RECENT INTERESTING AND USEFUL ENHANCEMENTS OF POLYPHASE FILTER BANKS

Parts of presentations made at the 23rd International Symposium on Wireless Personal Multimedia Communications (WPMC 2020 Virtual Edition)
18-21 October 2020
Okayama, Japan

Best Paper Award at Conference

Atlanta Section Chapter, COM19
Buenaventura Section Chapter, COM19
Coastal Los Angeles Sector Chapter, SP01, VT06/Com19
Seattle Sect Jt. Chapter, COM19/VT06/BT02/IT12/ITS38
What the Customer Wants

What ever you told the Customer, this is what he/she Heard:

More Bandwidth
More Dynamic Range
More Formats
More Options
More Bells
More Whistles
More
More
More
More
What the Customer Expects to Pay
When the Customer Wants it.
Size the Customer Wants.
When a sampled data filter reduces bandwidth, it makes sense that the system should also reduce sample rate. It would be interesting if the filter could do both; reduce BW and reduce sample rate.
Re-Sampling!

Does That Mean We Didn’t Do it Right the First time?
Common Down Sampling Filters

POLYPHASE FILTER

DYADIC HALF-BAND FILTER

HOGENAUER FILTER (CIC)

M-samples in
1-sample out
Dual LTI Filters Have Same Transfer Function

Dual Graphs:
Replace Nodes with Summing Junctions,
Replace Summing Junctions with Nodes,
Reverse Direction of Arrows.

Dual graph is
Seen as transposing
State transition matrix
Dual LTV Filters Perform Opposite Function

M-to-1 Down Sampler

1-to-M Up Sampler
Motivation For Using Multirate Filters
Processing Task:
Obtain Digital Samples of Complex Envelope Residing at Frequency $f_C$

Applause from Audience
When I presented this slide
At Signal Processing Society
Meeting at
Brigham Young University
Michael Rice was my host
See!
DSP Insertion in Communication Systems

Instinctive First Response:
Copy Legacy Analog Prototype

• We should avoid this approach!!
• If we don’t, we emulate an analog design!
• That is not the reason we invoke and apply DSP!
• DSP is inserted to improve performance and reduce cost!
• Analog prototype systems incorporate design compromises appropriate for the time they were made!
• We don’t want to perpetuate those compromises!
• We have access to tools and resources not available to past designers!
Signal Conditioning for DSP Receiver

1. Low Pass Filter

2. fs

3. DSP PROCESS

4. fs/2
Duplicate Analog Processing in DSP

Ignoring Good Advice!
Fundamental Operations
Select Frequency,
Limit Bandwidth,
Select Sample Rate

\[ s(t) \rightarrow s(n) \rightarrow s(n)e^{-j\theta_0 n} \rightarrow r(n) \rightarrow r(nM) \]

ADC

LO

LOWPASS FILTER

CLK

M:1
Spectral Description
Fundamental Operation

INPUT ANALOG FILTER RESPONSE

CHANNEL OF INTEREST

TRANSLATED SPECTRUM

OUTPUT DIGITAL FILTER RESPONSE

FILTERED SPECTRUM

SPECTRAL REPLICATES AT DOWN-SAMPLED RATE
Signal and Filter are at Different Frequencies
Which One to Move??

Second Option

First Option
Down Sample Complex Digital IF

Low Pass Filter

Band Pass Filter

$e^{j\omega_0n}$

$e^{-j\omega_0n}$

$f_s$ $\frac{f_s}{2}$ $\frac{f_s}{M}$

$\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$
Fundamental Operation Modified

s(t) → ADC → s(n) → Bandpass Filter → r(n)e^{j\theta_0n} → r(n) → r(nM)

h(n)e^{j\theta_0n}

CLK → Bandpass Filter

LO → Bandpass Filter

M:1
Equivalency Theorem

\[ r(n) = s(n)e^{-j\theta_0 n} * h(k) \]

\[ = \sum_k s(n-k)e^{-j\theta_0 (n-k)} h(k) \]

\[ = e^{-j\theta_0 n} \sum_k s(n-k)h(k)e^{j\theta_0 k} \]

\[ = e^{-j\theta_0 n} \{ s(n) * h(n)e^{j\theta_0 n} \} \]

I learned this from Irwin Jacobs in 1970 when I was a grad student at UCSD.
Communication Systems
Principles of Communication Engineering
Wozencraft and Jacobs
Signal Flow Description of Equivalency Theorem
Reorder Translate and Resample
SPECTRAL DESCRIPTION

REORDERED FUNDAMENTAL OPERATION
Successive Transformations to turn Sampled Data Version of Edwin Armstrong’s Heterodyne Receiver to Tuned Radio Frequency (TRF) Receiver to Aliased TRF Receiver.

Any Multiple of Output Sample Rate Aliases to Baseband

Can build DSP receiver Without a digital Down Converter

Now Problem Of computing Output samples Destined to be Be discarded
Coefficient Assignment of Polyphase Partition

For M-to-1 resample start at Index r and Increment by M
For 3-to-1 resample start at index r and increment by 3

This mapping from 1-D to 2-D is used by Cooley-Tukey FFT.
Polyphase Filters and CT-FFT are kissing cousins!
Polyphase Partition of Low Pass Filter
1-Path to M-Path Transformation

\[ H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n} \]

\[ H(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r + nM)Z^{-(r+nM)} \]

\[ H(Z) = \sum_{r=0}^{M-1} Z^{-r} \sum_{n=0}^{N-1} h(r + nM)Z^{-nM} \]

M-Path Partition Supports M-to-1 Down Sample
Also Supports Rational Ratio
M-to-Q and M-to-Q/P Down Sample!
Polyphase Partition of Band Pass Filter  
1-Path to M-Path Transformation

Modulation Theorem of Z-Transform

\[ G(Z) = \sum_{n=0}^{N-1} h(n) e^{j\theta_k n} Z^{-n} = \sum_{n=0}^{N-1} h(n)(e^{-j\theta_k} Z)^{-n} = H(e^{-j\theta_k} Z) \]

\[ G(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r + nM) e^{j\theta_k (r+nM)} Z^{-(r+nM)} \]

\[ G(Z) = \sum_{r=0}^{M-1} e^{j\theta_k r} Z^{-r} \sum_{n=0}^{N-1} h(r + nM)e^{j\theta_k nM} Z^{-nM} \]

\[ G(Z) = \sum_{r=0}^{M-1} e^{\frac{2\pi jkr}{M}} Z^{-r} \sum_{n=0}^{N-1} h(r + nM)Z^{-nM} \]

\[ M \cdot \theta_k = k \cdot 2\pi \]

or \[ \theta_k = k \cdot \frac{2\pi}{M} \]
Polyphase Band Pass Filter and M-to-1 Resampler
Noble Identity: Commute M-units of Delay followed by M-to-1 Down Sample

M-Units of Delay at Input Rate Same as 1-Unit of Delay at Output Rate
Apply Noble Identity to Polyphase Partition

We Reduce Sample Rate
M-to-1 Prior to Reducing Bandwidth
(Nyquist is Raising His Eyebrows!)

We Intentionally Alias the Spectrum.
(Were you Paying Attention
in school when they discussed the
importance of anti-aliasing filters?)

M-fold Aliasing!
M-Unknowns!
M-Paths supply M-Equations
We can the separate Aliases!
Move Phase Spinners to Output of Polyphase Filter Paths

Want Phase Spinners as far away from resampler as possible
Polyphase Partition with Commutator 
Replacing the “r” Delays in the “r-th” Path

Note: We don’t assign Phase Spinners to Select Desired Center Frequency Till after Down Sampling And Path Processing

This Means that The Processing for every Channel is the same till the Phase Spinners

No longer LTI, Filter now has M-Different Impulse Responses! Now LTV or PTV Filter.
Rather than selecting center frequency at input and reduce sample rate at output, we reverse the order, reduce sample rate at input and select center frequency at output. We perform arithmetic operations at low output rate rather than at high input rate!
Reorder Filter and Resample

Hmm... this is very good stuff....
A Sad but True Story!

• In 1983 I designed a 65536 channelizer for GTE in San Jose in response to a Request for Proposal (RFP) for Multi channel receiver for US Navy.

• My Design aliased all 65 k channels to baseband by down sampling prior to any signal processing. I then used the phase rotators of the IFFT to extract (un-alias) the separate channels.

• I was sure we would be famous. We were revolutionizing channelizers! And GTE was assured to be awarded the contract!

• The proposal was returned to us by the reviewer with big red letters across the face of the proposal saying…

• 😞 “Those Who Don’t Understand The Nyquist Theorem Shouldn’t be Doing Signal Processing”

What I Learned is that if you are smarter than your reviewer, you are both in trouble.
Aliases spin at different rates. All multiples of the M roots of unity. Extract a particular alias by de-spinning it prior to summing the paths. Only the non-spinning alias survives summation. All other aliases destructively cancel!

Frequency bands preserve Their complex envelope when translated by heterodyne or by aliasing!
Polyphase Partition

\[ \mathbf{h}(n) \]

Polyphase

\[ \mathbf{h}(n) \]

M-2

Polyphase

\[ \mathbf{h}(n) \]

1

Polyphase

\[ \mathbf{h}(n) \]

M-1

Polyphase

\[ \mathbf{h}(n) \]

0

\[ e^{-j0 \theta_k} \]

\[ e^{-j1 \theta_k} \]

\[ e^{-j(M-2) \theta_k} \]

\[ e^{-j(M-1) \theta_k} \]

Low Pass Filter

fs

fs/2

-\( fs/2 \)
Rather than select the band you want to move to baseband (Armstrong heterodyne), you move all bands to baseband and extract the alias of interest by phase coherent Summation which destructively cancels all other aliases.
Dual Channel Armstrong and Multirate Aliased Polyphase Receiver

Standard DDC

Polyphase DDC

1-Polyphase Filter

2-Polyphase Filters

1

2

M-to-1
M-Channel Polyphase Channelizer: M-path Filter and M-point FFT

\[ h(n) = h(r + nM) \]

Polyphase Partition

\[ h_0(n) \quad h_1(n) \quad h_2(n) \quad h_3(n) \quad \ldots \]

M-PNT FFT

FDM \( f_s \)

TDM

\[ h_r(n) = h(n + nM) \]
Advantage of Polyphase Filter over single stage tapped delay line implementation
Sample Rate Large Compared to Bandwidth

BW = 40 kHz
Δf = 40 kHz
fs = 4 MHz
Attn = 80dB

Nyquist Rate for Filter is 80 kHz or fs/50
Can Perform 50-to-1 Down Sample and Still Satisfy Nyquist

50-to-1 Ratio

fs > BW + Δf (2 sided BW + transition BW)
Reduce Sample Rate at Input to Filter: Very Efficient Implementation!

400 Weights

50 Path Polyphase Filter

80-kHz Output Sample Rate

4-MHz Input Sample Rate

50-to-1

4-MHz

8 taps

\[ \phi_0, \phi_1, \phi_2, \ldots, \phi_{49} \]

80-kHz

4-MHz

8-Tap Filter

Select Path Weights

50-Path Coefficient Bank

400 Weights

\[ \text{N/M} = \frac{400}{50} = 8 \text{-ops/Input} \]
Down Sample to Reduce Sample Rate in Proportion to Bandwidth Reduction and Up Sample to Preserve Input Sample Rate.

400-Tap Down Sample Filter Implemented with 8 Ops/Input

400-Tap Up-Sample Filter Implemented with 8 Ops/Output

16 Ops per Input-Output Sample
Replaces 400-Tap Requiring 400 Ops per Input Output
Efficient Polyphase Filter

16 Ops per Input-Output Sample Replaces 400-Tap Filter Requiring 400 Ops per Input Output Sample
Different Processes in Two Boxes: How can you tell which is which from outside box?

(The Wet Finger Test)

400 ops/input

4 MHz

400-Tap Low-Pass Filter

8-Tap Filter

Coefficient Bank

State Machine

Select

8 MHz

80-kHz

4 MHz

16-ops/input

Ouch

Ooh
Inner Filter:
1/M Length Reduction,
1/M Clock Reduction,
1/M² Workload Reduction

4000 tap Filter
Implemented with
18 Ops/Input
Standard M-Path Polyphase Analysis Channelizer

Channel Spacing from IFFT
Channel Bandwidth from Filter Prototype
Output Sample Rate for Input Commutator

\[ h_r(n) = h(r + nM) \]
Conventional Channelizer Center Frequencies Match Frequencies of M-Point IFFT, the integer Multiples of \( fs/M \), \( k \cdot fs/M \)
We Identify these Frequencies as the Even Multiples of \( fs/(2M) \), \( (2k) \cdot fs/(2M) \)

25 Channel Centers on Even Multiples of 12

Offset Channelizer Center Frequencies are Midway Between Frequencies of M-Point IFFT, the Integer plus \( 1/2 \) Multiples of \( fs/M \), \( (k+0.5) \cdot fs/M \)
We Identify these Frequencies as the Odd Multiples of \( fs/(2M) \), \( (2k+1) \cdot fs/(2M) \)
Receiver Channelizer Specifications

- Channel Spacing: 24.0 MHz
- Channel Bandwidth: 23.5 MHz
- Channel Sample Rate: 24.0 MHz
- Transition Bandwidth: 0.5 MHz
- Number of Channels: 24
- IFFT Size: 30
- Input Sample Rate: 720 MHz
- In-Band Ripple: 0.1 dB
- Stop band Attenuation: 50 dB
- Linear Phase FIR Filter

Number of Taps in Prototype
FIR Filter: 3720 Taps

\[ N_{Taps} \approx \frac{fs \cdot \text{atten (dB)}}{\Delta f \cdot 20} \]

Due to 1/f stopband slope

\[ = \frac{720 \cdot 50}{0.5 \cdot 20} = 1440 \cdot 2.5 = 3600 \text{ Taps} \]

Actual = 3720 Taps

Due to 1/f stopband slope
Frequency Response of Prototype Filter
In 30-Path Filter, 124 Taps per path at 24 MHz Sample Rate
First Option to Convert Channelizer from Even Indexed Bin Centers to Odd Indexed Bin Centers
Shift Input Spectra to Filter Spectral Locations

12 MHz Frequency Offset Heterodyne Operating at 720 MHz
To obtain Odd Indexed Frequency Bins

fs=720 MHz
Second Option to Convert Channelizer from Even Indexed Bin Centers to Odd Indexed Bin Centers
Shift Filter Spectra to Signal Spectral Locations

12 MHz Frequency Offset Heterodyne Embedded in M-Path Filter Arms
Operating at 24 MHz Sample Rate To obtain Odd Indexed Frequency Bins
(M/2)-to-1 Downsampler Non-Maximally Decimated M-Path Filter
Doubles Output Sample Rate

Output sample Rate is now Twice Channel Spacing
And is Twice Channel BW
With Increased Output Sample Rate we can Increase Transition Bandwidth

Original Transition BW was 0.5 MHz at 24 MHz Sample Rate

One Option is Change Transition BW to 12 MHz Which Would Reduce Filter Length by a Factor of 24

Second Option is Change Transition BW to 6 MHz Which Would let us see Stopband Level and Reduce Filter Length by a Factor of 12
Frequency Response of Prototype Filter
In 30-Path Filter, 6 Taps per path at 48 MHz Sample Rate

Significant Workload Reduction of 30-Path Filter

Originally 3720 Taps
124 Taps/Path

Now 180 Taps
6 Taps/Path
Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade Half Band FIR Filter on Each of the 24 Output Channels.

116 Tap Delay in FIR Filter
233 Taps have a Workload that is Larger than each path of original 30-Path Channelizer (124 Taps)

We are not Finished Shaving Workload
Two-Path Implementation of Half Band Filter
Reduces Sample Rate 2-to-1 While Reducing BW

Half of Weights in True Half Band Filter are Zero Upper Path has one Non Zero Weight

Lower Path has Even Symmetric Weights can Fold Filter and Operate Lower Path with 58 Multiplies per Output Sample at 24 MHz Rate

233 Taps

116 Odd Indexed Even Symmetric Coefficients
Following Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade Half Band Linear Phase IIR Filter on Each of the 24 Output Channels.

94 Tap Delay in IIR Filter

Lower Path of Linear Phase Recursive 2-Path Filter Operates with 47 Multiplies per Output Sample at 24 MHz Rate
Compare Traditional Tapped Delay Line Filter Implementation to Polyphase Filter Implementation

- **Input Bandwidth**

  - Traditional Input Filter: 660 Taps, 660 Multiplies
  - Channelizer Filter: 660 Taps

- **Channelized Bandwidth**

- **Output Filter**
  - 57.4 Multiplies
  - 8.7% of Workload
  - 91.3% Reduction
Frequency Domain Filtering With Cascade
M/2-to-1 Analysis and 1-to-M/2 Synthesis Channelizers

660 Tap Prototype Filter
60 Path Polyphase Partition
11 Coefficients per Path
Input Filter:
22 Operations Per Input Sample
22 Operations Per Output Sample

60 Point Good-Thomas Nested Winograd FFT
200 Real Multiples
Amortized over 15 Input Samples:
13.3 Operations per Input Sample
13.3 Operations per Output Sample

For Input and Output Polyphase Filter and IFFT:
70.6 Operations per Input-Output Sample Pair

Workload: 70.6/660 = 10.7% of 660 Coefficient Tapped Delay Line Filter With same Frequency Response!
Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade 40-Path Analysis and Synthesis Channelizers on Each of the 24 Output Channels

40 Path Filter, 6-taps per path
40 Point IFFT, 5*8 Good-Thomas, Winograd, 100 multiplies
Workload: 40-Path Filter, 12-multiplies per input
    40 Point IFFT 100/20, 5 Multiplies per 20 Inputs
34 multiplies per input output sample, Operating at 2.4 MHz Channelizer Rate
Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade 40-Path Analysis and 20 Path Synthesis Channelizers on Each of the 24 Output Channels

40 Path Filter, 6-taps per path
40 Point IFFT, 5*8 Good-Thomas, Winograd, 100 multiplies
Workload: 40-Path Filter, 12-multiplications per input
40 Point IFFT 100/20, 5 Multiplies per 20 Inputs
25 multiplies per input output sample, with 20 Path and 20 Point Output Filter
Following 30-Path Channelizer, we can Reduce Transition BW and Sample Rate with a Cascade 40-Path Analysis and 20-Path Synthesis Half Band Super Filter on Each of the 24 Output Channels.

Spectrum Synthesized from 20 1.2 MHz Channels of 40-Path Channelizer

Zoom to Pass Band Ripple

Zoom to Transition Bandwidth

25 multiplies per input-output sample, with 40 Path Input Analysis and 20 Point Output Synthesis Channelizer Synthesized Super Filter
Lesson Learned in Multirate Channelizers

- Don’t Design Filter with Narrow Transition BW and High Sample Rate.
- Solve Problem with Cascade of two Filters
  - First to Reduce BW and Sample Rate with Wide Transition BW
  - Second to Reduce Transition BW and Sample Rate at Reduced Sample Rate
- Design Signal Conditioning First Filter with
  - Reduced number of taps and Wider Transition BW
- Design second Filter to operate at its Reduced Nyquist Sample Rate
- Reduce Sample Rate to Nyquist Rate with Non Maximally Decimated M-Path Polyphase Filter
- Design and Operate Second Filter at Reduced Rate with Reduced Number of Coefficients
- If Required, Increase Sample Rate with M-Path Polyphase Filter
Analysis and Synthesis Channelizers with Even and Odd Indexed Bin Centers
M-Path Analysis Filter Bank with M to 1 Down Sampling
Maximally Decimated Filter Bank
M-Path Analysis Filter Bank with M/2 to 1 Down Sampling Non Maximally Decimated Perfect Reconstruction Filter Bank
M-Path Analysis Filter Bank with M/2 to 1 Down Sampling
Non Maximally Decimated Perfect Reconstruction Filter Bank

- 10 Taps/Path
- 80 Path Filter
- 40-to-1
- 160 MHz
- 80 Point Shift
- 80-Point Circular Buffer
- Odd Number of Bins [1, 3, 5, ..., 39]
- 80 Point IFFT
- 4 MHz
- 260 M
Even Indexed Channelizer

\[ H_1(f) \]

Odd Indexed Channelizer

\[ H_2(f) \]
Center Frequencies: Match Roots of $Z^N - 1$, N Roots of 1, $\exp(j \frac{2\pi k}{N})$
Center Frequencies: Match Roots of $Z^{N+1}$, N Roots of -1, $\exp(j \pi/N) \exp(j 2\pi k/N)$
M-Path Analysis Filter Bank with M to 1 Down Sampling
Maximally Decimated Perfect Reconstruction Filter Bank
Heterodyning Input Time Series Half a Bin Width to
Convert Even Indexed to Odd Indexed Bin Centers

\[ \exp(-j \frac{2\pi n}{2M}) \]
Construct both Even and Odd Interleaved Indexed Spectral Centers from Double Length Polynomial

Even Indexed Bins, \{... -4, -2, 0, +2, +4, ...\} \cdot \frac{2\pi}{N}

Located at Roots of \(Z^N - 1\)

Odd Indexed Bins, \{... -5, -3, -1, +1, +3, +5, ...\} \cdot \frac{2\pi}{N}

Located at Roots of \(Z^N + 1\)

Interleaved Indices, \{... -3, -2, -1, 0, +1, +2, +3, ...\} \cdot \frac{2\pi}{2N}

Located at Roots of \((Z^N - 1)(Z^{+N} + 1) = (Z^{2N} - 1)\)

This Option Requires a Double Length Filter and IFFT

We do away with the Complex Heterodyne of half a Bin Width of the Input Signal at the cost of a Double Length IFFT
Double Number of Center Frequencies
Center Frequencies: Match Roots of $Z^{2N} -1$, 2$N$ Roots of 1, $\exp(j 2\pi k/(2N))$
Another Approach

- When N is an Even Number
  - There is Root Symmetry about k=0 and k=N/2
  - If we Heterodyne DC to the Half Sample Rate
    DC is still on a root of $Z^{N-1}$
  - DFT Samples Above and Below k=N/2 are the same
    Samples Above and Below k=0, k=+1 and k=-1
- When N is an Odd Number
  - The Root Symmetry about k=0 Differs from k=N/2
  - There is no Root at k=N/2!
  - There is a Root Above and Below N/2 by ±1/2
Symmetry of Zeros at DC and at $fs/2$ of a 16 Point DFT
Lack of Symmetry of Zeros at DC and $fs/2$ of a 15 Point DFT
M-Path Channelizer For Even Indexed DFT Bin Centers
M-Path Channelizer For Odd Indexed DFT Bin Centers
Don’t Shift Input Spectrum Half a Bin, Shift to $fs/2$

$$\exp(-j\pi n)$$

Diagram showing DDS, M Path Filter, and M Point IFFT.
The Polyphase Filter Sees the Sign Changes of the Input Samples because the Array has an Odd Number of Samples.

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>sign</th>
<th>data</th>
<th>sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>new</td>
<td>n</td>
<td>+</td>
<td>old</td>
<td>n-15</td>
</tr>
<tr>
<td>n-1</td>
<td>n-16</td>
<td>+</td>
<td>n-2</td>
<td>n-17</td>
</tr>
<tr>
<td>n-2</td>
<td>n-18</td>
<td>+</td>
<td>n-3</td>
<td>n-19</td>
</tr>
<tr>
<td>n-3</td>
<td>n-20</td>
<td>+</td>
<td>n-4</td>
<td>n-21</td>
</tr>
<tr>
<td>n-4</td>
<td>n-22</td>
<td>+</td>
<td>n-5</td>
<td>n-23</td>
</tr>
<tr>
<td>n-6</td>
<td>n-24</td>
<td>+</td>
<td>n-7</td>
<td>n-25</td>
</tr>
<tr>
<td>n-7</td>
<td>n-26</td>
<td>+</td>
<td>n-8</td>
<td>n-27</td>
</tr>
<tr>
<td>n-8</td>
<td>n-28</td>
<td>+</td>
<td>n-9</td>
<td>n-29</td>
</tr>
<tr>
<td>n-9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15 New Inputs with sign

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>sign</th>
<th>data</th>
<th>sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>new</td>
<td>n+15</td>
<td>-</td>
<td>n+14</td>
<td>+</td>
</tr>
<tr>
<td>n+14</td>
<td>n-1</td>
<td>-</td>
<td>n+13</td>
<td>-</td>
</tr>
<tr>
<td>n+13</td>
<td>n-2</td>
<td>+</td>
<td>n+12</td>
<td>+</td>
</tr>
<tr>
<td>n+12</td>
<td>n-3</td>
<td>-</td>
<td>n+11</td>
<td>+</td>
</tr>
<tr>
<td>n+11</td>
<td>n-4</td>
<td>+</td>
<td>n+10</td>
<td>+</td>
</tr>
<tr>
<td>n+10</td>
<td>n-5</td>
<td>-</td>
<td>n+9</td>
<td>+</td>
</tr>
<tr>
<td>n+9</td>
<td>n-6</td>
<td>+</td>
<td>n+8</td>
<td>+</td>
</tr>
<tr>
<td>n+8</td>
<td>n-7</td>
<td>-</td>
<td>n+7</td>
<td>+</td>
</tr>
<tr>
<td>n+7</td>
<td>n-8</td>
<td>+</td>
<td>n+6</td>
<td>+</td>
</tr>
<tr>
<td>n+6</td>
<td>n-9</td>
<td>-</td>
<td>n+5</td>
<td>+</td>
</tr>
<tr>
<td>n+5</td>
<td>n-10</td>
<td>+</td>
<td>n+4</td>
<td>+</td>
</tr>
<tr>
<td>n+4</td>
<td>n-11</td>
<td>-</td>
<td>n+3</td>
<td>+</td>
</tr>
<tr>
<td>n+3</td>
<td>n-12</td>
<td>+</td>
<td>n+2</td>
<td>+</td>
</tr>
<tr>
<td>n+2</td>
<td>n-13</td>
<td>-</td>
<td>n+1</td>
<td>-</td>
</tr>
<tr>
<td>n+1</td>
<td>n-14</td>
<td>+</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15 New Inputs with sign
Perform a Non-Maximally decimated Filter with an Even Number of Input Points

Do 10-to-1 Down Sampling Rather than 15-to-1 Down Sampling
The Polyphase Filter No Longer Sees the Sign Changes of The Input Samples
Because the Input Array has an Even Number of Samples

<table>
<thead>
<tr>
<th></th>
<th>new</th>
<th>sign</th>
<th>old</th>
<th>sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>+</td>
<td>n-15</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-1</td>
<td>-</td>
<td>n-16</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-2</td>
<td>+</td>
<td>n-17</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-3</td>
<td>-</td>
<td>n-18</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-4</td>
<td>+</td>
<td>n-19</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-5</td>
<td>-</td>
<td>n-20</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-6</td>
<td>+</td>
<td>n-21</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-7</td>
<td>-</td>
<td>n-22</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-8</td>
<td>+</td>
<td>n-23</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-9</td>
<td>-</td>
<td>n-24</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-10</td>
<td>+</td>
<td>n-25</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-11</td>
<td>-</td>
<td>n-26</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-12</td>
<td>+</td>
<td>n-27</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-13</td>
<td>-</td>
<td>n-28</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-14</td>
<td>+</td>
<td>n-29</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>new+10</th>
<th>sign</th>
<th>old+5</th>
<th>sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>n+10</td>
<td>+</td>
<td>n-5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n+9</td>
<td>-</td>
<td>n-6</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n+8</td>
<td>+</td>
<td>n-7</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n+7</td>
<td>-</td>
<td>n-8</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n+6</td>
<td>+</td>
<td>n-9</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n+5</td>
<td>-</td>
<td>n-10</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n+4</td>
<td>+</td>
<td>n-11</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n+3</td>
<td>-</td>
<td>n-12</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n+2</td>
<td>+</td>
<td>n-13</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n+1</td>
<td>-</td>
<td>n-14</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>+</td>
<td>n-15</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-1</td>
<td>-</td>
<td>n-16</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-2</td>
<td>+</td>
<td>n-17</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>n-3</td>
<td>-</td>
<td>n-18</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>n-4</td>
<td>+</td>
<td>n-19</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Rotators of the Non-Maximally Decimated Filter with an Even Number of Input Points can be Embedded in the Filter Weights
Channelizer Input Spectrum and Channelizer Channel Responses with 10 MHz Channel Center Spacing

Frequency (MHz), Channel BW = 10 MHz, Channel Center Spacing 10 MHz, No Channel Offset
DC at Index 0 of 18-Point DFT

DC at Index 4.5 of 18-Point DFT
First Edition, Used Copies, Asking Prices on Amazon

Used - Good
$759.95
$3.99 delivery: March 2 - 17

Used - Acceptable
$1,064.33
$3.99 delivery: March 24 - April 5

Used - Good
$1,076.20
FREE delivery: March 3 - 11
Fastest delivery: March 2 - 9

Used - Acceptable
$1,324.00
$3.99 delivery: March 24 - April 5
Professor harris: may I be excused?
My brain is full!
Two Quick Examples of Applied Magic with Polyphase Filter Banks
Satellite Broadcasts
384 MP3 Channels to Earth Stations
Demodulate all MP3 Channels
Remodulate as FM Channels
Task: Replace Legacy Transceiver

What size room is required to house new DSP based Transceiver?
Equipment Bay: 192-Stereo FM Modulators
Conversation with Client!

• How big a room will we need to house the DSP version of this Transceiver?
• My Answer: I think it will fit on one chip.
• Response: Don’t be Absurd: Can’t Pack a Room on a Single Chip!

• Results: 48-Analog Devices Blackfin Processors to Demodulate 192 MP3 Stereo Channels.
• 1 Virtex V-4 for 192 Digital Stereo FM Modulators and 256 Channel Channelizer @ 293 kHz Bandwidth per Channel. (60% of Chip)
A Smaller Package

2-U High, Full Rack Width

H  3.5 in, 8.89 cm
W  17.0 in, 43.18 cm
D  9.4 in, 23.88 cm
How to Pack a Room of Analog FM Modulators into a Xilinx FPGA

You are likely familiar with the way that digital television is transmitted from satellites as multi-channel MPEG (Moving Picture Experts Group) compressed video to a cable head end where the multiple channels are demodulated. The MPEG streams are decoded and then remodulated as channelized analog NTSC (National Television Standards Committee) or PAL (Phase Alternating Line) television signals for insertion in a cable distribution plant.

Similarly, high-quality stereo audio is transmitted from a satellite or multi-channel MPEG (Moving Picture Experts Group) compressed audio to a cable head end where the multiple channels are demodulated. The MPEG streams are decoded and then remodulated as channelized analog FM signals for insertion in a cable distribution plant.

Figure 1 - Equipment bay containing legacy transponder equipment.

DSP techniques replace a legacy multi-channel analog modulator.
Signal Specifications; Complex Input Samples, Independent Upper and Lower 95 MHZ Channels

Channel-A

Channel-B

Filter Specifications

80.0 dB Out-of-Band Attenuation

0.1 dB In-Band Ripple

1401 Tap Filter

DOCSIS 3.0
Six Channels, 96 MHz wide Channels, 94 MHz Occupied, 16*102.4 MHz = 1.6384 GHz sample rate
Occupied band 108 to 684 MHz: stopband -80 dB, passband 0.05 dB
Complex Input Signal
Require 2-Filters for Upper Band
And 2-Filters for Lower Band
2800 ops/Input
1401 Tap Direct Filter Response, (5600 Mults for Complex Input)

Channel Response

60-Path Ripple 10-Path Ripple

Outer Tier Filter 30

Inner Tier Filter

Outer Tier Channel (Dashed Black), Inner Tier Channel (Black)

60-Path Outer 10-Path Inner
Dilbert, is it true that DSP makes the world go around but multirate signal processing supplies the music for the ride?
Can There be any Doubt???
SOFTWARE DEFINED RADIO MAN

Is Open For Questions