(Nyquist is Raising His Eyebrows!)

We Intentionally Alias the Spectrum. (Were you Paying Attention in school when they discussed the importance of anti-aliasing filters?)

M-fold Aliasing! M-Unknowns! M-Paths supply M-Equations We can the separate Aliases!

Move Phase Spinners to Output of Polyphase Filter Paths



Want Phase Spinners as far away from resampler as possible

Polyphase Partition with Commutator Replacing the "r" Delays in the "r-th" Path



Note: We don't assign Phase Spinners to Select **Desired Center Frequency** Till after Down Sampling And Path Processing

This Means that The Processing for every Channel is the same till the Phase Spinners

No longer LTI, Filter now has M-Different Impulse Responses! Now LTV or PTV Filter.

Armstrong to Tuned RF with Alias Down Conversion to Polyphase Receiver



Rather than selecting center frequency at input and reduce sample rate at output, we reverse the order, reduce sample rate at input and select center frequency at output. We perform arithmetic operations at low output rate rather than at high input rate!
Reorder Filter and Resample



A Sad but True Story!

- In 1983 I designed a 65536 channelizer for GTE in San Jose in response to a Request for Proposal (RFP) for Multi channel receiver for US Navy.
- My Design aliased all 65 k channels to baseband by down sampling prior to any signal processing. I then used the phase rotators of the IFFT to extract (un-alias) the separate channels.
- I was sure we would be famous. We were revolutionizing channelizers! And GTE was assured to be awarded the contract!
- The proposal was returned to us by the reviewer with **big red** letters across the face of the proposal saying...

What I Learned is that if you are smarter than your reviewer, you are both in trouble.





Single Channel Armstrong and Multirate Aliased Polyphase Receiver



Rather than select the band you want to move to baseband (Armstrong heterodyne), you move all bands to baseband and extract the alias of interest by phase coherent Summation which destructively cancels all other aliases.

Dual Channel Armstrong and Multirate Aliased Polyphase Receiver



M-Channel Polyphase Channelizer: M-path Filter and M-point FFT



Advantage of Polyphase Filter over single stage tapped delay line implementation

4.5.5 11.15

Homey Spector





Nyquist Rate for Filter is 80 kHz or fs/50 Can Perform 50-to-1 Down Sample and Still Satisfy Nyquist Reduce Sample Rate at Input to Filter: Very Efficient Implementation!



Down Sample to Reduce Sample Rate in Proportion to Bandwidth Reduction and Up Sample to Preserve Input Sample Rate.



16 Ops per Input-Output Sample Replaces 400-Tap Requiring 400 Ops per Input Output

47

Efficient Polyphase Filter



16 Ops per Input-Output Sample Replaces 400-Tap Filter Requiring 400 Ops per Input Output Sample **48**







Inner Filter: 1/M Length Reduction, 1/M Clock Reduction, 1/M² Workload Reduction

Standard M-Path Polyphase Analysis Channelizer

Channel Spacing from IFFT Channel Bandwidth from Filter Prototype Output Sample Rate for Input Commutator



Conventional Channelizer Center Frequencies Match Frequencies of M-Point IFFT, the integer Multiples of fs/M, k·fs/M

We Identify these Frequencies as the Even Multiples of fs/(2M), (2k)·fs/(2M)



Offset Channelizer Center Frequencies are Midway Between Frequencies of M-Point IFFT, the Integer plus 1/2 Multiples of fs/M, (k+0.5)·fs/M We Identify these Frequencies as the Odd Multiples of fs/(2M), (2k+1)·fs/(2M)

Receiver Channelizer Specifications

Channel Spacing: 24.0 MHz Channel Bandwidth: 23.5 MHz Channel Sample Rate: 24.0 MHz > Transition Bandwidth: 0.5 MHz > Number of Channels: 24 > IFFT Size: 30 Input Sample Rate: 720 MHz ➢ In-Band Ripple: 0.1 dB Stop band Attenuation: 50 dB Linear Phase FIR Filter

Number of Taps in Prototype FIR Filter: 3720 Taps

$_{N}$ \sim fs attent dB
$h_{Taps} = \frac{1}{\Delta f} \frac{1}{20}$
_ 720 50
0.5 20
= 1440 · 2.5 = 3600 <i>Taps</i>
Actual = 3720 Taps
Due to 1/f stopband slope

Frequency Response of Prototype Filter In 30-Path Filter, 124 Taps per path at 24 MHz Sample Rate



First Option to Convert Channelizer from Even Indexed Bin Centers to Odd Indexed Bin Centers Shift Input Spectra to Filter Spectral Locations

12 MHz Frequency Offset Heterodyne Operating at 720 MHz To obtain Odd Indexed Frequency Bins



Second Option to Convert Channelizer from Even Indexed Bin Centers to Odd Indexed Bin Centers Shift Filter Spectra to Signal Spectral Locations

12 MHz Frequency Offset Heterodyne Embedded in M-Path Filter Arms Operating at 24 MHz Sample Rate To obtain Odd Indexed Frequency Bins



(M/2)-to-1 Downsampled Non-Maximally Decimated M-Path Filter Doubles Output Sample Rate



With Increased Output Sample Rate we can Increase Transition Bandwidth



Original Transition BW was 0.5 MHz at 24 MHz Sample Rate

One Option is Change Transition BW to 12 MHz Which Would Reduce Filter Length by a Factor of 24

Second Option is Change Transition BW to 6 MHz Which Would let us see Stopband Level and Reduce Filter Length by a Factor of 12

Frequency Response of Prototype Filter In 30-Path Filter, 6 Taps per path at 48 MHz Sample Rate



Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade Half Band FIR Filter on Each of the 24 Output Channels



116 Tap Delay in FIR Filter 233 Taps have a Workload that is Larger than each path of original 30-Path Channelizer (124 Taps)

We are not Finished Shaving Workload Two-Path Implementation of Half Band Filter Reduces Sample Rate 2-to-1 While Reducing BW

Half of Weights in True Half Band Filter are Zero Upper Path has one Non Zero Weight

Lower Path has Even Symmetric Weights can Fold Filter and Operate Lower Path with 58 Multiplies per Output Sample at 24 MHz Rate



Following Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade Half Band Linear Phase IIR Filter on Each of the 24 Output Channels



94 Tap Delay in IIR Filter

Lower Path of Linear Phase Recursive 2-Path Filter Operates with 47 Multiplies per Output Sample at 24 MHz Rate Compare Traditional Tapped Delay Line Filter Implementation to Polyphase Filter Implementation



Frequency Domain Filtering With Cascade M/2-to-1 Analysis and 1-to-M/2 Synthesis Channelizers



660 Tap Prototype Filter
60 Path Polyphase Partition
11 Coefficients per Path
Input Filter:
22 Operations Per Input Sample
22 Operations Per Output Sample

60 Point Good-Thomas Nested Winograd FFT200 Real MultiplesAmortized over 15 Input Samples:13.3 Operations per Input Sample13.3 Operations per Output sample

For Input and Output Polyphase Filter and IFFT: 70.6 Operations per Input-Output Sample Pair

Workload: 70.6/660 = 10.7% of 660 Coefficient Tapped Delay Line Filter With same Frequency Response!

Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade 40-Path Analysis and Synthesis Channelizers on Each of the 24 Output Channels



40 Path Filter, 6-taps per path
40 Point IFFT, 5*8 Good-Thomas, Winograd, 100 multiplies
Workload: 40-Path Filter, 12-multiplies per input
40 Point IFFT 100/20, 5 Multiplies per 20 Inputs
34 multiplies per input output sample, Operating at 2.4 MHz Channelizer Rate

Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade 40-Path Analysis and 20 Path Synthesis Channelizers on Each of the 24 Output Channels



48-MHz

100 M

40 Path Filter, 6-taps per path 40 Point IFFT, 5*8 Good-Thomas, Winograd, 100 multiplies Workload: 40-Path Filter, 12-multiplies per input 40 Point IFFT 100/20, 5 Multiplies per 20 Inputs 25 multiplies per input output sample, with 20 Path and 20 Point Output Filter Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade 40-Path Analysis and 20-Path Synthesis Half Band Super Filter on Each of the 24 Output Channels



Spectrum Synthesized from 20 1.2 MHz Channels of 40-Path Channelizer

25 multiplies per input-output sample, with 40 Path Input Analysis and 20 Point Output Synthesis Channelizer Synthesized Super Filter

Lesson Learned in Multirate Channelizers

- Don't Design Filter with Narrow Transition BW and High Sample Rate. Solve Problem with Cascade of two Filters \succ First to Reduce BW and Sample Rate with Wide Transition BW Second to Reduce Transition BW and Sample Rate at Reduced Sample Rate Design Signal Conditioning First Filter with Reduced number of taps and Wider Transition BW Design second Filter to operate at its Reduced Nyquist Sample Rate Reduce Sample Rate to Nyquist Rate with Non Maximally **Decimated M-Path Polyphase Filter** Design and Operate Second Filter at Reduced Rate
 - with Reduced Number of Coefficients
- If Required, Increase Sample Rate with M-Path Polyphase Filter

Analysis and Synthesis Channelizers with Even and Odd Indexed Bin Centers M-Path Analysis Filter Bank with M to 1 Down Sampling Maximally Decimated Filter Bank



M-Path Analysis Filter Bank with M/2 to 1 Down Sampling Non Maximally Decimated Perfect Reconstruction Filter Bank



M-Path Analysis Filter Bank with M/2 to 1 Down Sampling Non Maximally Decimated Perfect Reconstruction Filter Bank






Center Frequencies: Match Roots of Z^{N} -1, N Roots of 1, exp(j 2π k/N)



Center Frequencies: Match Roots of Z^{N+1} , N Roots of -1, exp(j π /N) exp(j 2π k/N)



M-Path Analysis Filter Bank with M to 1 Down Sampling Maximally Decimated Perfect Reconstruction Filter Bank Heterodyning Input Time Series Half a Bin Width to Convert Even Indexed to Odd Indexed Bin Centers





Construct both Even and Odd Interleaved Indexed Spectral Centers from Double Length Polynomial

Even Indexed Bins, {... -4, -2, 0, +2, +4, ...} $\cdot \frac{2\pi}{N}$

Located at Roots of $(Z^N - 1)$

Odd Indexed Bins, {... -5, -3, -1, +1, +3, +5, ...} $\cdot \frac{2\pi}{N}$

Located at Roots of $(Z^N + 1)$

Interleaved Indicies, {... -3, -2, -1, 0, +1, +2, +3, ...} $\cdot \frac{2\pi}{2N}$

Located at Roots of $(Z^{N} - 1)(Z^{+N} + 1) = (Z^{2N} - 1)$

This Option Requires a Double Length Filter and IFFT

We do away with the Complex Heterodyne of half a

Bin Width of the Input Signal at the cost of a Double Length IFFT

Double Number of Center Frequencies

Center Frequencies: Match Roots of Z^{2N}-1, 2N Roots of 1, exp(j 2π k/(2N))





Another Approach

- ➤ When N is an Even Number
- There is Root Symmetry about k=0 and k=N/2
- If we Heterodyne DC to the Half Sample Rate DC is still on a root of Z^N-1
- DFT Samples Above and Below k=N/2 are the same Samples Above and Below k=0, k=+1 and k=-1
- When N is an Odd Number
- ➤ The Root Symmetry about k=0 Differs from k=N/2
- ➤ There is no Root at k=N/2!
- ➤ There is a Root Above and Below N/2 by ±1/2

Symmetry of Zeros at DC and at fs/2 of a 16 Point DFT



Lack of Symmetry of Zeros at DC and fs/2 of a 15 Point DFT



M-Path Channelizer For Even Indexed DFT Bin Centers



M-Path Channelizer For Odd Indexed DFT Bin Centers Don't Shift Input Spectrum Half a Bin, Shift to fs/2



The Polyphase Filter Sees the Sign Changes of The Input Samples Because the Array has an Odd Number of Samples



15

New

Inputs

with

sign



new		old		
<mark>n+15</mark>	-	n	+	
<mark>n+14</mark>	+	n-1	-	
<mark>n+13</mark>	-	n-2	+	
<mark>n+12</mark>	+	n-3	-	
<mark>n+11</mark>	-	n-4	+	
<mark>n+10</mark>	+	n-5	-	
<mark>n+9</mark>	-	n-6	+	
<mark>n+8</mark>	+	n-7	-	
<mark>n+7</mark>	-	n-8	+	
<mark>n+6</mark>	+	n-9	-	
<mark>n+5</mark>	-	n-10	+	
<mark>n+4</mark>	+	n-11	-	
<mark>n+3</mark>	-	n-12	+	
<mark>n+2</mark>	+	n-13	-	
n+1	-	n-14	+	

Perform a Non-Maximally decimated Filter with an Even Number of Input Points



The Polyphase Filter No Longer Sees the Sign Changes of The Input Samples Because the Input Array has an Even Number of Samples



The Rotators of the Non-Maximally Decimated Filter with an Even Number of Input Points can be Embedded in the Filter Weights











River Publishers Series in Sighal, Image, and Speech Processing



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Professor harris: may I be excused? My brain is full!

Two Quick Examples of Applied Magic with Polyphase Filter Banks



Sitting over Japan is a geosynchronous satellite beaming down 192 stereo MP3 Signals.

Satellite Broadcasts 384 MP3 Channels to Earth Stations

Demodulate all MP3 Channels Remodulate as FM Channels

> Task: Replace Legacy Transceiver

What size room is required to house new DSP based Transceiver? 97

Equipment Bay: 192-Stereo FM Modulators



Conversation with Client!

- How big a room will we need to house the DSP version of this Transceiver?
- My Answer: I think it will fit on one chip.
- Response:

Don't be Absurd: Can't Pack a Room on a Single Chip!

- Results: 48-Analog Devices Blackfin Processors to Demodulate 192 MP3 Stereo Channels.
- 1 Virtex V-4 for 192 Digital Stereo FM Modulators and 256 Channel Channelizer @ 293 kHz Bandwidth per Channel. (60% of Chip)



A Smaller Package



2-U High, Full Rack Width

H 3.5 in, 8.89 cm
W 17.0 in, 43.18 cm
D 9.4 in, 23.88 cm

How to Pack a Room of Analog FM Modulators into a Xilinx FPGA

DSP techniques replace a legacy multi-channel analog modulator.

by Fred Honts Professor Son Diego State Delwestly And Americ@ation adv

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You are likely familiar with the way that digiral colorision is transmitted from astellizes as multi-channel MPEG (Motion Picture Experts Group) compressed video to a cable head and where the multiple channels are demodulated. The MPEG streams are decoded and then remodulated as channelized analog NTSC (National Televition Seandards Committee) or PAL (Phase Alternating Lines) televition signals for insertion in a cable distribution plane.

Similarly, high-quality storeo audio is transmitted from a satellite as multi-channel MP3 (MPEG Layer-3) compressed audio to a cable head end where the multiple channels are demodulated. The MP3 streams are decoded and then remodulated as channelited analog FM signals for intertion in a cable distribution plant.



Figure 1 – Equipment bay constituting legacy transactives equipment





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Diagan Valeit Signum Concepts 工程电道 degen valeiteithignamamarytaan

无线

Wade Lowdern lik Signum Concepts BRID:1717 webs lowing the Unignament optimum

非可能熟悉数字电相的传播方式。值号以 多位道 MPEG (初初回像专家说) 压缩相 教教式从卫星传送到电缆头编程。成后对 多个做道进行相遇。MPEG 滚旋解码后 被 里解调制力依道化模拟 MTSC (国家电相称 准委员会) 或 PAL (进行假福)电相做号 并独入到电缆分配设备中。

与此指似。 高品质的立体声音频说 与 以多依差 NP3 (MPES Leyer-3) 压 康音频格式 从卫星传送到电缆头着器。 武后对多个 依 差近行解消。 MP3 流经过解码点被重 新调 制力依差化模拟 FM 他与并强入到电缆分 配设备中。



题1-8合作统收发展 设备的设备是

D37 (\$4.8)

101

...

...



Six Channels, 96 MHz wide Channels, 94 MHz Occupied, 16*102.4 MHz = 1.6384 GHz sample rate Occupied band 108 to 684 MHz: stopband -80 dB, passband 0.05 dB

Spectrum: 1401-Tap Filter





Spectrum Direct Implementation (Red), Synthesized Implementation (Blue), Outer Tier Channel (Dashed Black), Inner Tier Channel (Black)





Spectrum Direct Implementation (Red), Synthesized Implementation (Blue), Outer Tier Channel (Dashed Black), Inner Tier Channel (Black)

Dilbert, is it true that DSP makes the world go around but multirate signal processing supplies the music for the ride?

Can There be any Doubt???



SOFTWARE DEFINED RADIO MAN Is Open For Questions

