IEEE Presentation
March 18, 2021

The ABCs of OFDM

Part 1

by Dr. Bernard Sklar

bsklar@ieee.org

IEEE Presentation

March 18, 2021

Or

How to Battle the Awful Effects of Multipath

DISTORTION

OFDM's main function is to manipulate orthogonal sinusoids.

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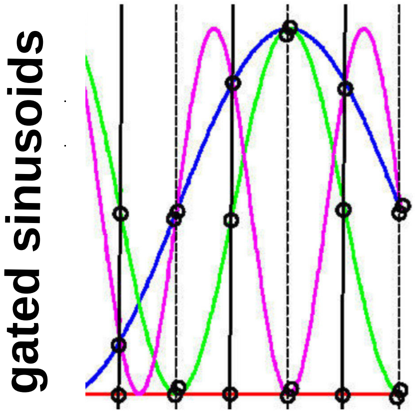
Abstract: The main benefit of OFDM is its ability to cope with Severe multipath channel conditions without needing Complex Equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a High-data-rate signal into Smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:



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Why is this useful?

Because sinusoids are amazing.

The steady-state response of a multipath channel yields NO DISTORTION to a fixed frequency sinusoid.

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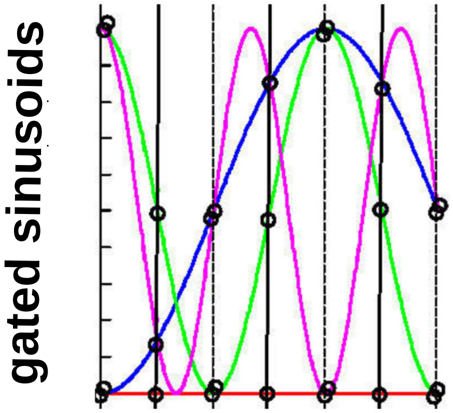

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Saying this more precisely

A steady-state sinusoid plus its channel-induced echoes into any LTI system yields an undistorted sinusoid at the output.

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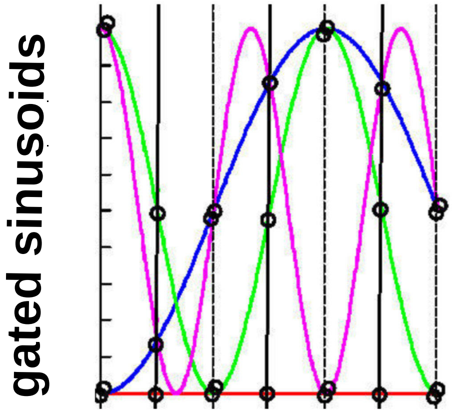

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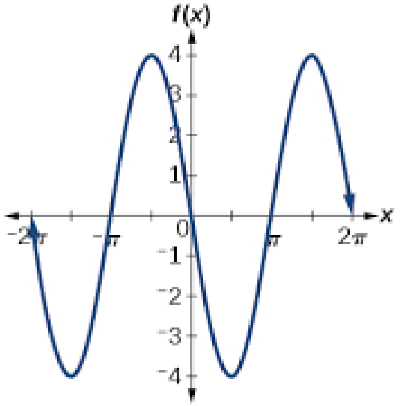
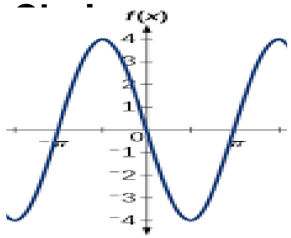


channel induced
+ Echo + Echo + Echo

Linear Time Invariant System

=



Same frequency. Change in amplitude and/or phase.

No other waveform has this property.

Do you see why this is true?
Because any other waveform is comprised of two or more sinusoids at different frequencies, each changed uniquely. Therefore the shape of the composite will typically change (suffer distortion).

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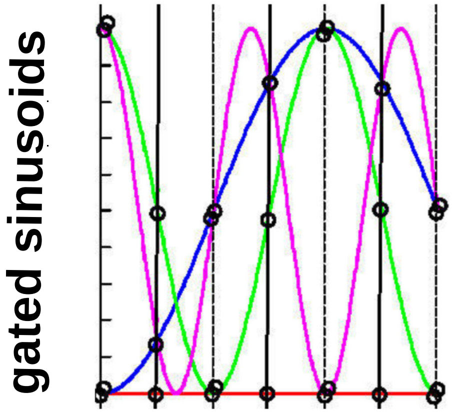

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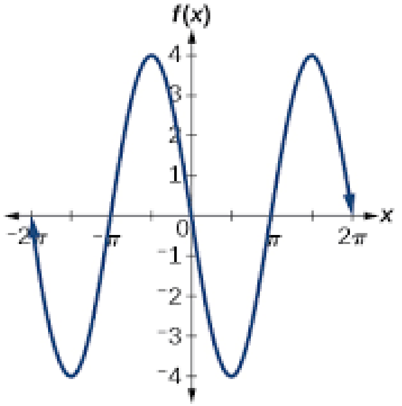

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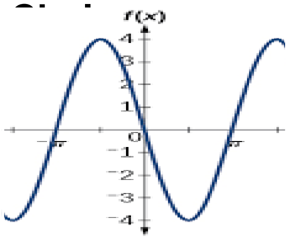

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A remarkable taken-for-granted property

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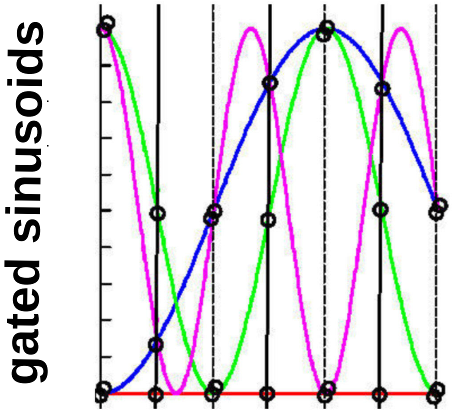

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OFDM's main function is to manipulate orthogonal sinusoids.

Why is this useful?

Because sinusoids are amazing.

The steady-state response of a multipath channel yields NO DISTORTION to a fixed frequency sinusoid.

A steady-state sinusoid plus its echoes is an undistorted sinusoid.

That's why we love OFDM.

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WARNING

Dr. Sklar's presentation can be hazardous to your well being. You are about to experience his quirks.

(1) He wastes no paper. His slides are busy and cluttered.

(2) He often repeats himself. But, he only does so for really important stuff.

(3) He talks loud and fast. Well ... he is from the Bronx.

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Understanding the Malady

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Almost 60 years ago

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With these tools

OFDM can accomplish what seems impossible.

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elegantly

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Really?
How complex can they be?

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How Difficult is it to Equalize a Rich-Multipath Channel?

In today's world of computational power, it just means that we need to invert a large dimensional matrix. We can do that. Right?

Part of the equalization problem is that we don't have the matrix to invert. We try to form the matrix or its inverse from the raw noisy, corrupted received signal.

This takes time and undesired-overhead. In a wide bandwidth channel, the time to learn the channel and to form the equalizer weights, and apply the equalizer process may be longer than the message we are trying to move through the channel.

As bandwidth increases, the task becomes much more difficult.

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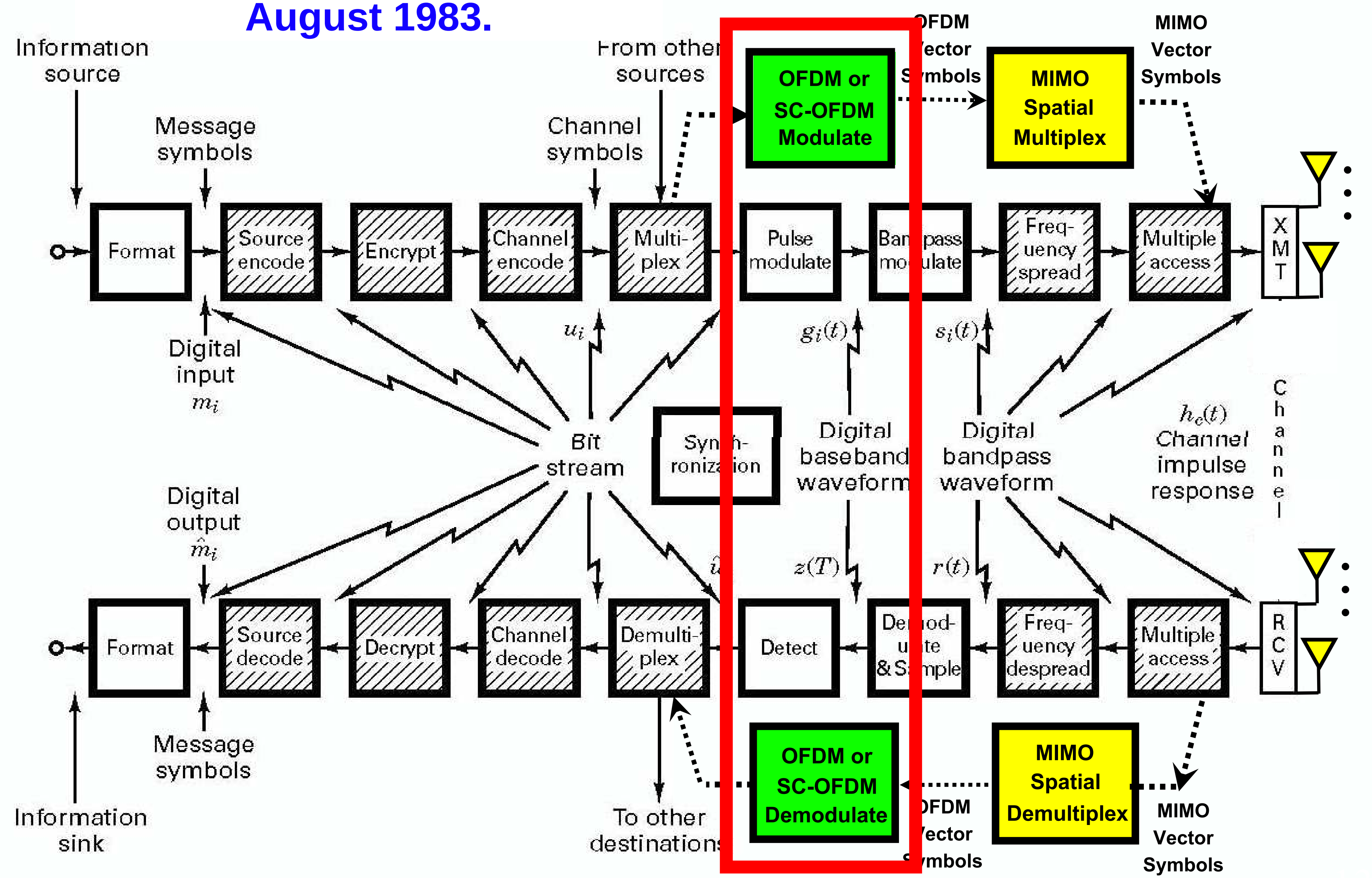
Divide & Conquer

in other words

Partition

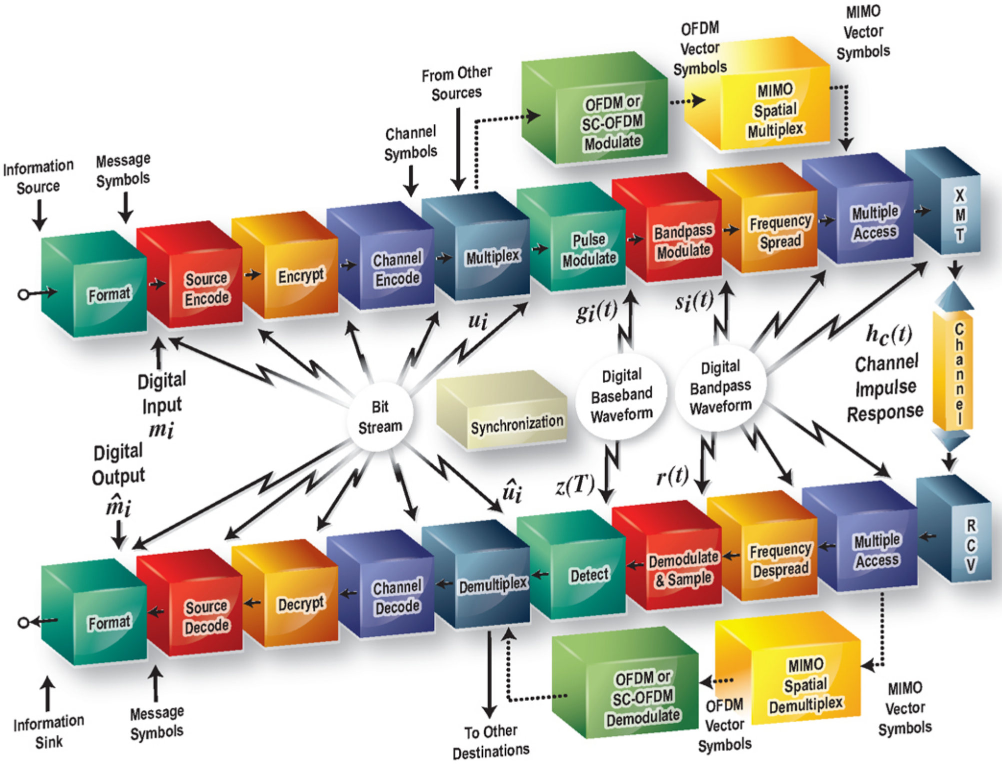
Sklar signature slide:
First version appeared in IEEE Communications Magazine, August 1983.



THIRD EDITION

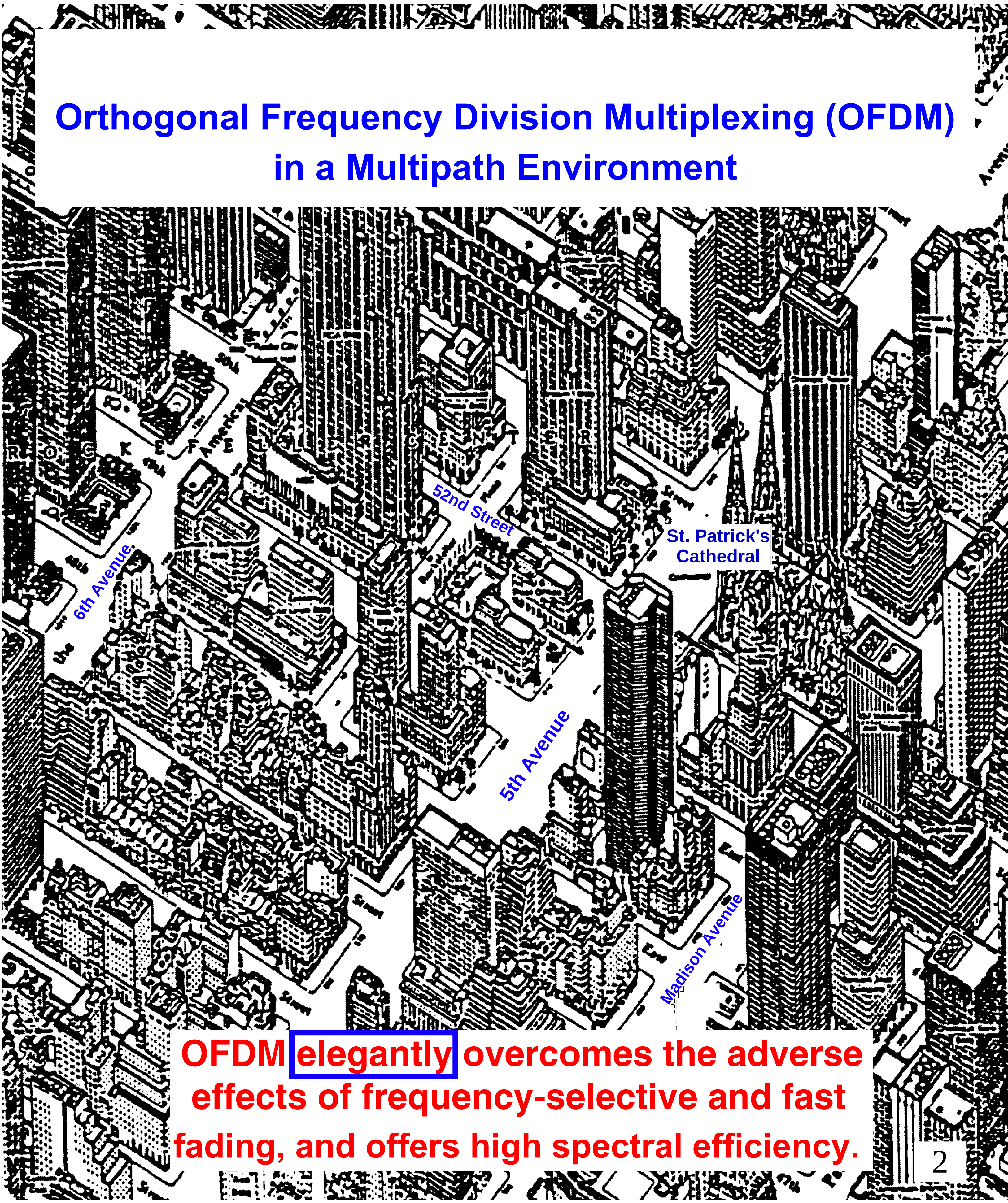
Digital Communications

Fundamentals and Applications



Bernard Sklar *and* **fred harris**

Orthogonal Frequency Division Multiplexing (OFDM) in a Multipath Environment



OFDM elegantly overcomes the adverse effects of frequency-selective and fast fading, and offers high spectral efficiency.

Qualcomm Completes Acquisition of Flarion Technologies

JAN 19, 2006

SAN DIEGO

Qualcomm Incorporated (Nasdaq: QCOM), a leading developer and innovator of Code Division Multiple Access (CDMA) and other advanced wireless technologies, today announced that it has completed the acquisition of Flarion Technologies, a pioneer and leading developer of Orthogonal Frequency Division Multiplex Access (OFDMA) technology and the inventor of FLASH-OFDM® technology for mobile broadband Internet protocol (IP) services. The acquisition expands Qualcomm's already extensive portfolio of OFDMA intellectual property and enhances the Company's engineering team with expertise in OFDMA technology and products.

Historical Background 1957

Ref: Prof. Carlo Regazzoni, Corso di Comunicazioni Mobili

Multi-carrier modulation techniques are considered to be **fourth generation (4G)** communication systems used for fixed and mobile digital transmission.

But the idea of multi-carrier modulation dates back to the **end of the fifties** (Doelz, Heald, Martin, Proceedings of IRE, May 1957, pp. 656-661).

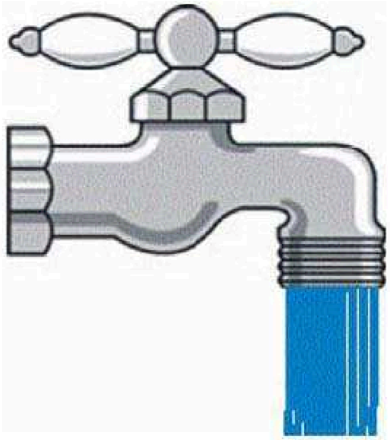
This work showed a **practical implementation** of a digital transmission system, called **KINEPLEX,** with **bit multiplexing on orthogonal carriers,** which is the basic principle of OFDM.

KINEPLEX demodulator

KINEPLEX S/P converter

Note that Kineplex was a method for sharing the channel BW. It was NOT a process to invert the effects of channel distortion.

FDM: One Wide Band Pipe
OFDM: Many Parallel Narrow Band Pipes



Partitioning

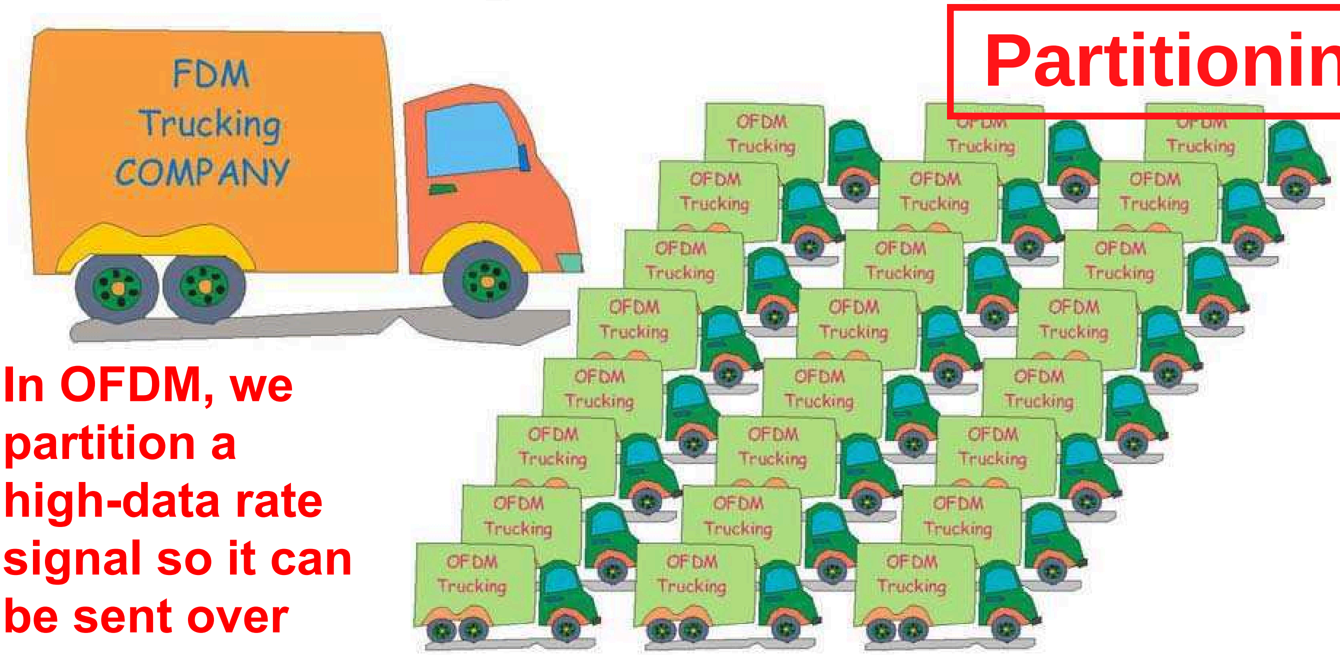
Wideband

Narrow band

REF: Charan Langton, www.complextoreal.com

Charan Langton www.complextoreal.com

FDM: One Big Delivery Vehicle
OFDM: Many Small Delivery Vehicles



Partitioning

In OFDM, we partition a high-data rate signal so it can be sent over many low-rate subchannels.

Charan Langton, www.complextoreal.com

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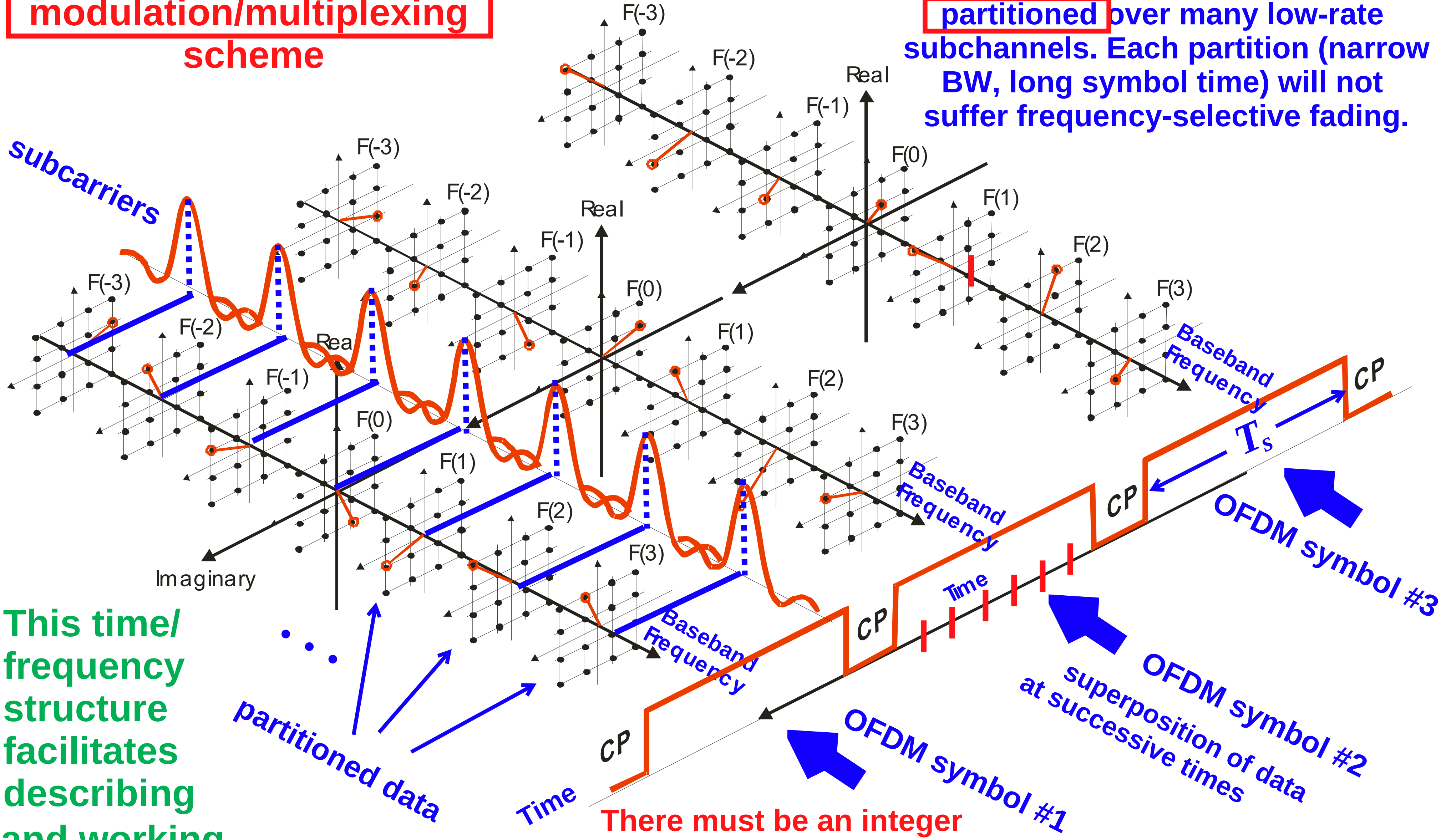
OFDM
Time/Frequency Relationship

The Big Picture

2. **OFDM is a multi-carrier modulation/multiplexing scheme**

3. **Big Picture:**
High data-rate messages are partitioned over many low-rate subchannels. Each partition (narrow BW, long symbol time) will not suffer frequency-selective fading.



1. **This time/ frequency structure facilitates describing and working with OFDM.**

There must be an integer number of cycles of each subcarrier (sinusoid) contained in the interval T_S

2. **OFDM is a multi-carrier modulation/multiplexing scheme**

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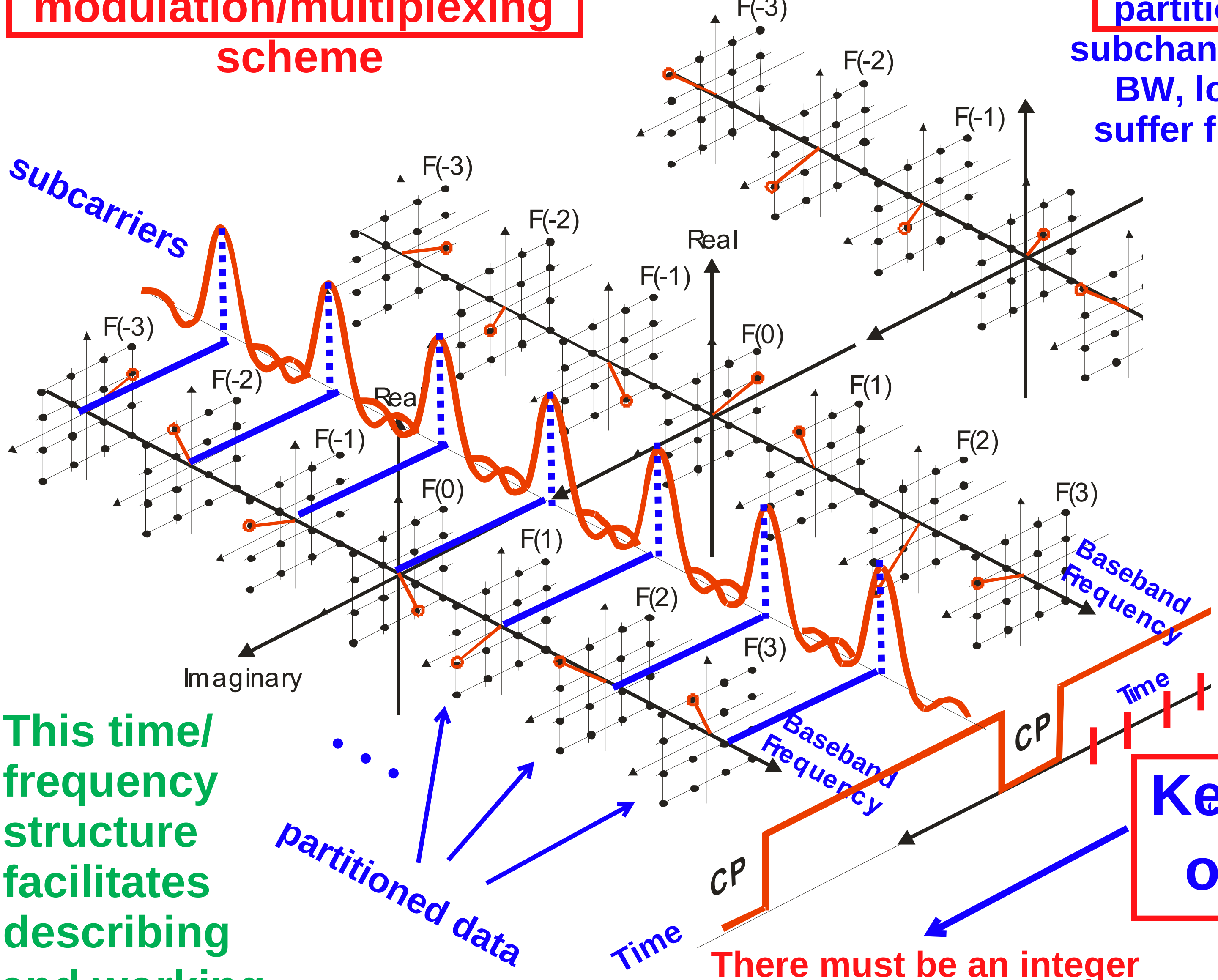

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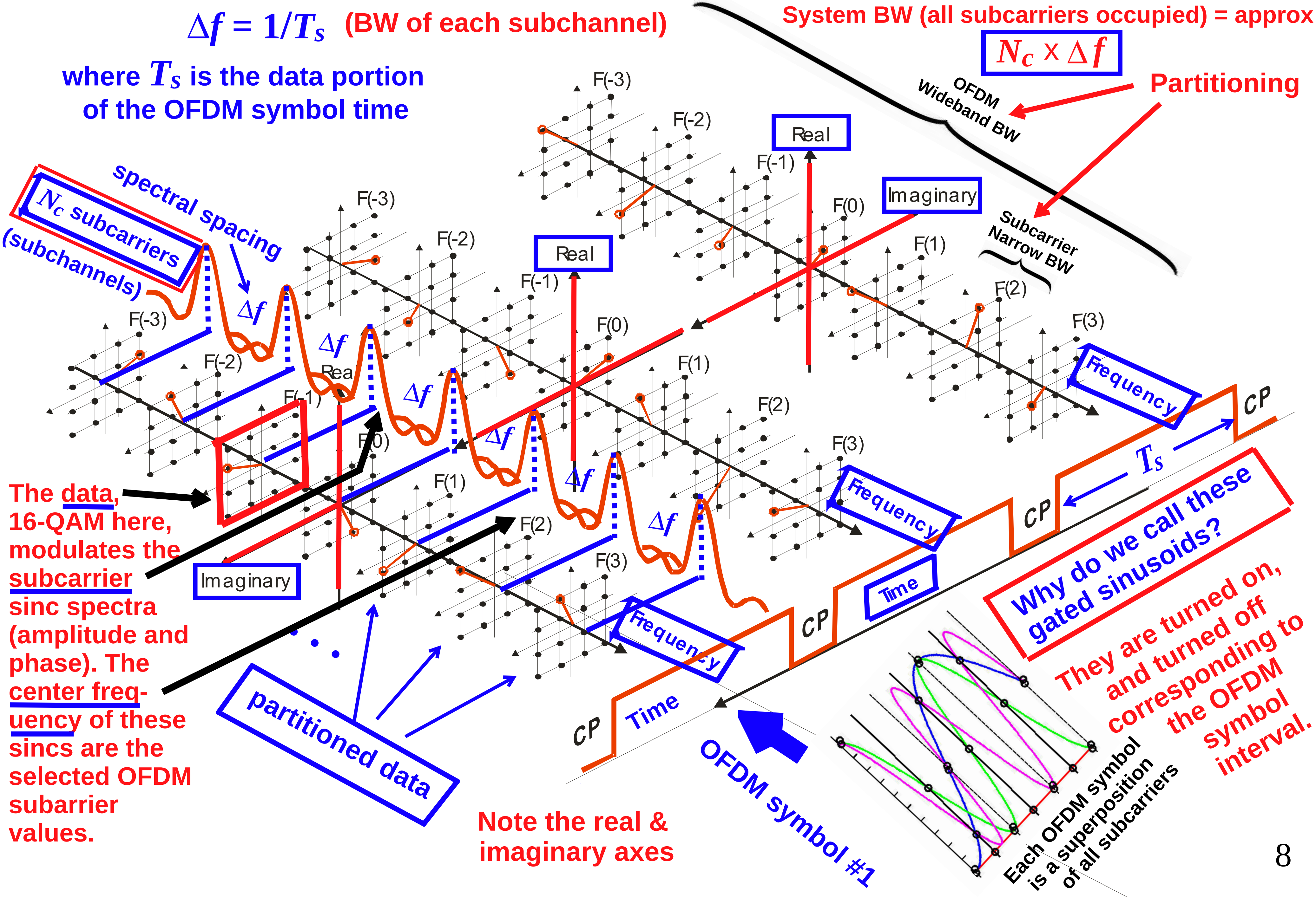
Key Characteristic of Orthogonality

$\Delta f = 1/T_S$ **(BW of each subchannel)**

where T_S is the data portion of the OFDM symbol time

System BW (all subcarriers occupied) = approx $\boxed{N_c \times \Delta f}$

Partitioning



spectral spacing

N_c subcarriers (subchannels)

The data, 16-QAM here, modulates the subcarrier sinc spectra (amplitude and phase). The center frequency of these sincs are the selected OFDM subarrier values.

partitioned data

Note the real & imaginary axes

OFDM symbol #1

Why do we call these gated sinusoids?

They are turned on, and turned off corresponding to the OFDM symbol interval.

Each OFDM symbol is a superposition of all subcarriers

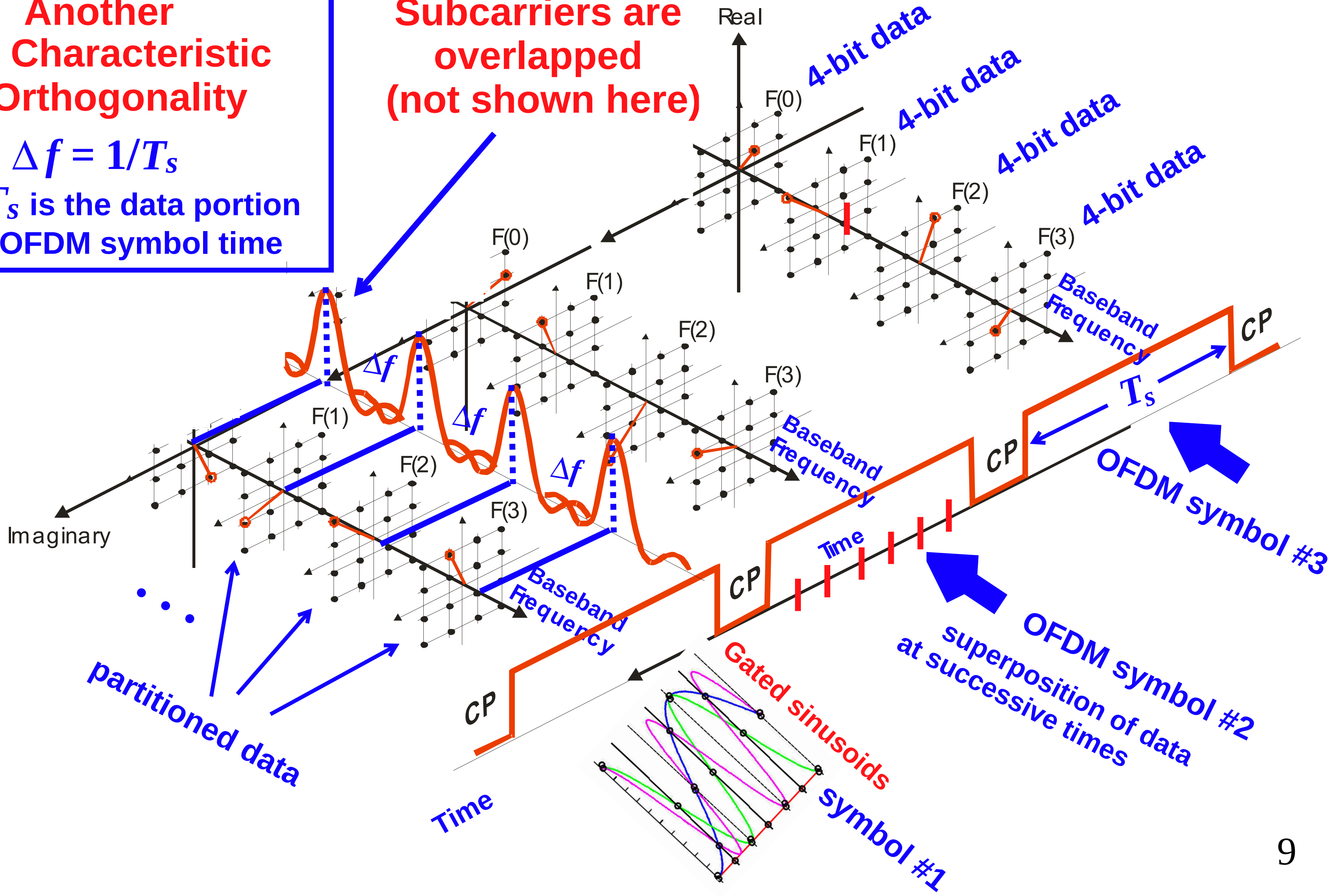
An OFDM system, with $\boxed{N_c = 4}$ subcarriers and 16-QAM modulation

Another Key Characteristic of Orthogonality

$$\Delta f = 1/T_S$$

where T_S is the data portion of the OFDM symbol time

Subcarriers are overlapped (not shown here)



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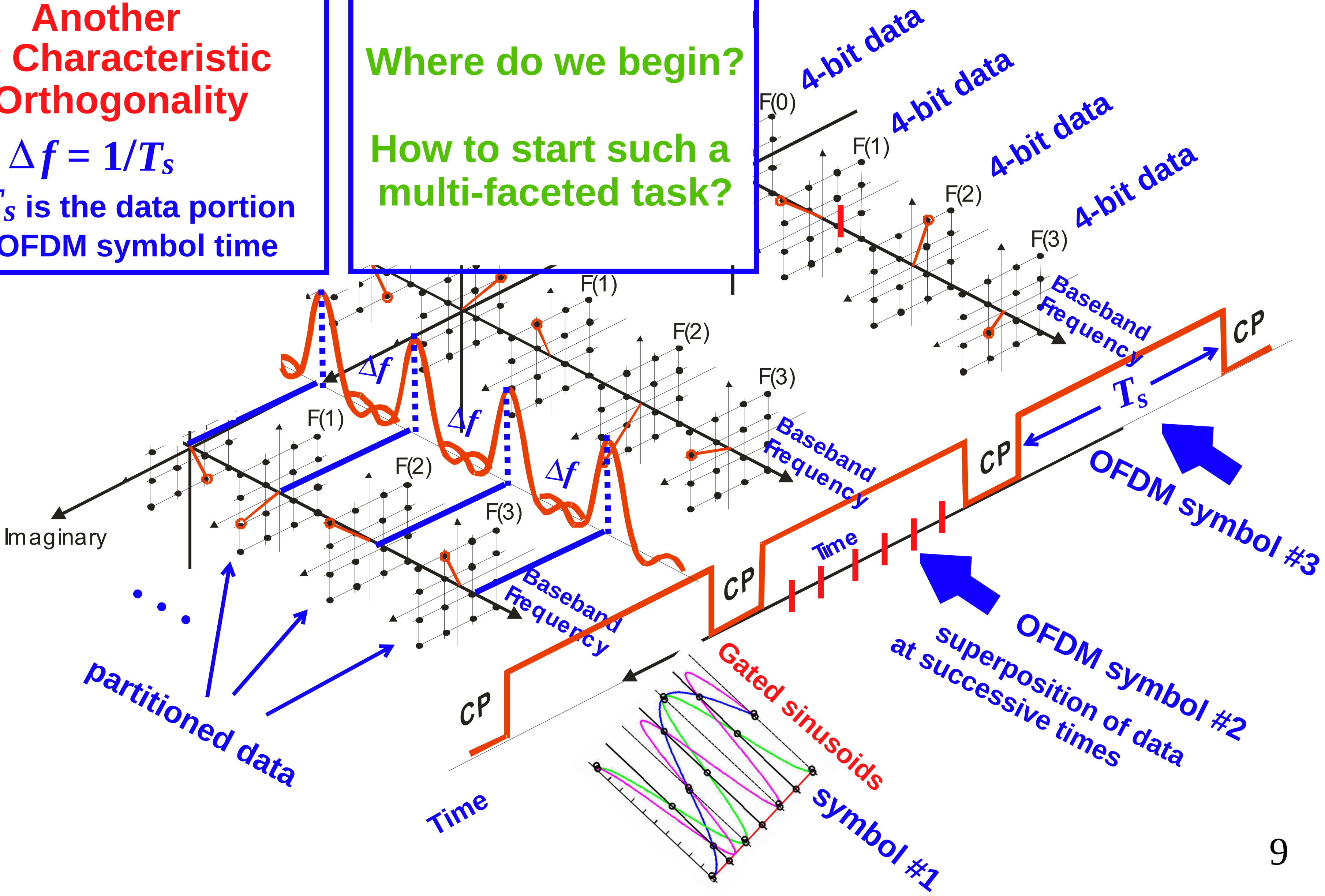
Where do we begin?

How to start such a multi-faceted task?



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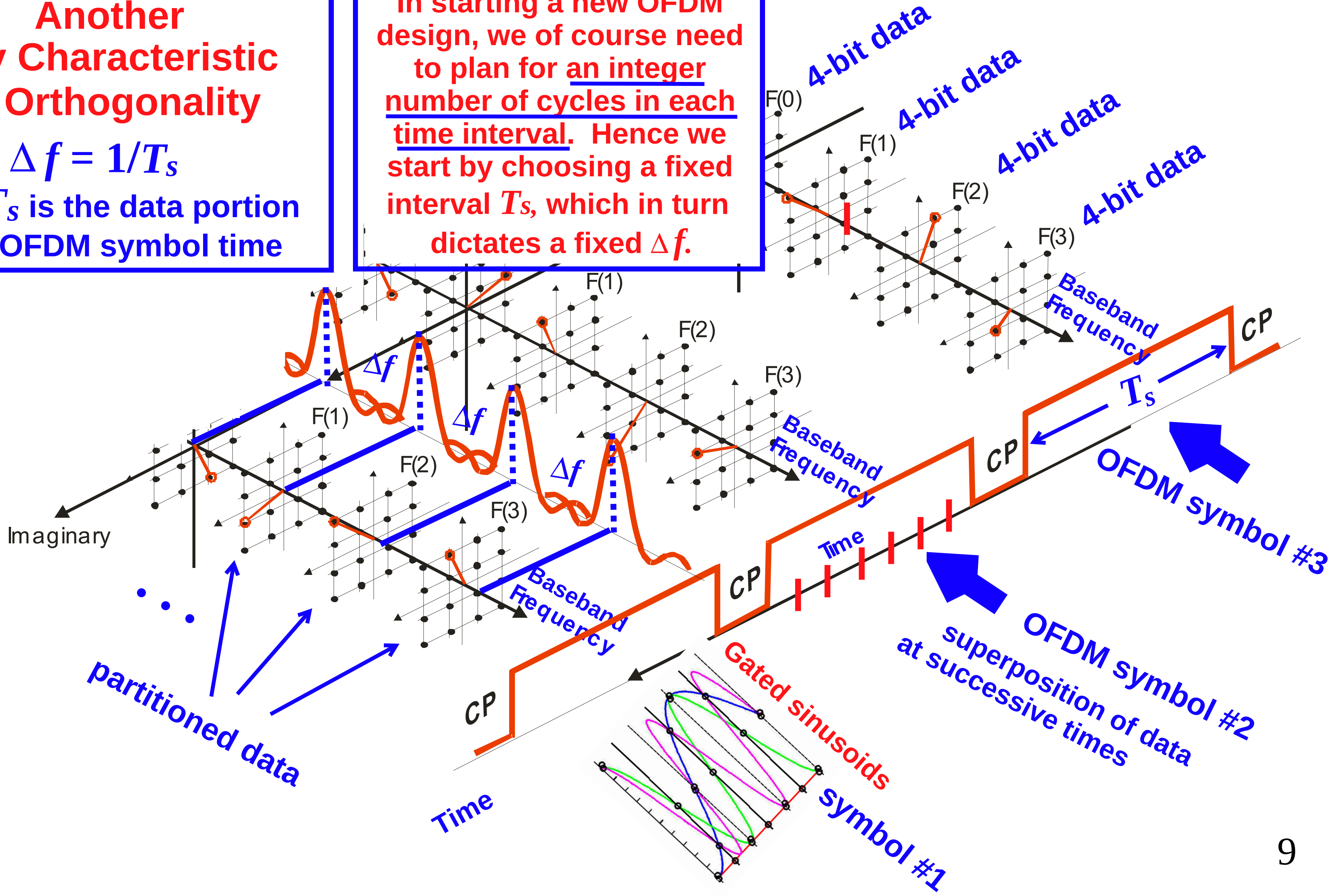
Another
Key Characteristic
of Orthogonality

$$\Delta f = 1/T_S$$

where T_S is the data portion of the OFDM symbol time

In starting a new OFDM design, we of course need to plan for an integer number of cycles in each time interval. Hence we start by choosing a fixed interval T_S, which in turn dictates a fixed Δf.



An OFDM system, with $\boxed{N_c = 4}$ subcarriers and 16-QAM modulation

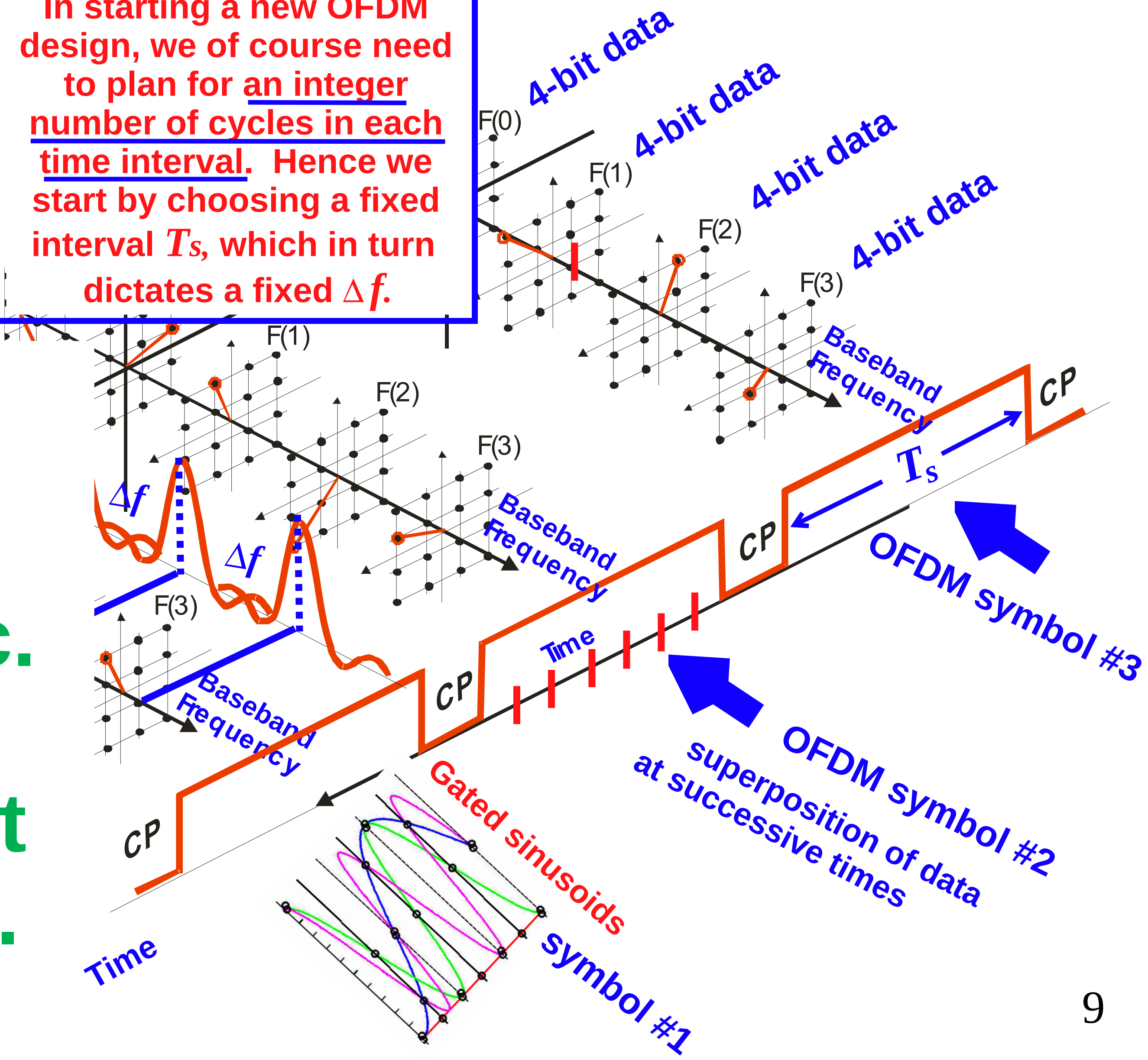
Another Key Characteristic of Orthogonality

$$\Delta f = 1/T_S$$

where T_S is the data portion of the OFDM symbol time

In starting a new OFDM design, we of course need to plan for an integer number of cycles in each time interval. Hence we start by choosing a fixed interval T_S, which in turn dictates a fixed Δf.

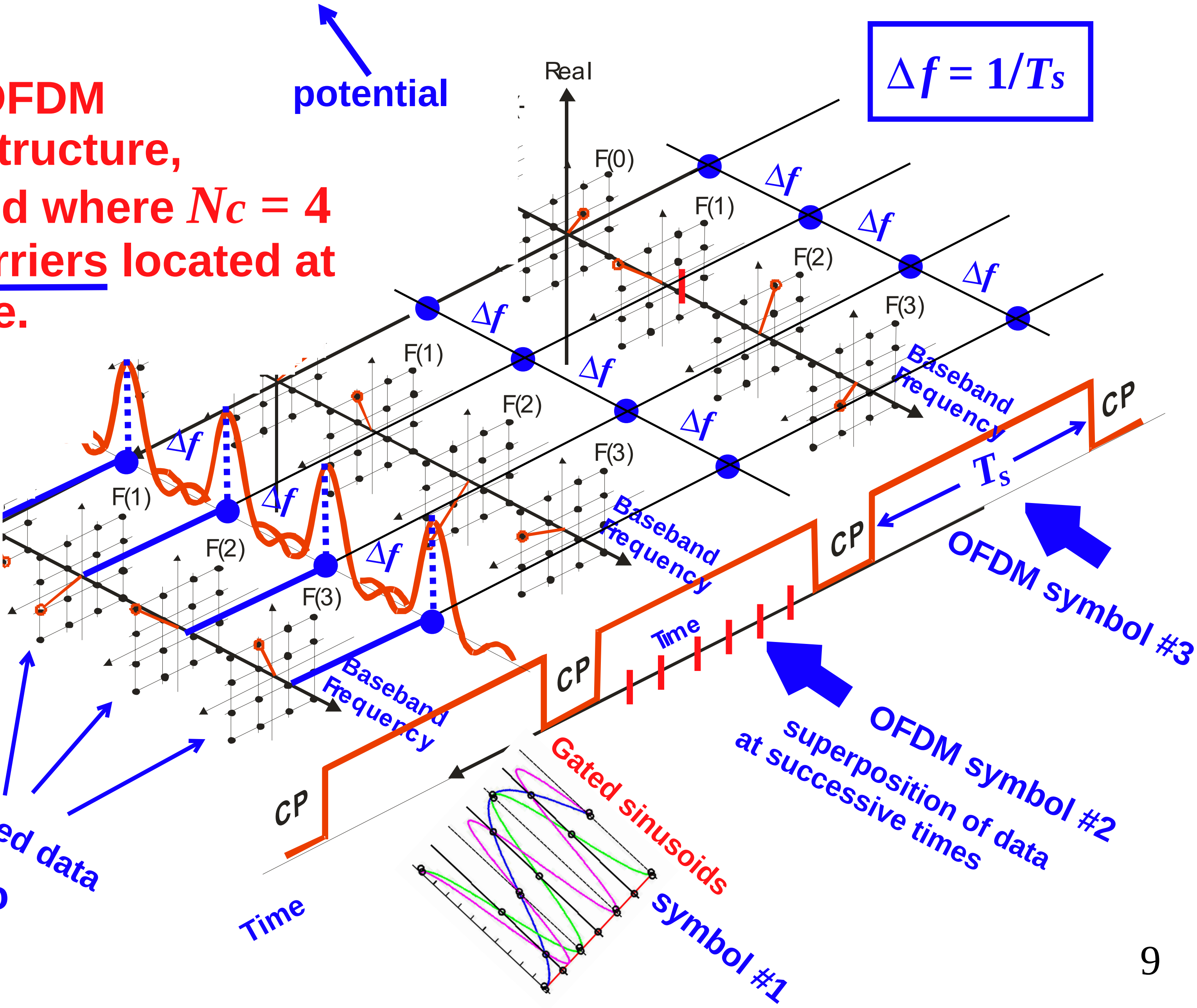
Note what is meant by Characteristic. It means necessary, but not sufficient.



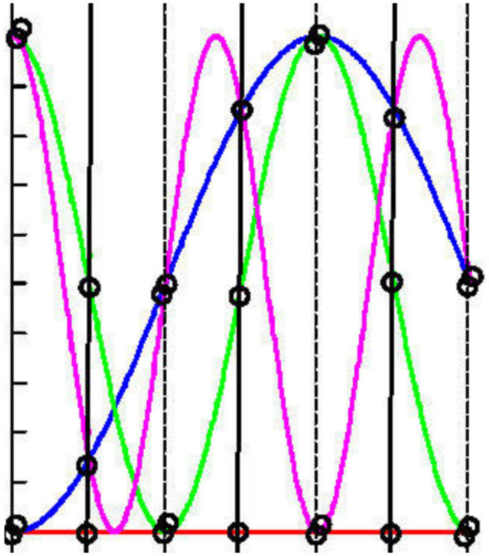
An OFDM system, with $\boxed{N_c = 4}$ subcarriers and 16-QAM modulation

Example of the OFDM time/frequency structure, focusing on a grid where $N_c = 4$ candidate subcarriers located at each symbol time.

The subcarrier's amplitude can be zero, as seen in 8 unoccupied grid points here. They have the potential to be assigned.



Once T_s is chosen, so too is Δf chosen.

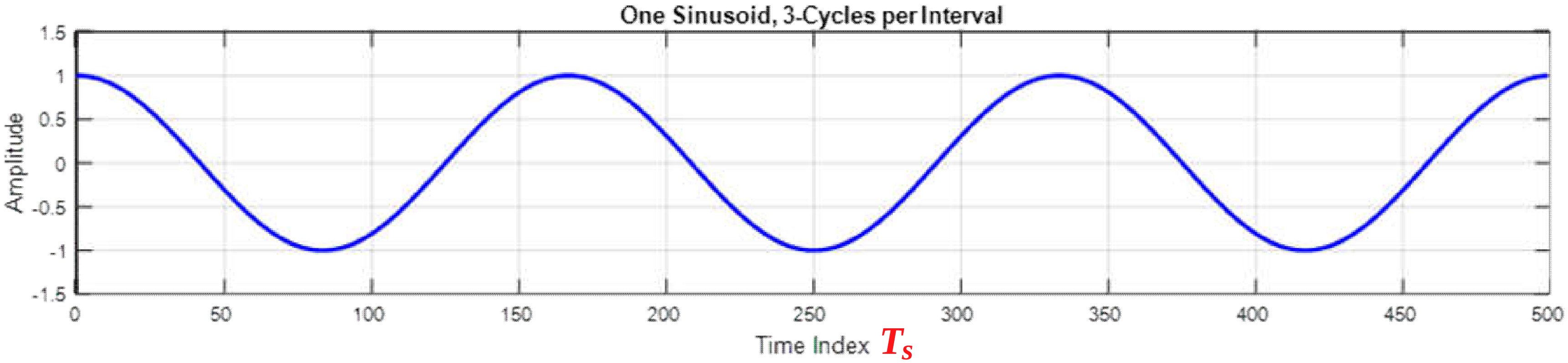


Examining the Spectra of Gated Sinusoids having an integer number of cycles per interval

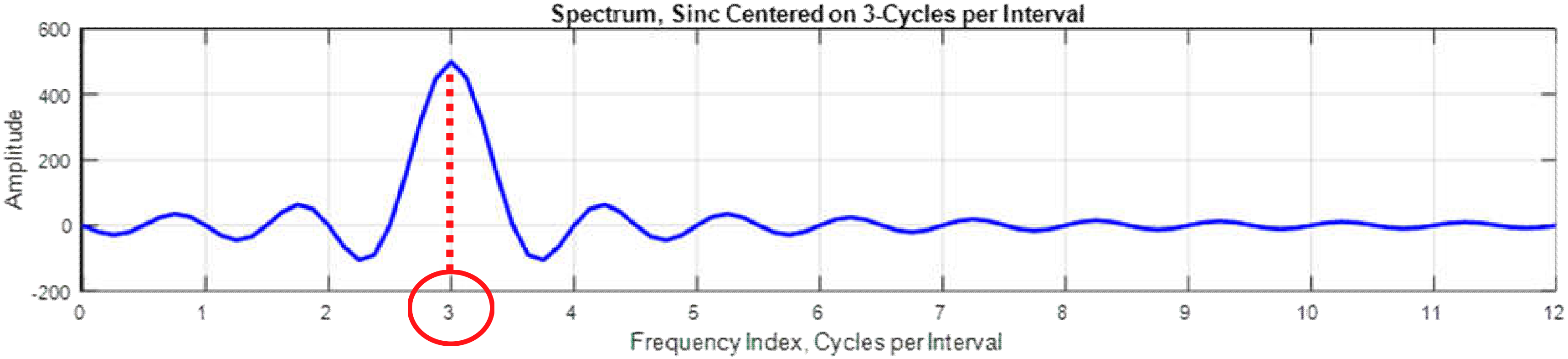
Characteristic of orthogonal sinusoids: An integer number of cycles per interval

One OFDM sinusoidal subcarrier (3 Cycles per interval)

One sinusoid


One Sinusoid, 3-Cycles per Interval

Sinc function spectrum

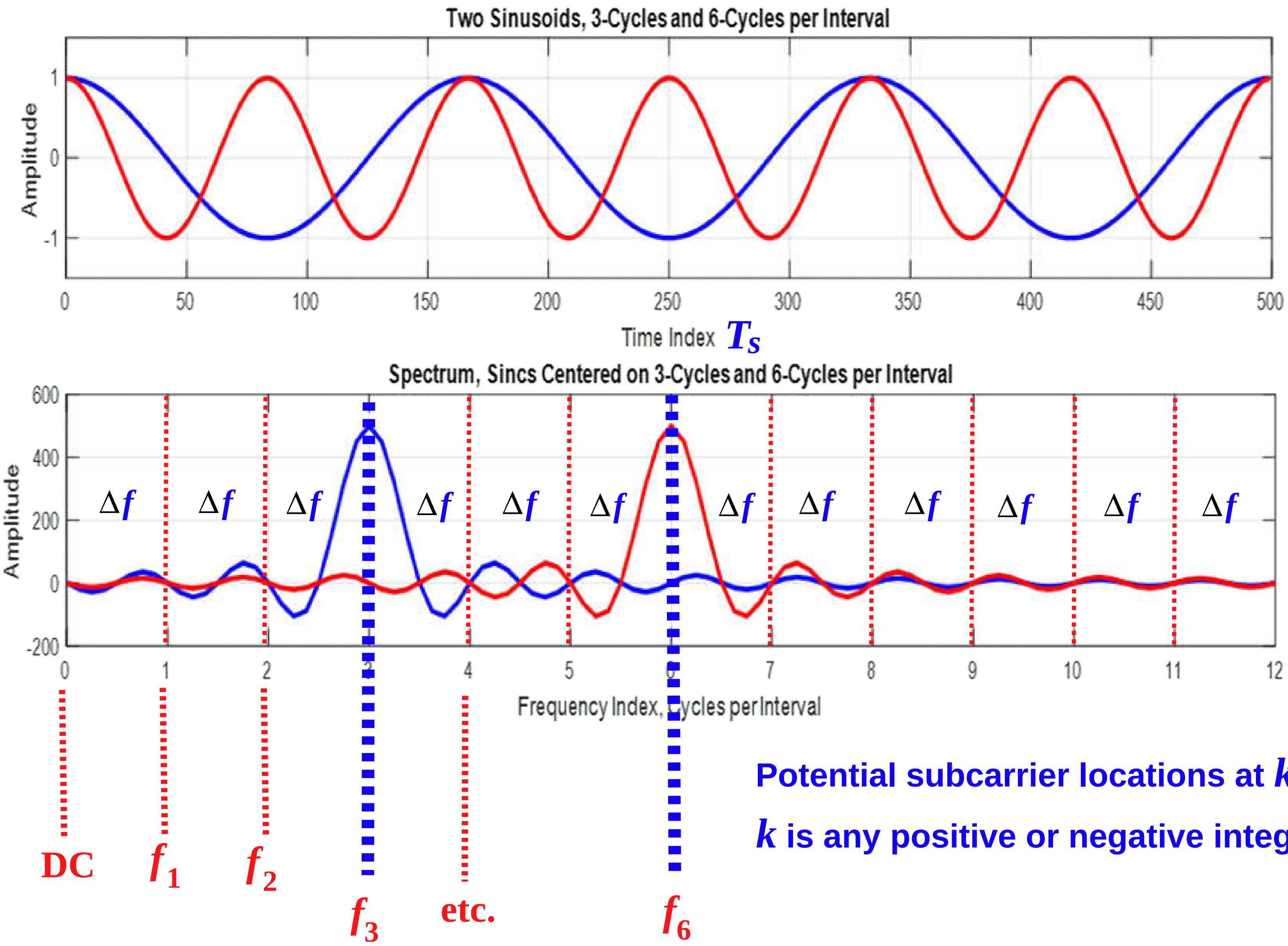

Spectrum, Sinc Centered on 3-Cycles per Interval

3 cycles per T_s

Two OFDM sinusoidal subcarriers (3 & 6 Cycles per interval)

Two sinusoids

Sinc function spectra

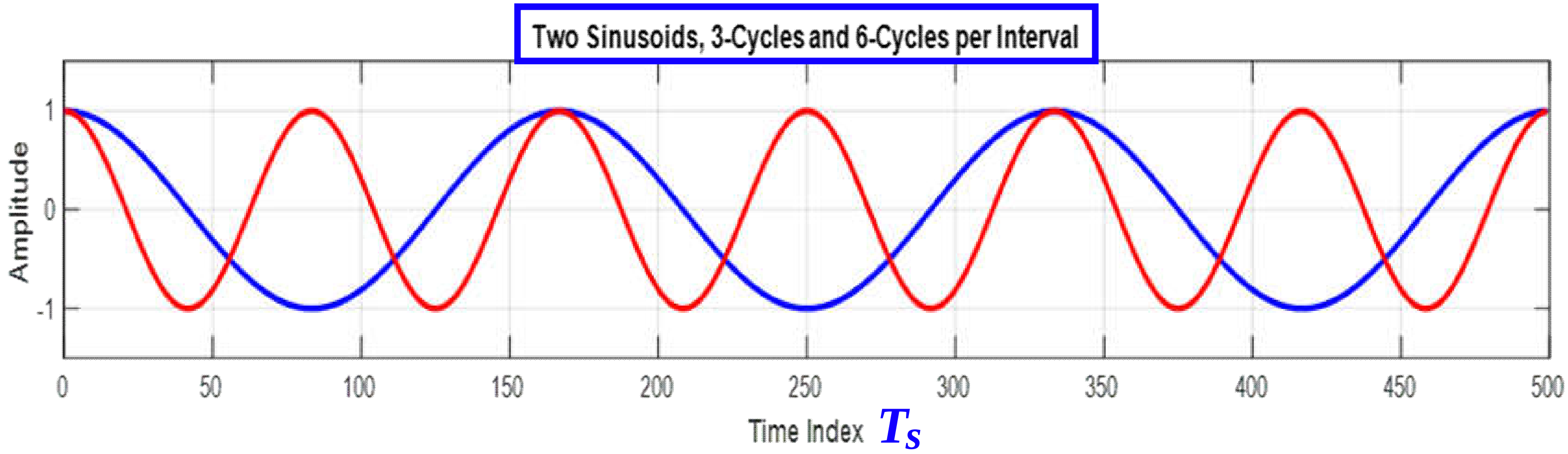


Potential subcarrier locations at $k\,\Delta f$

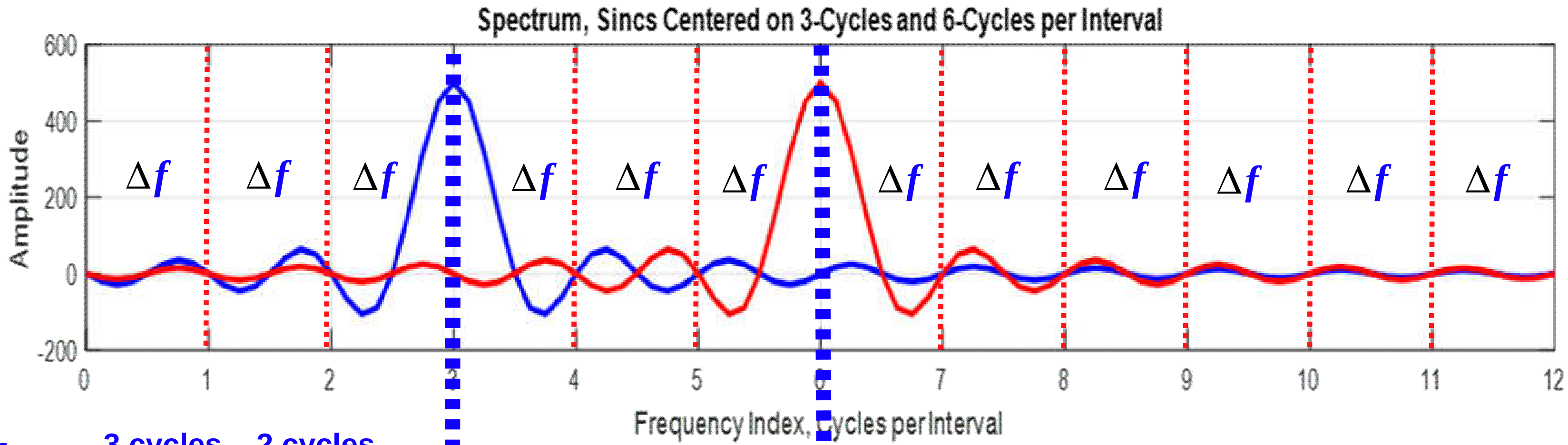
k is any positive or negative integer.

Two OFDM sinusoidal subcarriers (3 & 6 Cycles per interval)

Two sinusoids


Two Sinusoids, 3-Cycles and 6-Cycles per Interval

Sinc function spectra


Spectrum, Sincs Centered on 3-Cycles and 6-Cycles per Interval

$$\Delta f = f_3 - f_2 = \frac{3\text{ cycles} - 2\text{ cycles}}{500}$$

$$= 1/500 = 1/T_s$$

check

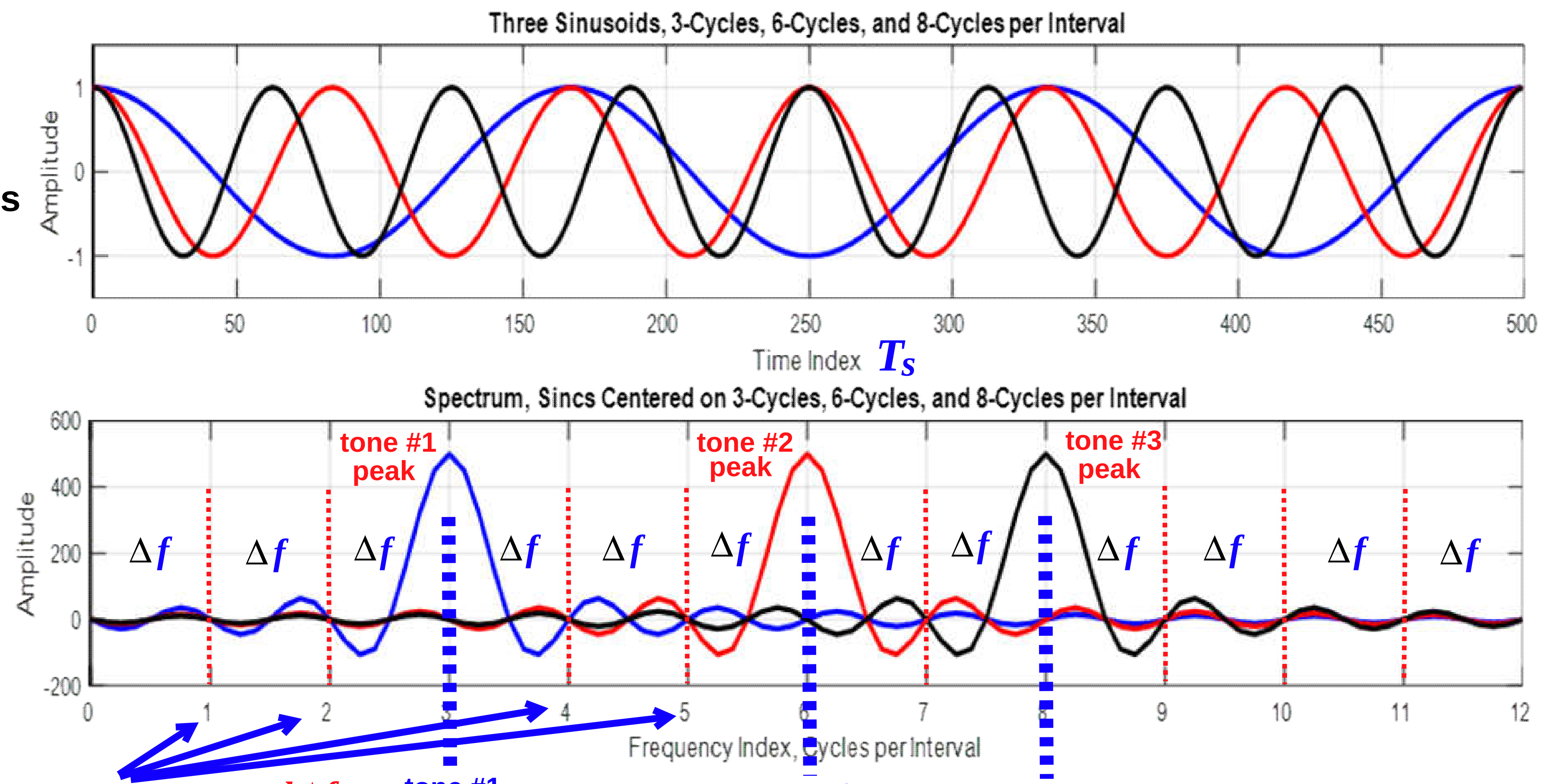
Potential subcarrier locations at $k\,\Delta f$

k is any positive or negative integer.

Three OFDM sinusoidal subcarriers (3, 6, & 8 Cycles per interval)

Three sinusoids

Sinc function spectra



tone #1 center freq

tone #2 center freq

tone #3 center freq

Potential subcarrier locations at $k\Delta f$.
k is any positive or negative integer.

When the gated sinusoids have an integer number of cycles per pulse-time interval, the main-lobe peak of each spectral sinc function is aligned with the nulls of neighboring spectral sidelobes. Note that the difference-frequency Δf between potential subcarriers is fixed. Such subcarriers have random amplitudes (including zero). In other words, even though Δf and T_S are fixed, the selected subcarriers may be arbitrarily chosen according to a plan such as the one above.

An OFDM system, with $\boxed{N_c = 4}$ subcarriers and 16-QAM modulation

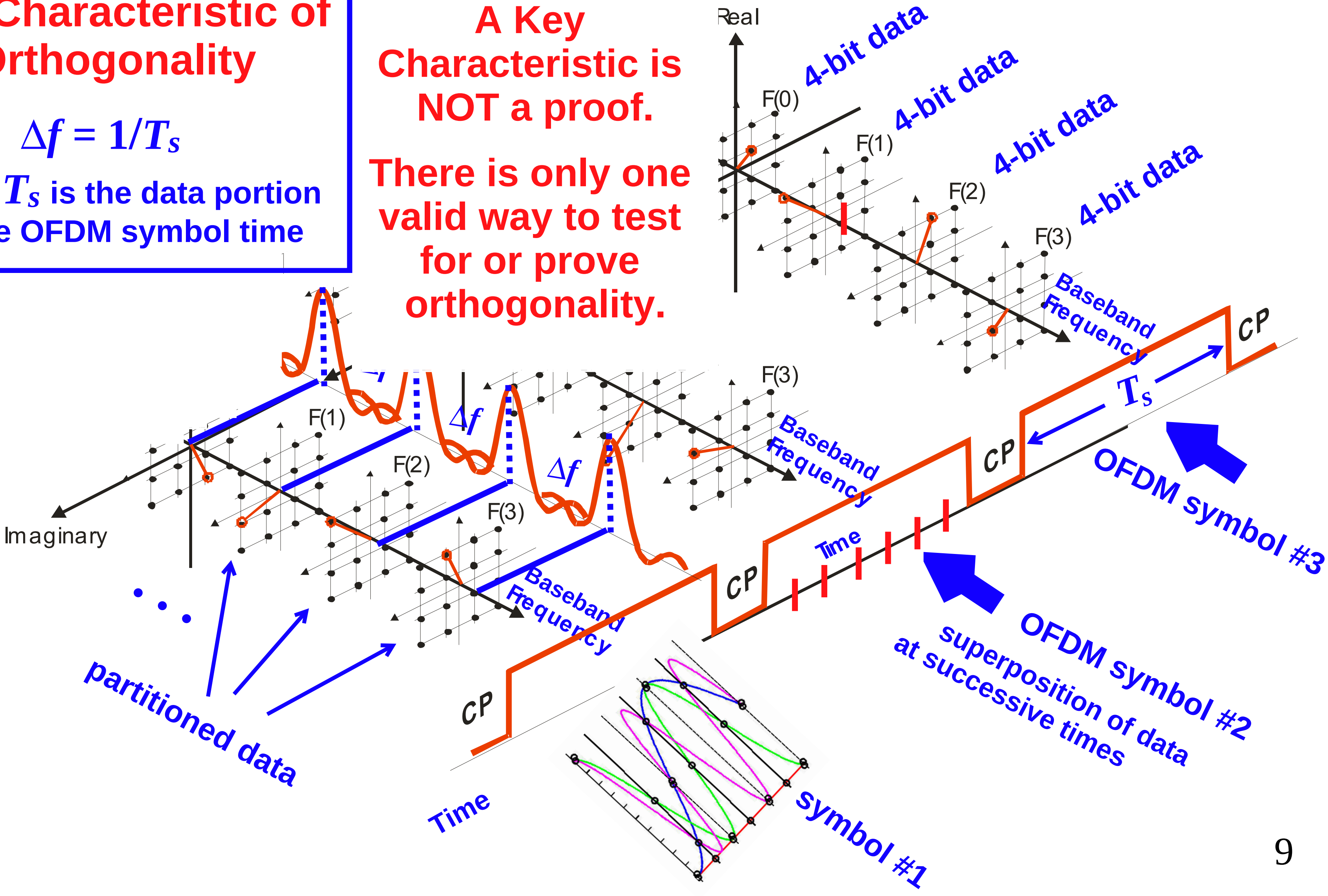
Key Characteristic of Orthogonality

$$\Delta f = 1/T_s$$

where T_s is the data portion of the OFDM symbol time

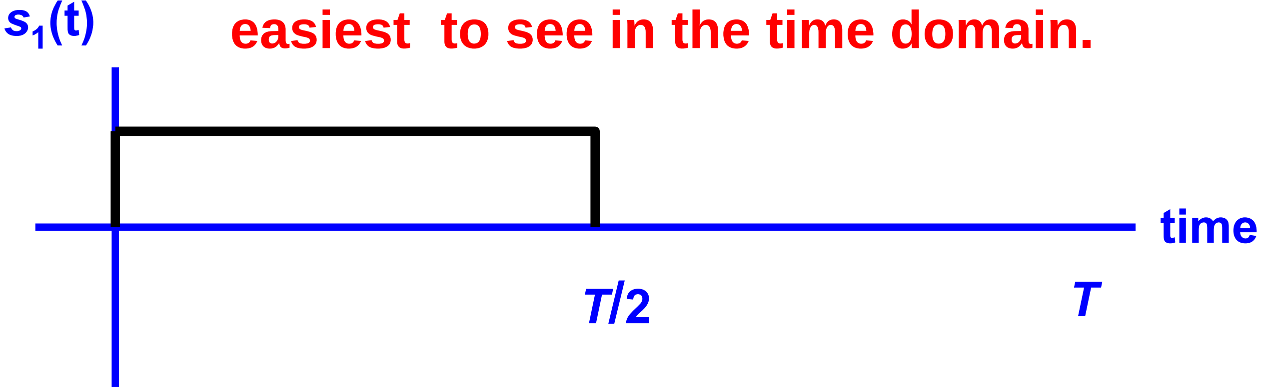
A Key Characteristic is NOT a proof.

There is only one valid way to test for or prove orthogonality.

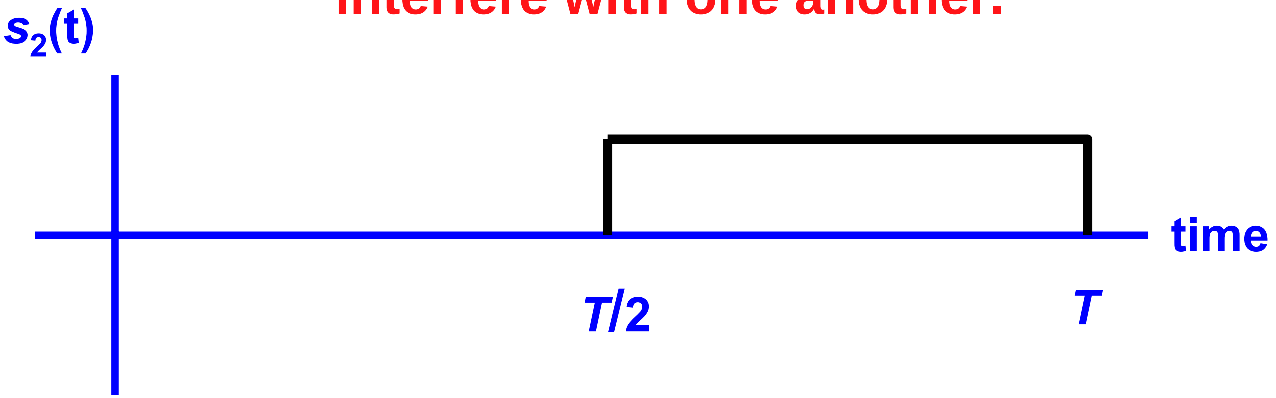


Test for Orthogonality

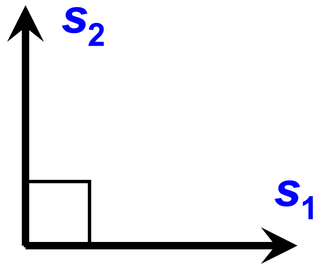
Orthogonality in the time domain assures orthogonality in the frequency domain, and vice versa. The property is easiest to see in the time domain.



S_1 and S_2 cannot possibly interfere with one another.



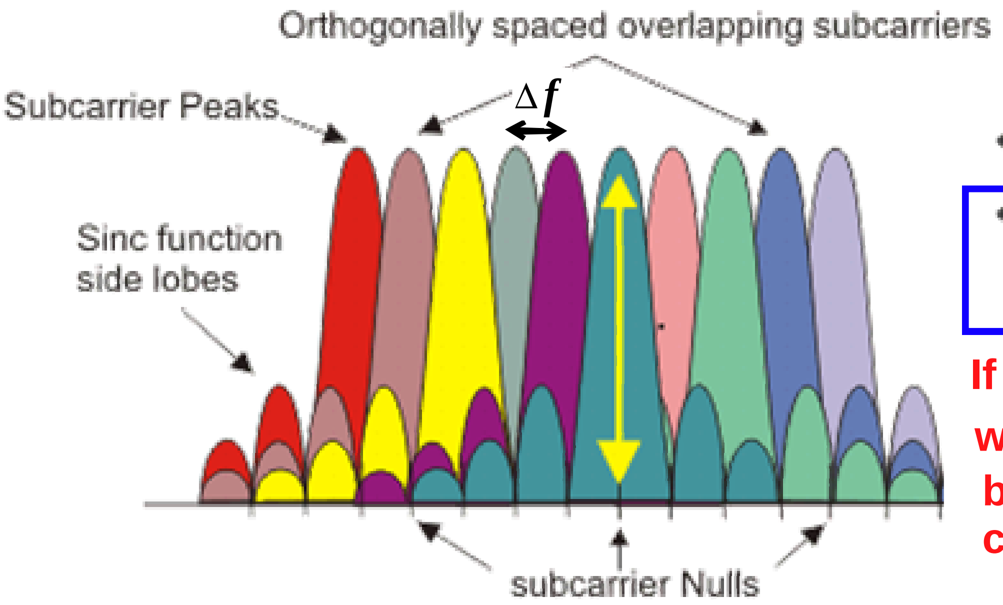
Vector Representation



This is the test

$$\boxed{\int_0^T s_1(t)\, s_2(t)\, dt = 0}$$

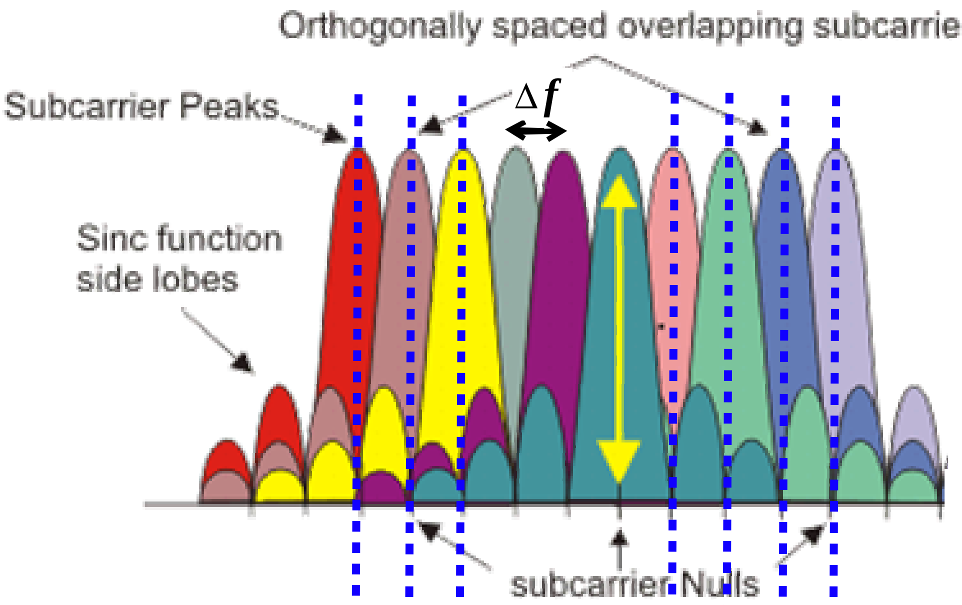
Cross-Correlation Inner Product equals zero.



OFDM Signal Frequency Spectra

- Closely spaced subcarriers overlap
- Note that subcarrier nulls correspond to peaks of adjacent subcarriers for Zero Inter-Carrier-Interference.

If signals are orthogonal, then $\Delta f = 1/T_s$ where Δf is the fixed frequency interval between any 2 adjacent spectra of candidate subcarriers.



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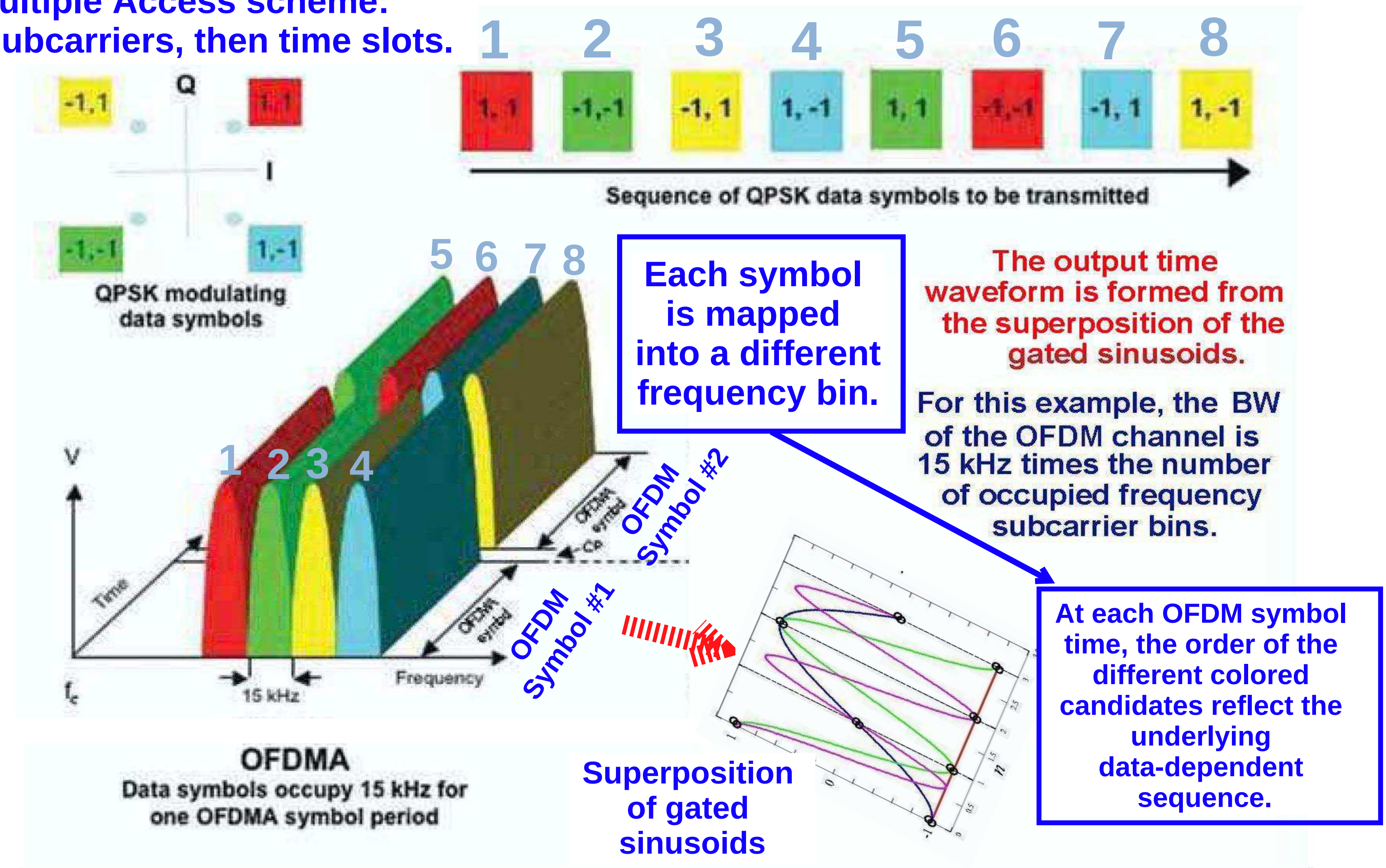
If signals are orthogonal, then $\Delta f = 1/T_s$ where Δf is the fixed frequency interval between any 2 adjacent spectra of candidate subcarriers.

If the spectral sincs are orthogonal, then each subcarrier peak is aligned with the nulls of its candidate neighbors.

Loading messages into the OFDM Multiple Access scheme: first subcarriers, then time slots.



Each symbol is mapped into a different frequency bin.

The output time waveform is formed from the superposition of the gated sinusoids.

For this example, the BW of the OFDM channel is 15 kHz times the number of occupied frequency subcarrier bins.

At each OFDM symbol time, the order of the different colored candidates reflect the underlying data-dependent sequence.

Superposition of gated sinusoids

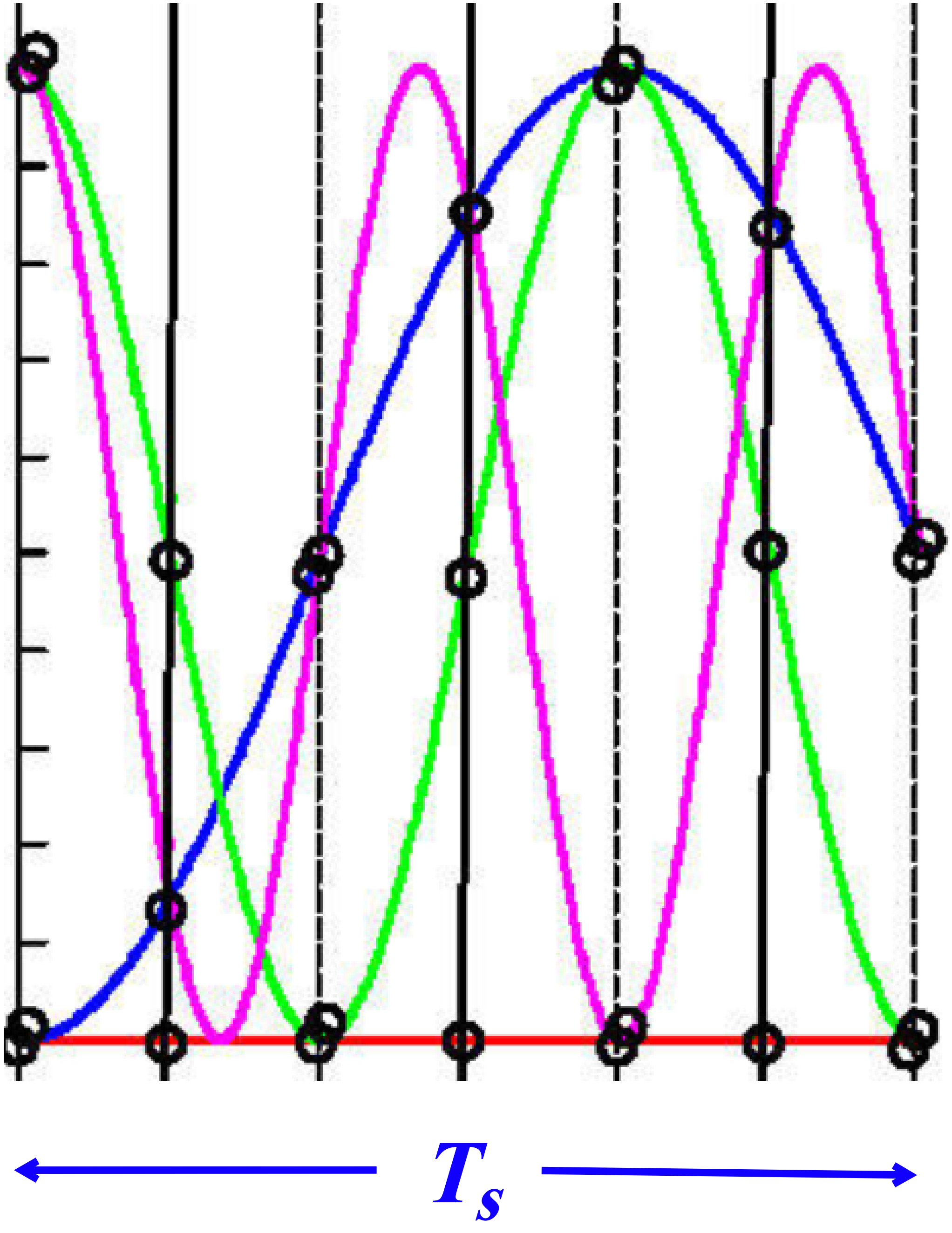
OFDMA
Data symbols occupy 15 kHz for one OFDMA symbol period

Figure 2 *Comparison of OFDMA and SC-FDMA transmitting a series of QPSK data symbols*

For an IDEAL CHANNEL, sampling the SC-OFDM output waveform yields the original 2-space points. Not so with OFDM.

**Ref: White Paper, "De-mystifying SC-FDMA, The New LTE Upli "
Agilent Technologies, April 2008**

Gated Sinusoids



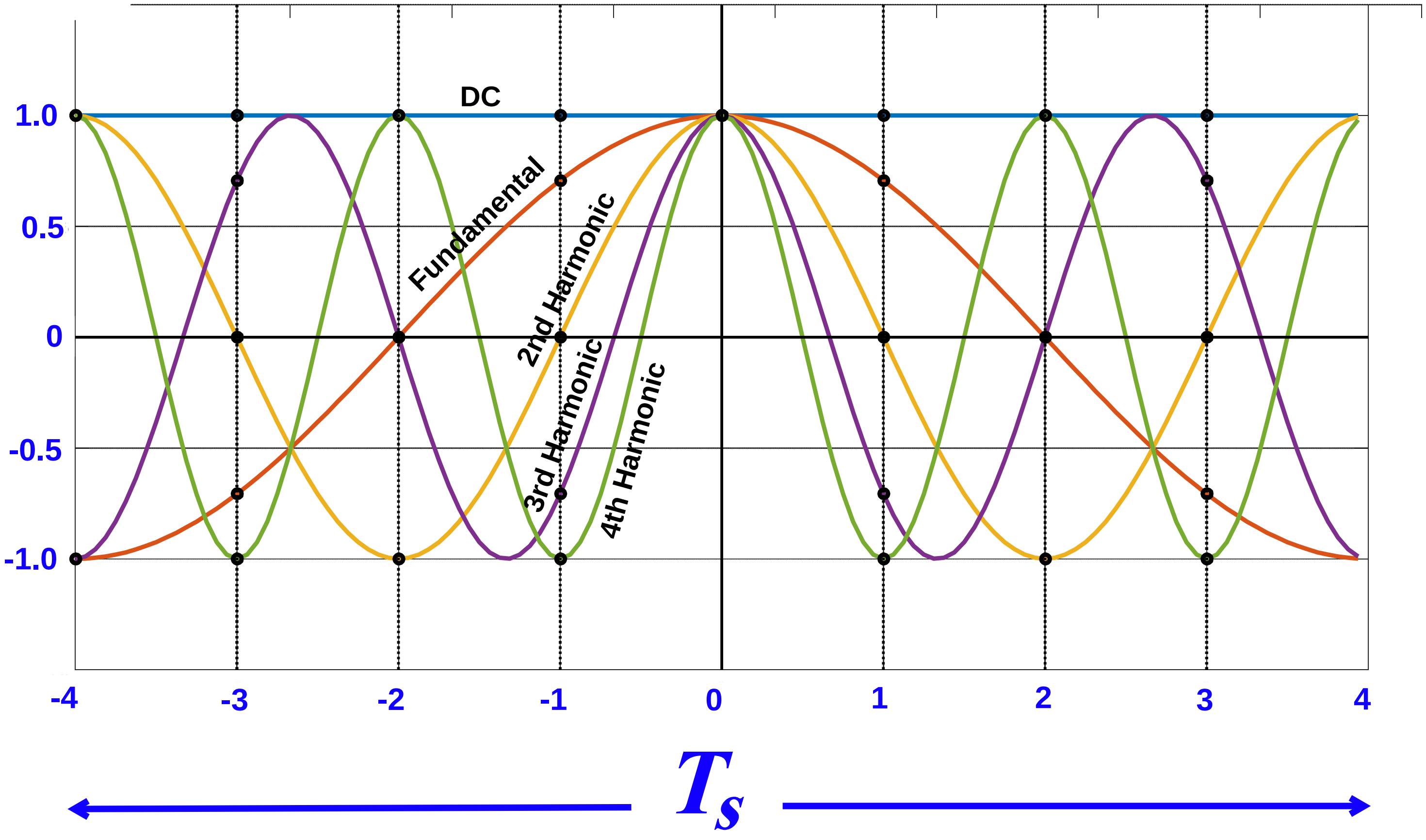
This sketch of gated sinusoids is not accurate since it violates an important orthogonality rule:

Each harmonic must display an integer number of cycles during the pulse interval.

It's a colorful convenient sketch to act as a logo for "gated sinusoids". But it is not precise.

Gated Sinusoids and the OFDM Pulse T_s

- **To maintain orthogonality, each harmonic displays an integer number of cycles during the pulse inter**val. Also the frequency spacing Δf between any 2 adjacent tones equals $1/T_s$ and Test must be satisfied.
  - **The Fourier Transform of such gated sinusoids is a spectral sequence of sinc functions with zeros equally spaced at** Δf Hz.



IEEE Virtual Presentation
The ABCs of OFDM
By Dr. Bernard Sklar

Part 1 March 18, 2021
Part 2 March 25, 2021

Abstract: The main benefit of OFDM is its ability to cope with Severe multipath channel conditions without needing Complex Equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a High-data-rate signal into Smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

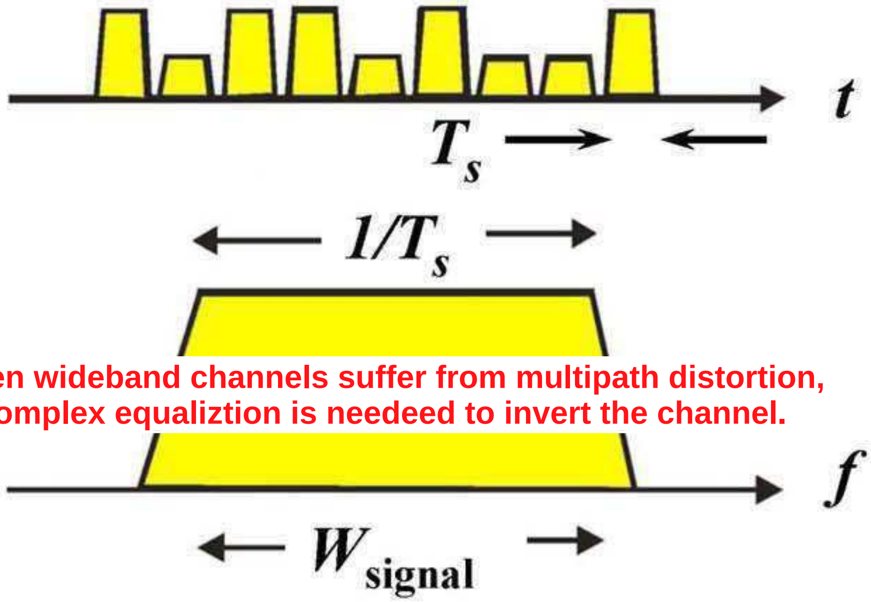
- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

Single-Carrier versus Multi-Carrier Systems

Single-Carrier versus Multi-Carrier System

with idealized shapes

Short Symbol = Large BW

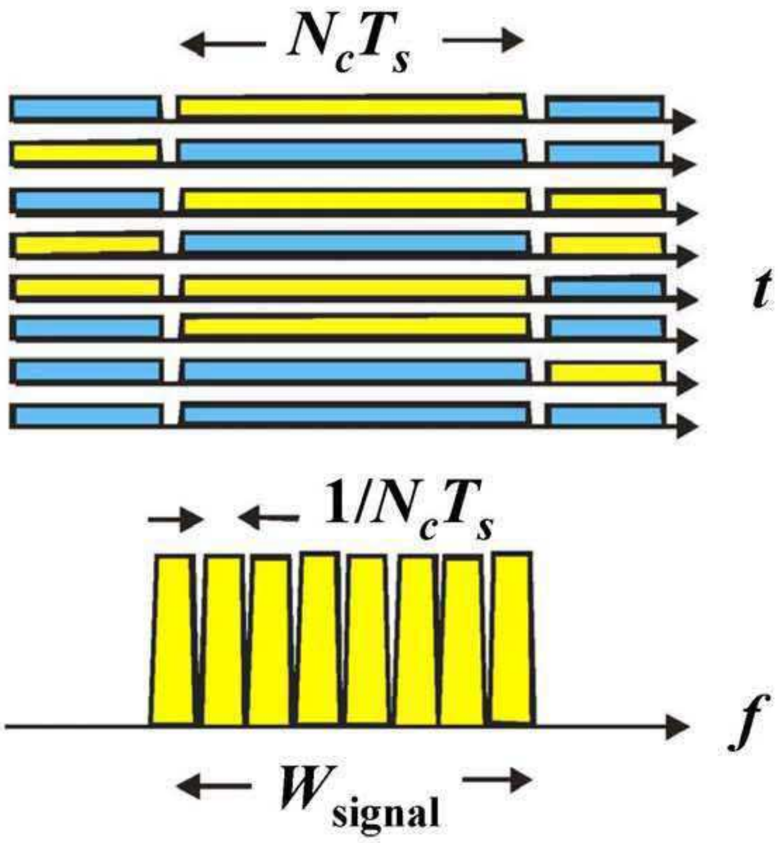


When wideband channels suffer from multipath distortion, complex equaliztion is needeed to invert the channel.

(a) Single-Carrier System

One Wide-band Carrier

Long Symbol = Small BW



(b) Conventional Multi-Carrier System

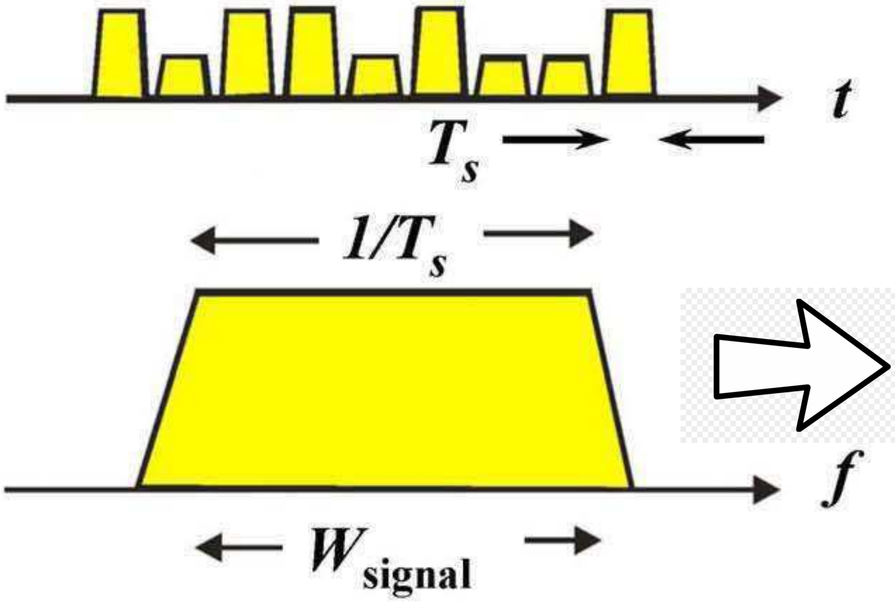
N_c carriers share the same bandwidth

Multiple Narrow-band Carriers

Single-Carrier versus Multi-Carrier System

with idealized shapes

Short Symbol = Large BW

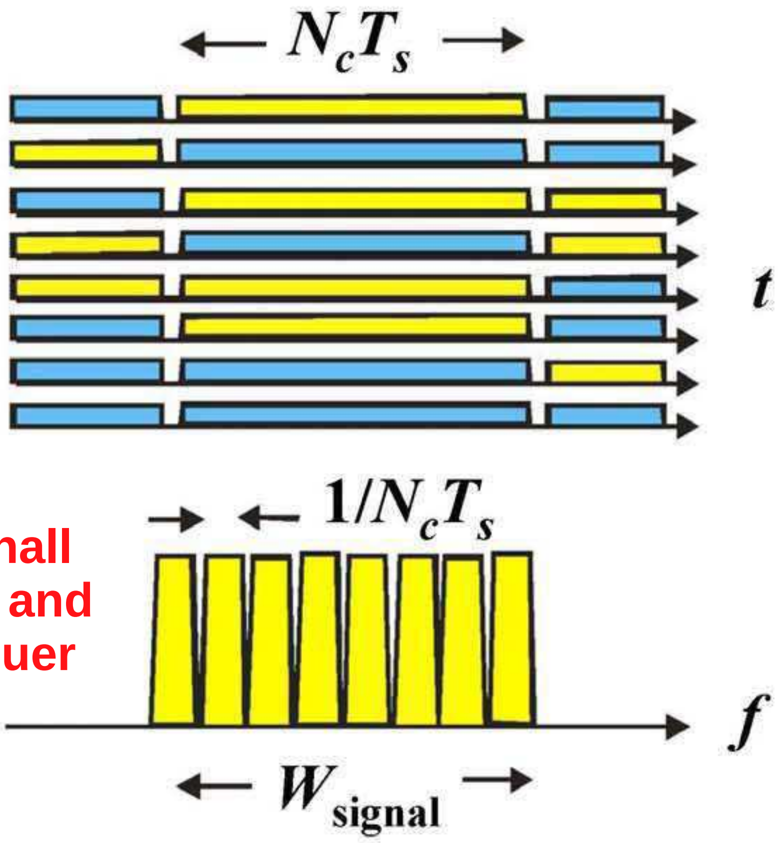


(a) Single-Carrier System

One Wide-band Carrier

We shall divide and conquer

Long Symbol = Small BW

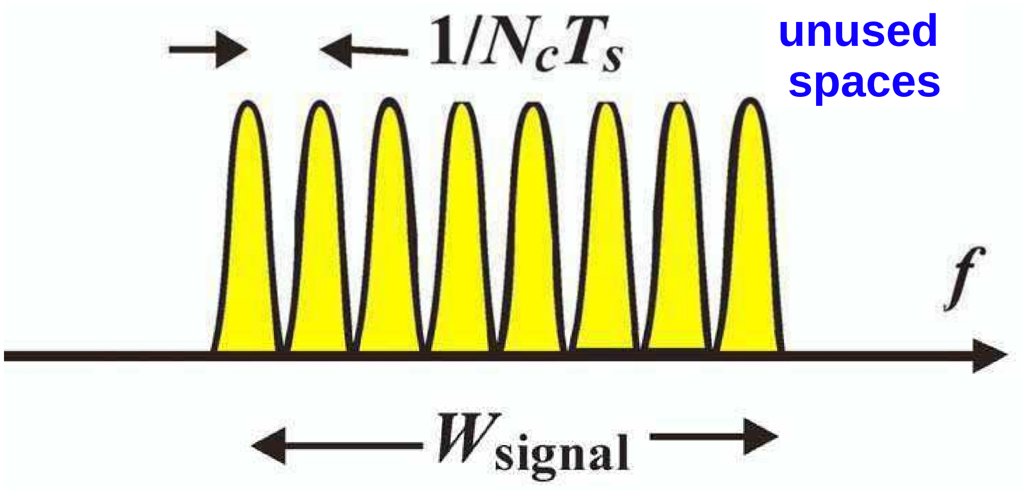


(b) Conventional Multi-Carrier System

N_c carriers share the same bandwidth

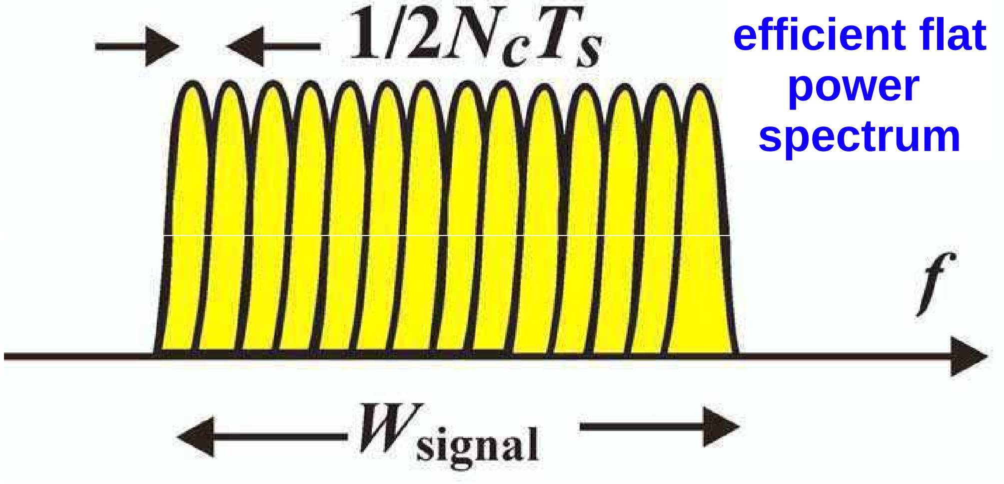
Multiple Narrow-band Carriers

Conventional Multi-Channel versus OFDM Multi-Channel System



Note the unused spaces

(a) Conventional multi-channel system



Note the more efficient flat power spectrum

(b) OFDM multi-channel system

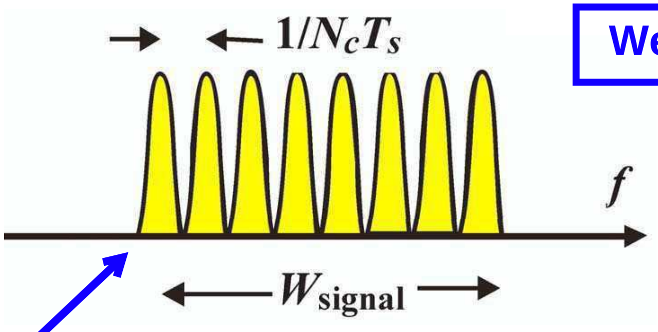
- Conventional multi-channel systems (eg: AM and FM radios) have non-overlapping adjacent channels. Channels are separated by more than their two-sided bandwidth.

- OFDM multi-channel systems:

  have 50% overlap of adjacent channels. Available bandwidth is used twice. Channels are separated by half their 2-sided BW.

Such BW efficiency (utilization) is concomitant with minimally spaced orthogonal. Orthogonal spectra cannot overlap more than 50%

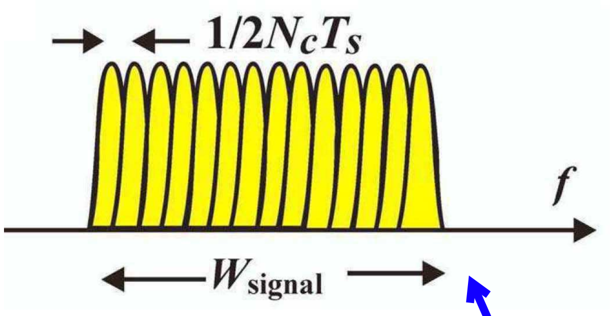
Maximizing Bandwidth Utilization



We want to whiten the signaling space

Revisiting Shannon Capacity

$$C = W \log_2 \left(1 + \mathrm{SNR}\right)$$

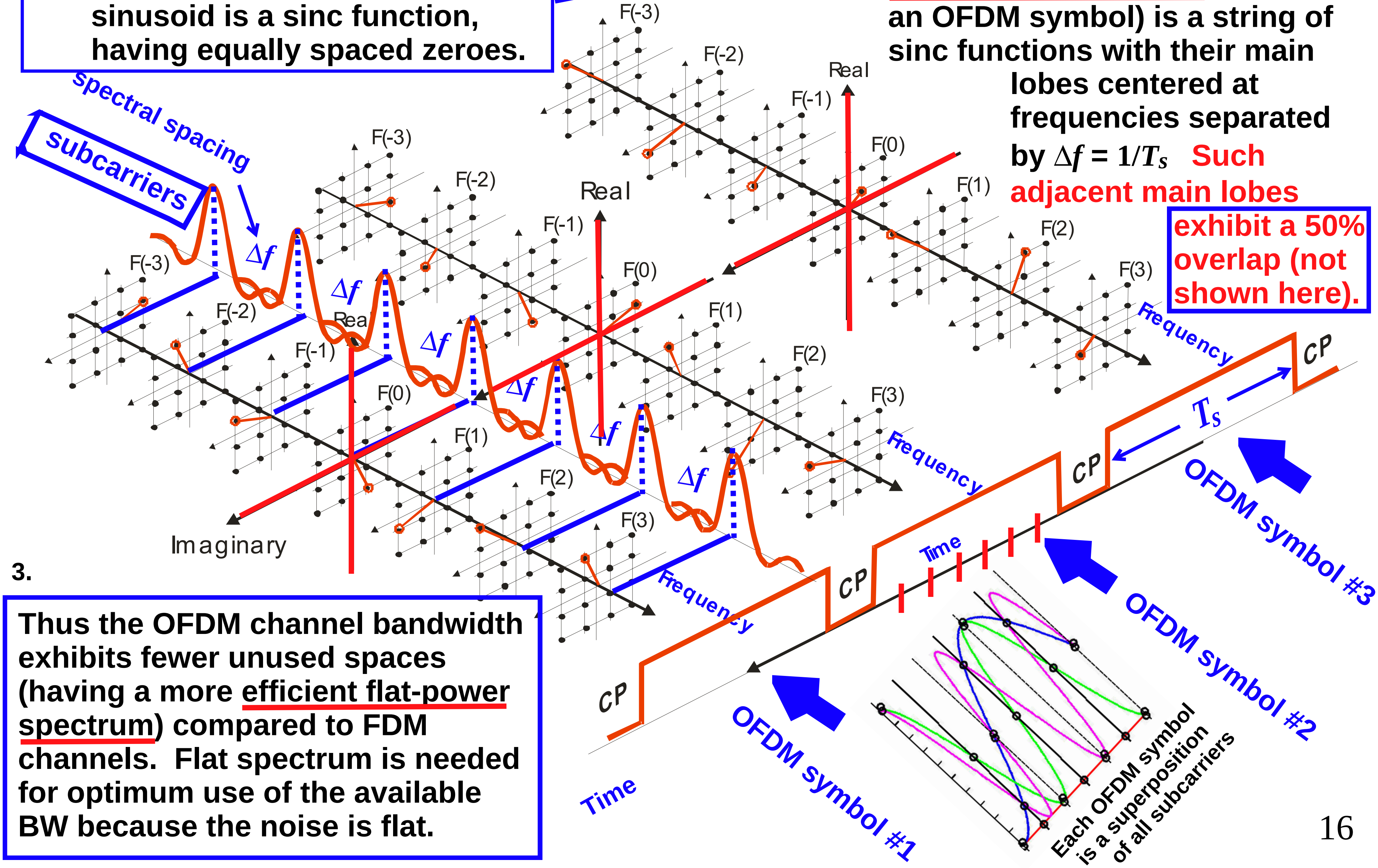


We usually focus on white noise. Here we examine how channel capacity is influenced by the whiteness of the signal power spectrum.

- *W* is the signal bandwidth (BW), generally assumed to have a flat power spectrum, meaning that the BW is fully and equally populated (white) across the band.
- One might use modulation types such that the signal is not white, for example: frequency modulation (FM), any signaling with non-overlapped sinc spectra.
- Then the BW could have regions containing little or no signal energy.
- The capacity would be reduced by a factor (dependent on the BW utilization).
- Historical Example: In 1995 European countries began to replace analog FM radio transmission with an OFDM system called DAB (digital audio broadcasting). Norway and England were the first countries to convert.
- By 2017 most all of Europe and Australia had converted. DAB replaced the BW-inefficient FM with a digital OFDM technology. The benefit was more capacity by removing spaces in the system BW (whitening), greater digital flexibility, and reduction of adjacent channel interference (ACI) due to lower intermods.

1. **The Fourier Transform (FT) of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.**

2. **The FT of the superposition of many gated sinusoids (within an OFDM symbol) is a string of sinc functions with their main lobes centered at frequencies separated by $\Delta f = 1/T_s$ Such adjacent main lobes exhibit a 50% overlap (not shown here).**

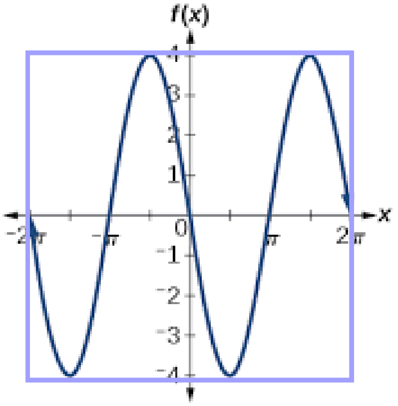


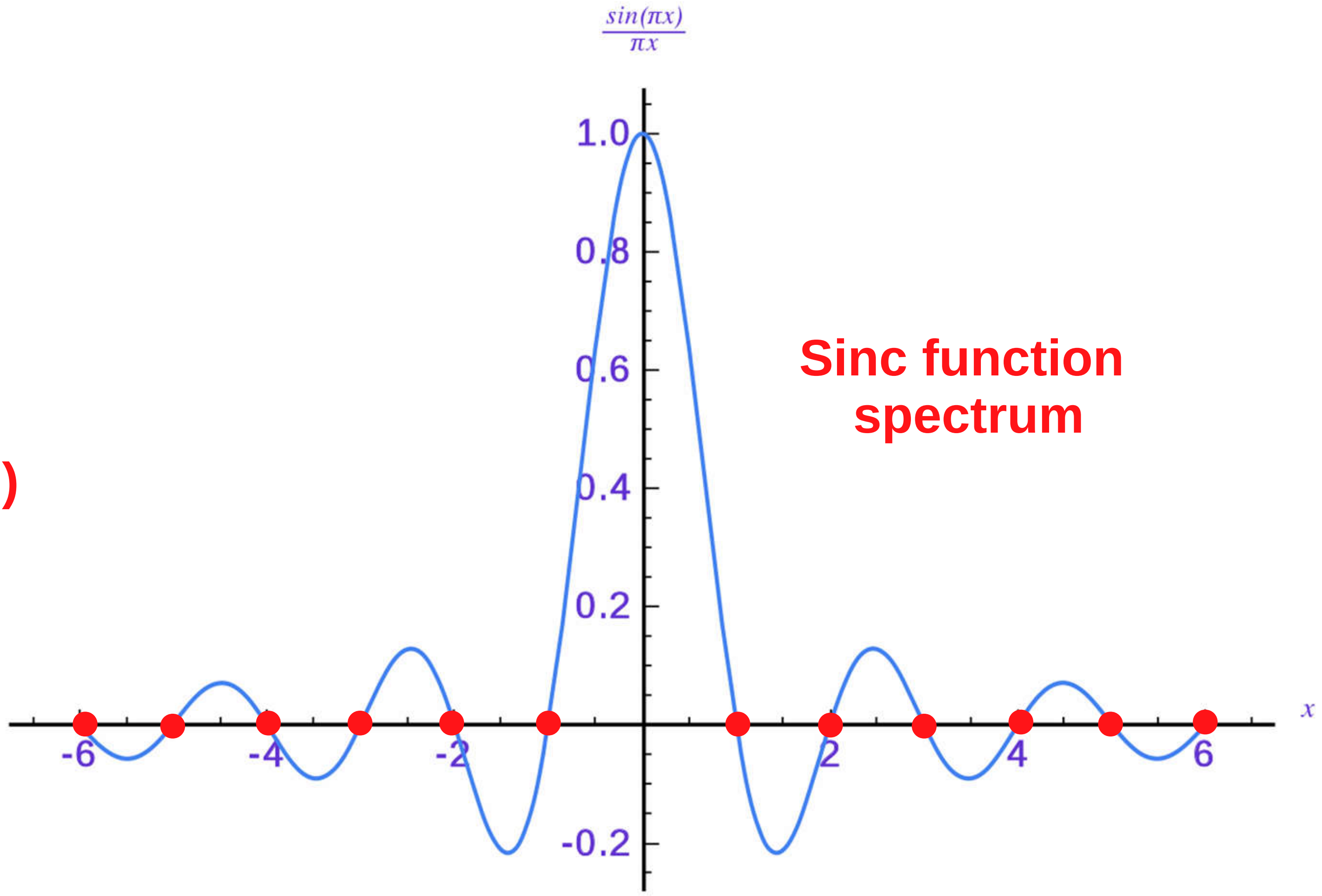
3.

Thus the OFDM channel bandwidth exhibits fewer unused spaces (having a more efficient flat-power spectrum) compared to FDM channels. Flat spectrum is needed for optimum use of the available BW because the noise is flat.

The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

Gated Sinusoid (one sinusoid with an integer number of cycles)





Sinc function spectrum

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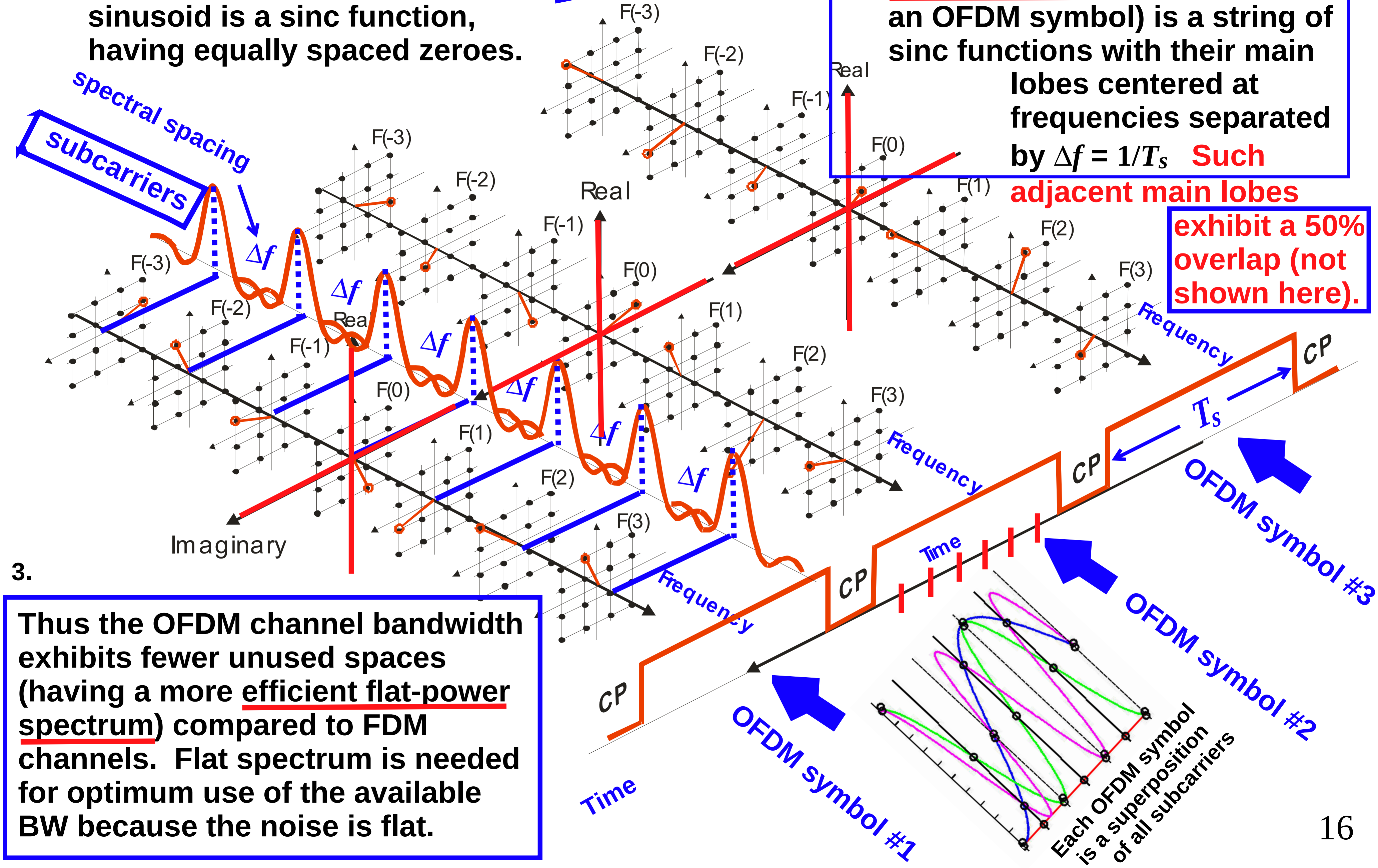

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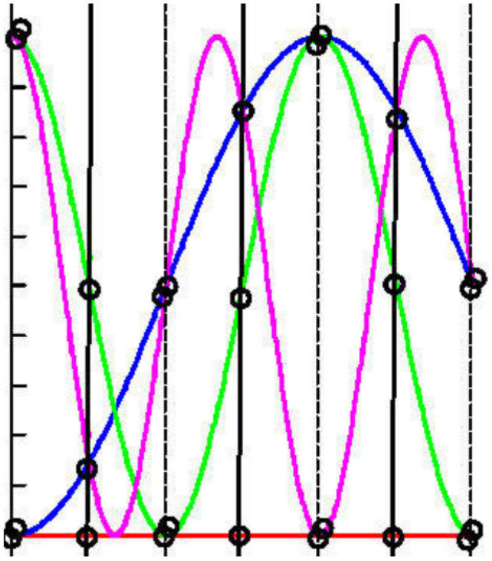
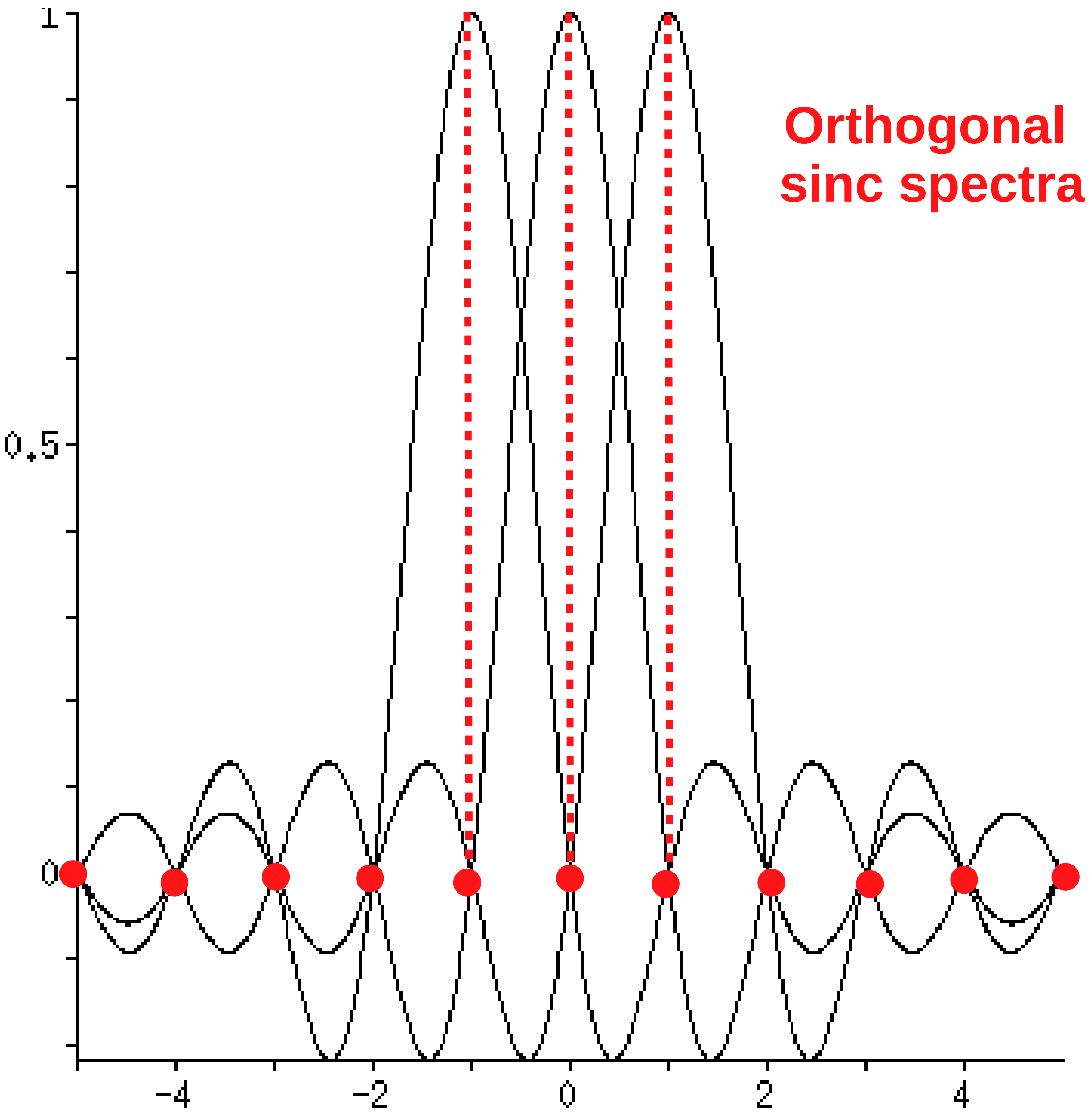
Thus the OFDM channel bandwidth exhibits fewer unused spaces (having a more efficient flat-power spectrum) compared to FDM channels. Flat spectrum is needed for optimum use of the available BW because the noise is flat.

The Fourier Transform of the superposition of many gated sinusoids is a string of sinc functions with their main lobes centered at frequencies separated by $\Delta f = 1/T_s$

Gated Sinusoids (multiple sinusoids, each with an integer number of cycles within the fixed interval T_s)

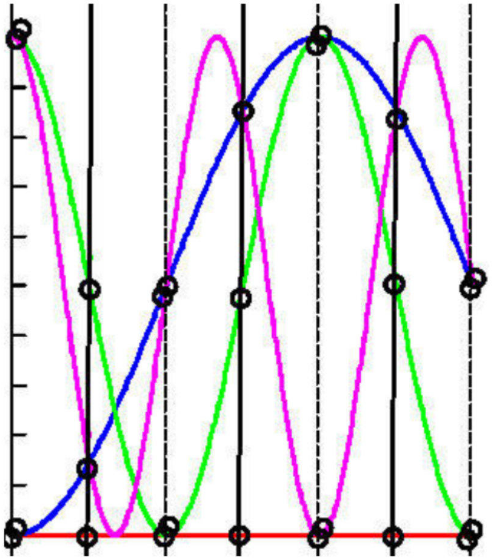


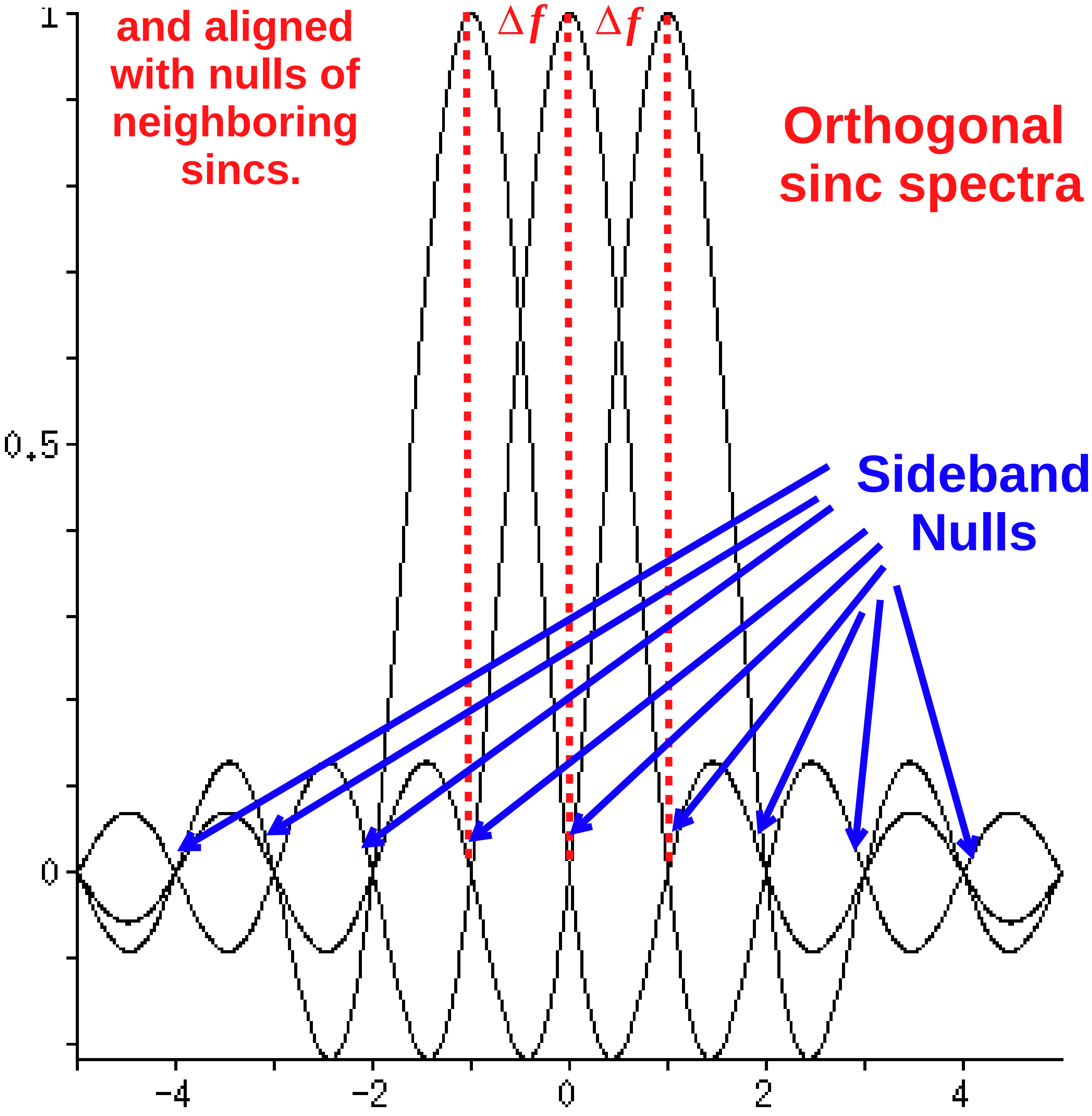


Orthogonal sinc spectra

The Fourier Transform of the superposition of many gated sinusoids is a string of sinc functions with their main lobes centered at frequencies separated by $\Delta f = 1/T_s$

Gated Sinusoids (multiple sinusoids, each with an integer number of cycles within the fixed interval T_s)

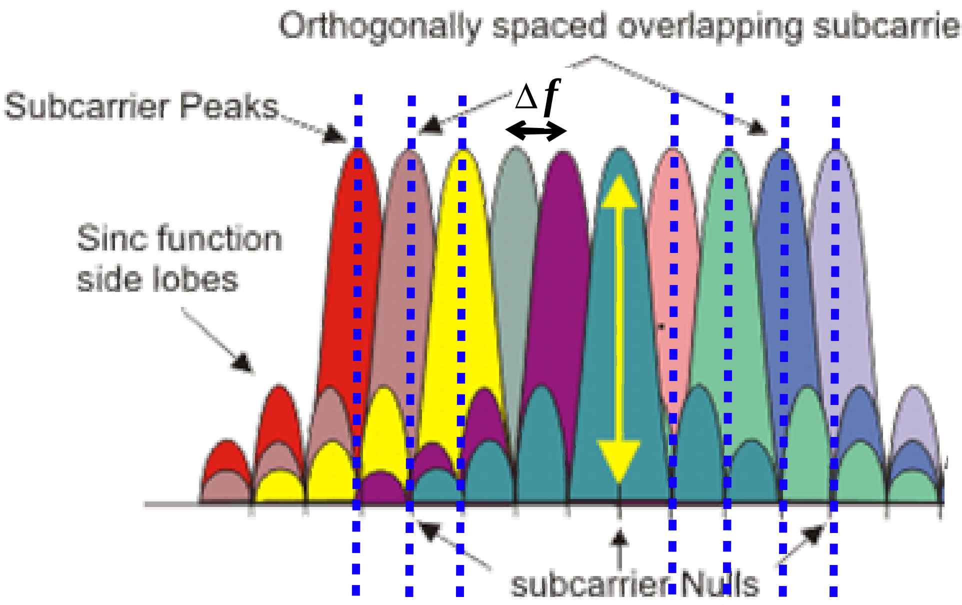




and aligned with nulls of neighboring sincs.

Orthogonal sinc spectra

Sideband Nulls



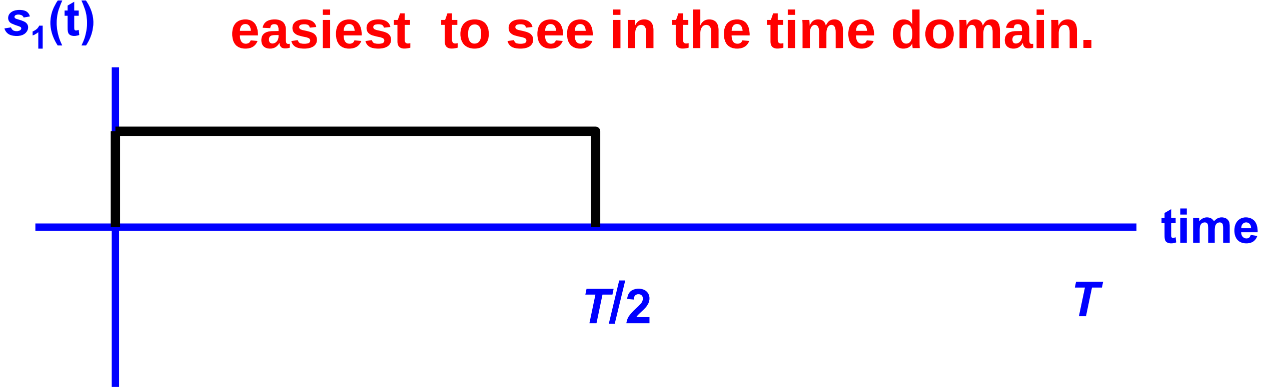
- Closely spaced subcarriers overlap
- Note that subcarrier nulls correspond to peaks of adjacent subcarriers for Zero Inter-Carrier-Interference.

If signals are orthogonal, then $\boxed{\Delta f = 1/T_s}$ where Δf is the fixed frequency interval between any 2 adjacent spectra of candidate subcarriers.

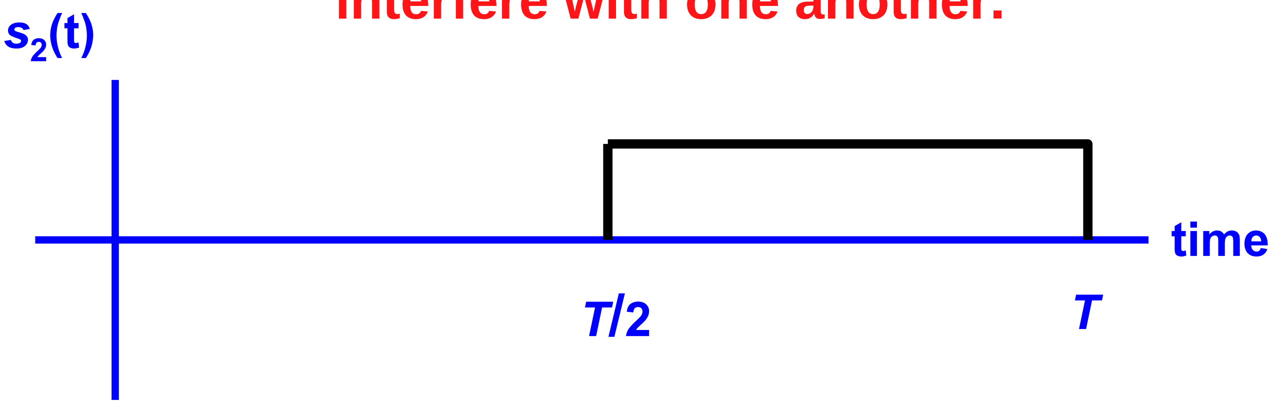
If the spectral sincs are orthogonal, then each subcarrier peak is aligned with the nulls of its candidate neighbors.

Reminder: Testing for Orthogonality

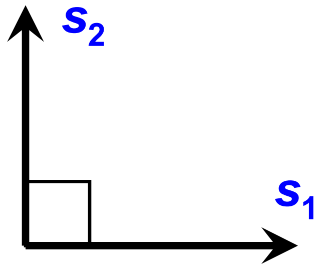
Orthogonality in the time domain assures orthogonality in the frequency domain, and vice versa. The property is easiest to see in the time domain.



S_1 and S_2 cannot possibly interfere with one another.



Vector Representation



This is the test

$$\boxed{\int_0^T s_1(t)\, s_2(t)\, dt = 0}$$

Cross-Correlation Inner Product equals zero.

Sinusoids are Amazing!

The Amazing Uniqueness of Sinusoids

- An input sinusoid into any linear time-invariant (LTI) filter yields (in steady state) an output sinusoid which is an exact copy of the input sinusoid except for its amplitude and phase.

- This is not true for any other waveform shape

AMAZING

because other waveform shapes are comprised of 2 or more sinusoids of different frequencies. The amplitude and phase response at each frequency will typically be different for some of the LTI filters. Hence any shape (other than a sinusoid) into some of the LTI filters will yield different (distorted) outputs.

- Summing the sinusoids of different frequencies is different from summing delayed and scaled versions of a single sinusoid.

- Summing the sinusoids of different frequencies allows us to obtain waveforms of most any arbitrary shape we desire.

A sinudoidal shape is truly a FUNDAMENTAL element in nature. Note that OFDM exploits this feature (basic message is a gated sinusoid). At the receiver the DFT plays the part of the matched filter.

OFDM's main function is to manipulate orthogonal sinusoids.

Why is this useful?

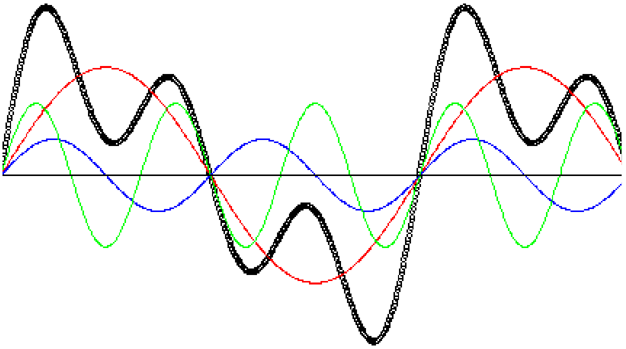
Because sinusoids are amazing.

The steady-state response of a multipath channel yields
NO DISTORTION
to a fixed frequency sinusoid.
There will only be changes in amplitude and phase.

The Uniqueness of (Gated) Sinusoids

- Can the steady-state response (ssr) of an imperfect channel (having multipath echoes) introduce distortion to a sinusoid?
- The answer is NO.

  No matter how bad the channel is, for a sinusoid of a given frequency, the ssr is just an amplitude & phase variation of that sinusoid. This is only true for steady state responses; it is not the case for a time-varying channel. A sinusoid plus its echoes = an undistorted sinusoid.
- How about the steady-state channel response to any arbitrary waveform? Any waveform is just a combination of sinusoids. Shouldn't we claim that the steady-state response to any waveform will see no distortion?
- Each one of the component sinusoids making-up any waveform will not be distorted (just changed in amplitude/phase). But the amplitude/phase changes for the sinusoids at different frequencies will see different amplitude/phase changes, such that the sum of components (overall arbitrary waveform) will be distorted.



At the OFDM receiver, the DFT plays the part of the MF, thereby identifying the correct sinusoid. (subcarrier).

In OFDM we manipulate orthogonal sinusoids.

The Uniqueness of (Gated) Sinusoids

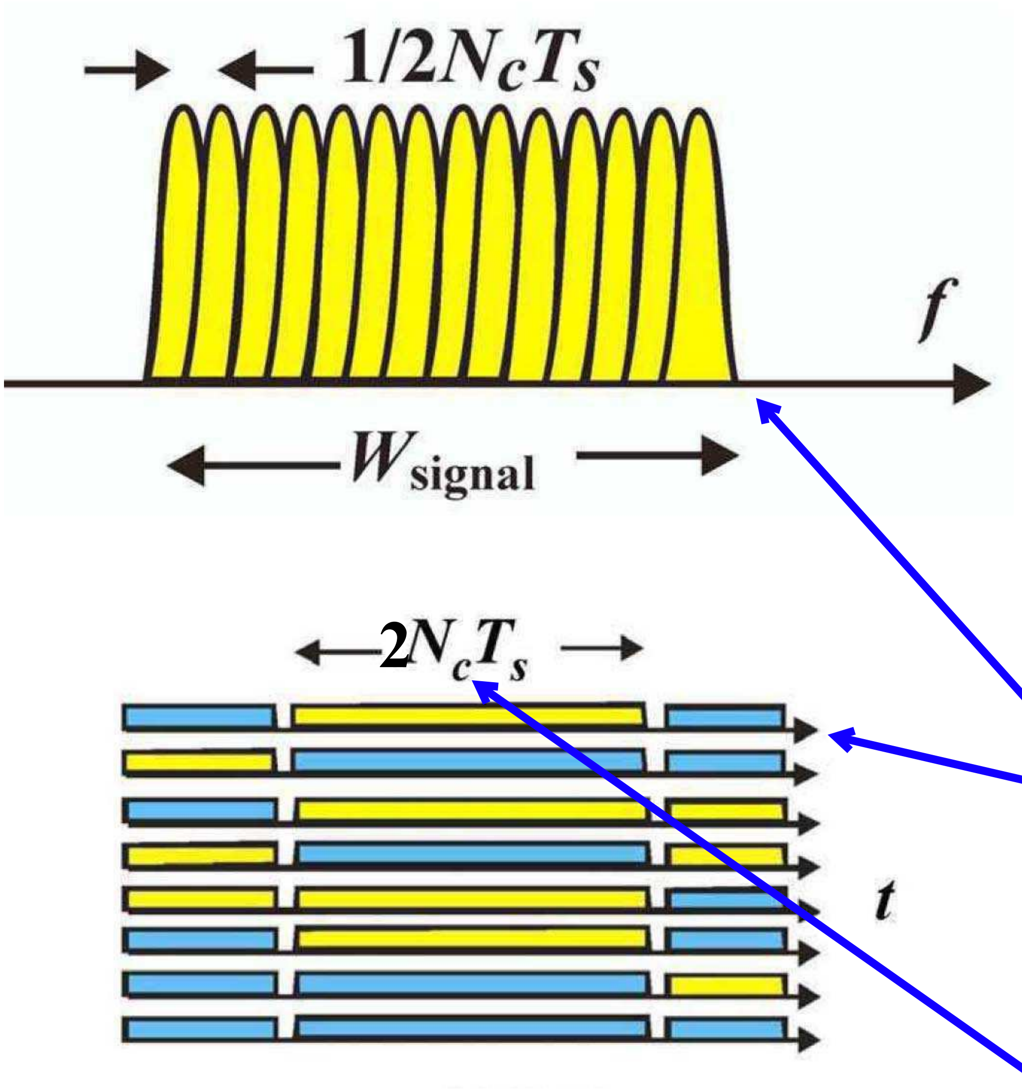
No matter how bad the channel is, for a sinusoid of a given frequency, the ssr is just an amplitude & phase variation of that sinusoid.
This is only true for steady state responses; it is not the case for a time-varying channel.

A sinusoid plus its echoes = an undistorted sinusoid.

more precisely

A steady-state sinusoid plus its channel-induced echoes into any LTI system yields an undistorted sinusoid at the output.

Long Pulses are the Key to OFDM providing Mitigation for Multipath



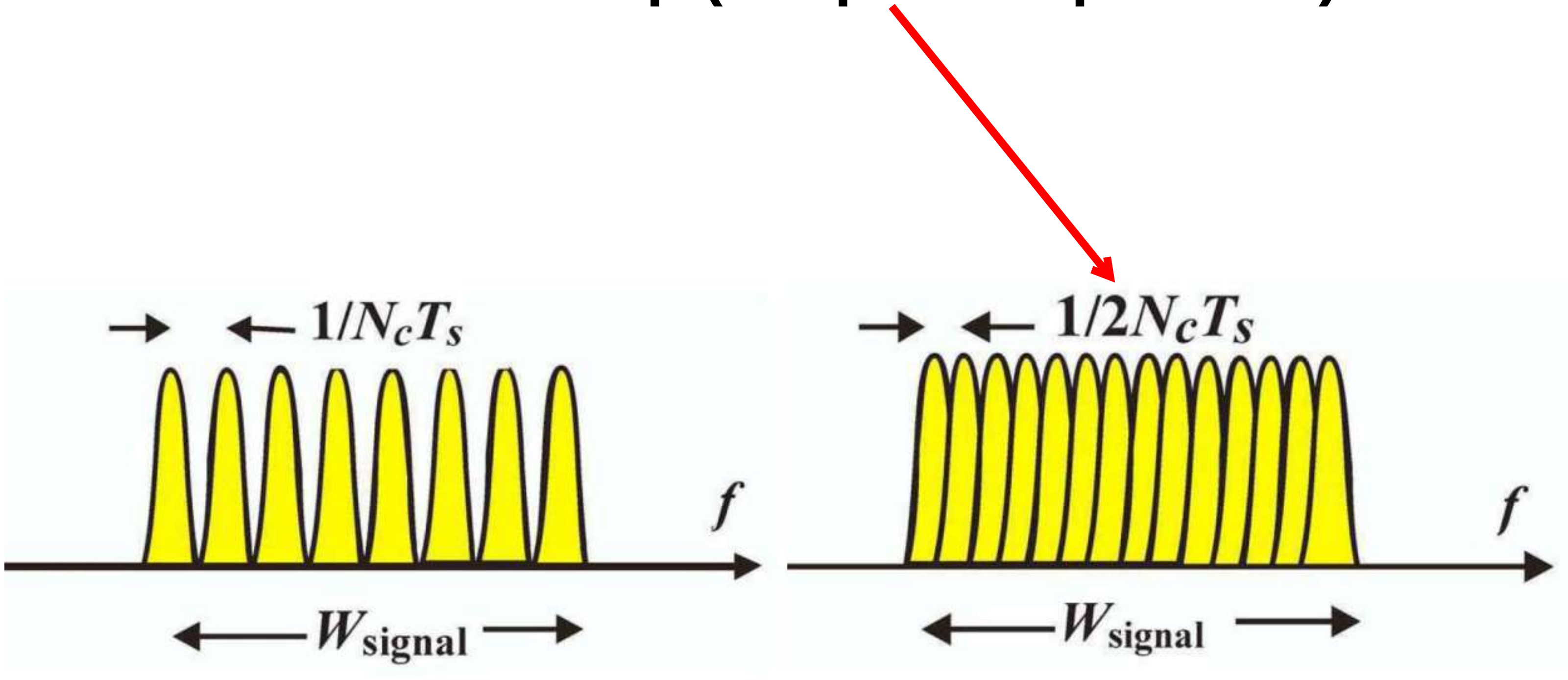
In OFDM, a wideband symbol sequence is partitioned onto narrowband subcarriers with a 50% spectral overlap, via the IDFT operation.

Prior to the partitioning and transform, the wideband input signals are made up of short pulses.

After the transform, the narrowband output signals are made up of long pulses.

The length of the pulses is proportional to the number of subcarriers.

The factor 2 corresponds to the improved BW efficiency (utilization) with 50% channel overlap (flat power spectrum)



(a) Conventional multi-channel system

(b) OFDM multi-channel system

But bandwidth efficiency is NOT the main beneficial attribute of OFDM.

It is the elegence in Mitigating Multipath degradation.

IEEE Virtual Presentation
The ABCs of OFDM
By Dr. Bernard Sklar

Part 1 March 18, 2021
Part 2 March 25, 2021

Abstract: The main benefit of OFDM is its ability to cope with Severe multipath channel conditions without needing Complex Equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a High-data-rate signal into Smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

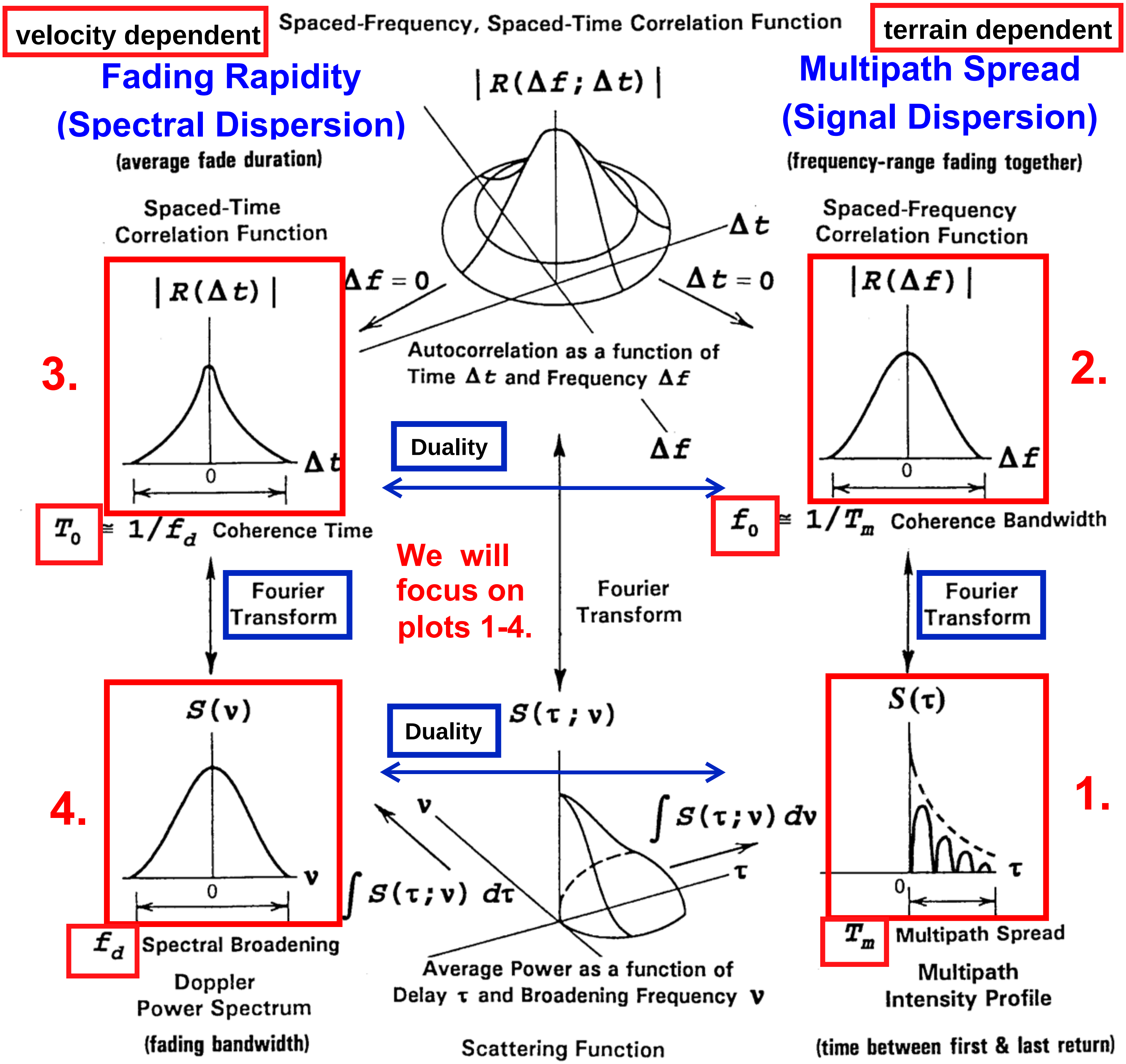
The 4 Key WSSUS Functions

(wide-sense stationary
uncorrelated scatterers)

Relationships Among the Channel Correlation Functions and Power Spectra

Portrayal of the four key WSSUS functions by Paul Green:

Relationships Among the Channel Correlation Functions and Power Spectra



velocity dependent

Fading Rapidity

(Spectral Dispersion)

(average fade duration)

Spaced-Frequency, Spaced-Time Correlation Function

$|R(\Delta f;\Delta t)|$

terrain dependent

Multipath Spread

(Signal Dispersion)

(frequency-range fading together)

Spaced-Time Correlation Function

Spaced-Frequency Correlation Function

3.

Autocorrelation as a function of Time Δt and Frequency Δf

2.

Duality

$T_0 \cong 1/f_d$ Coherence Time

$f_0 \cong 1/T_m$ Coherence Bandwidth

Fourier Transform

We will focus on plots 1-4.

Fourier Transform

Fourier Transform

4.

Duality

1.

f_d Spectral Broadening

Doppler Power Spectrum

(fading bandwidth)

Average Power as a function of Delay τ and Broadening Frequency ν

Scattering Function

T_m Multipath Spread

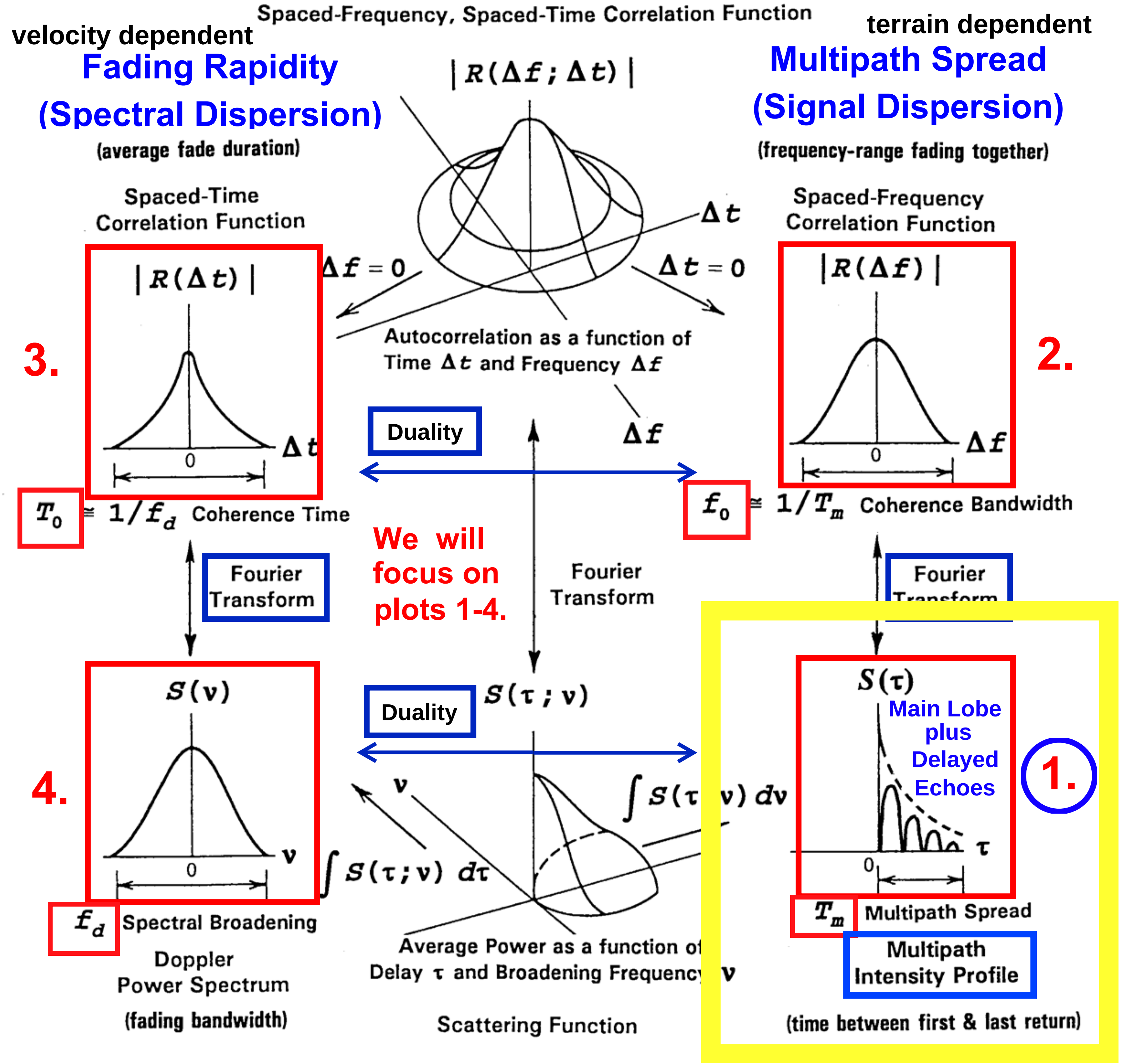
Multipath Intensity Profile

(time between first & last return)

REF: PROAKIS, J.G., DIGITAL COMMUNICATIONS, McGRAW-HILL BOOK COMPANY, NEW YORK, 1983, Green, P. E., Jr., "Radar Astronomy Measurement Techniques ," MIT Lincoln Laboratory, Tech Report #282, December 1962.

Multipath Spread T_m

Relationships Among the Channel Correlation Functions and Power Spectra



Spaced-Frequency, Spaced-Time Correlation Function

velocity dependent

Fading Rapidity

(Spectral Dispersion)

(average fade duration)

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Multipath Spread

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$f_0 \cong 1/T_m$ Coherence Bandwidth

We will focus on plots 1-4.

Fourier Transform

Fourier Transform

Fourier Transform

4.

Duality

1.

Main Lobe plus Delayed Echoes

T_m Multipath Spread

Multipath Intensity Profile

(time between first & last return)

f_d Spectral Broadening

Doppler Power Spectrum

(fading bandwidth)

Average Power as a function of Delay τ and Broadening Frequency ν

Scattering Function

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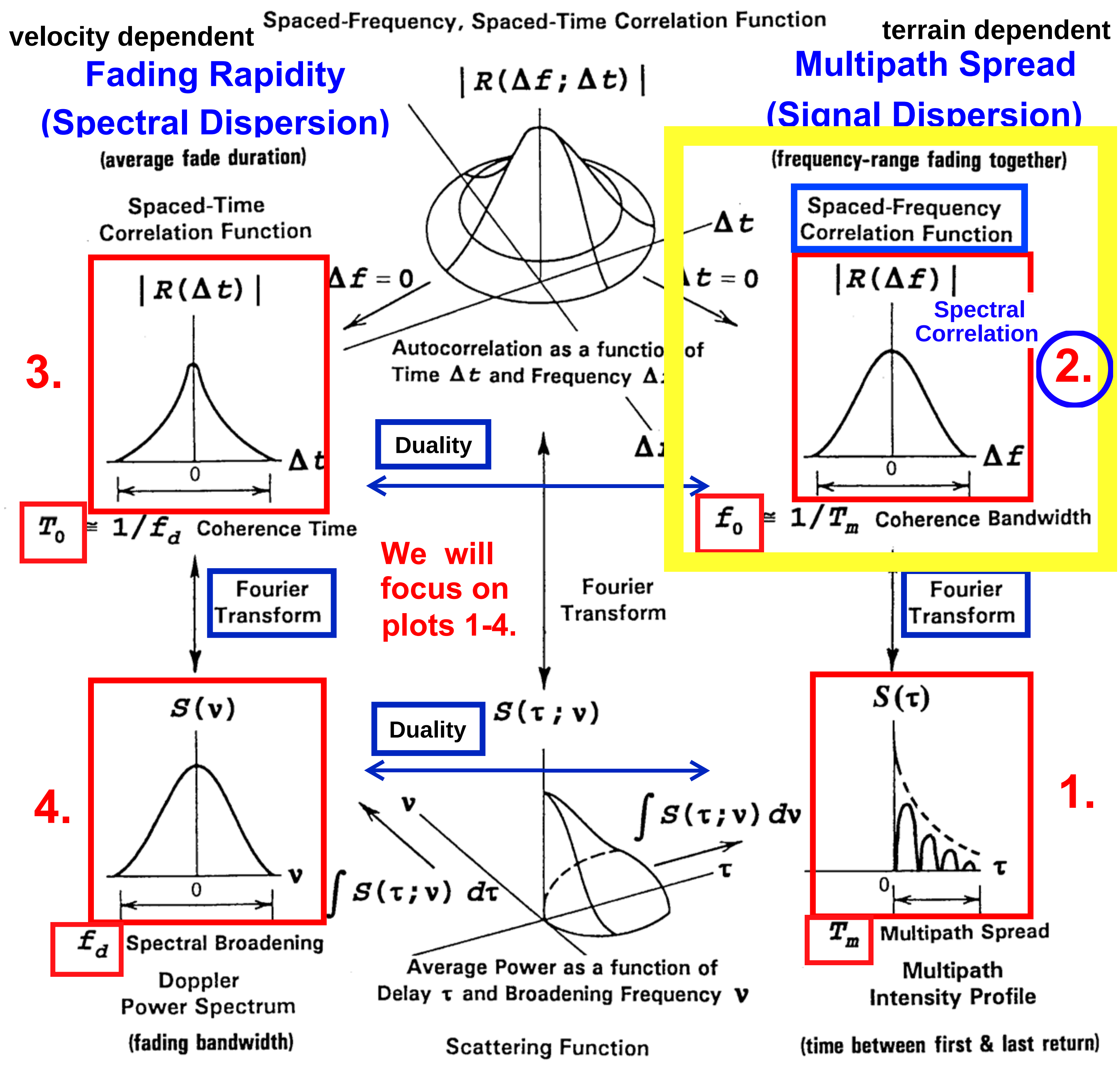
Coherence Bandwidth f_0

Relationships Among the Channel Correlation Functions and Power Spectra



velocity dependent

Fading Rapidity

(Spectral Dispersion)

(average fade duration)

Spaced-Frequency, Spaced-Time Correlation Function

terrain dependent

Multipath Spread

(Signal Dispersion)

(frequency-range fading together)

Spaced-Time Correlation Function

Spaced-Frequency Correlation Function

Spectral Correlation

Autocorrelation as a function of Time Δt and Frequency Δf

Duality

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Fourier Transform

Fourier Transform

Fourier Transform

We will focus on plots 1-4.

Duality

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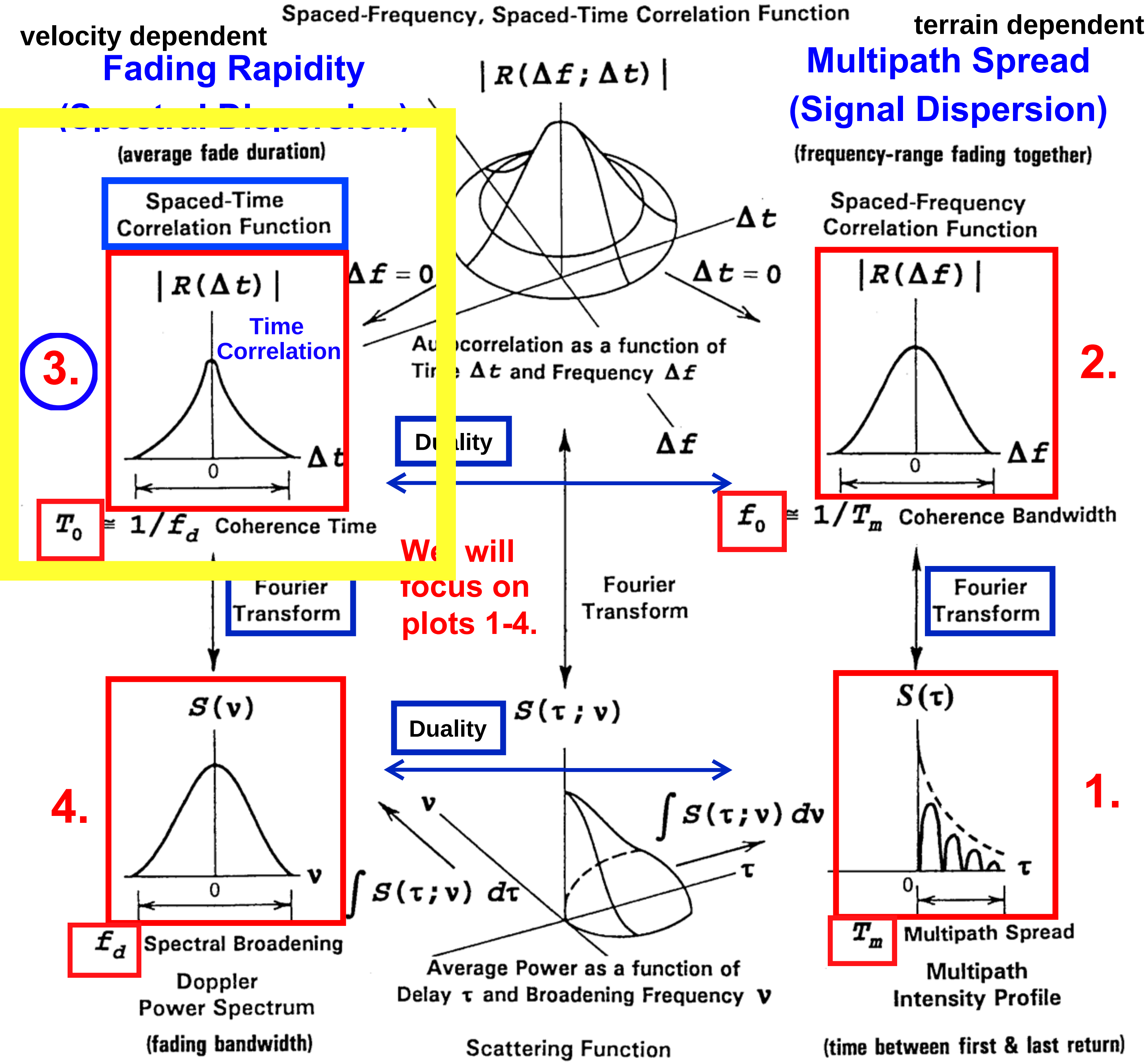
Coherence Time T_0

Relationships Among the Channel Correlation Functions and Power Spectra



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$|R(\Delta f;\Delta t)|$

Au ocorrelation as a function of Ti e Δt and Frequency Δf

velocity dependent

Fading Rapidity

(Spectral Dispersion)

(average fade duration)

Spaced-Time Correlation Function

$|R(\Delta t)|$

Time Correlation

3.

$T_0 \cong 1/f_d$ Coherence Time

terrain dependent

Multipath Spread

(Signal Dispersion)

(frequency-range fading together)

Spaced-Frequency Correlation Function

$|R(\Delta f)|$

2.

$f_0 \cong 1/T_m$ Coherence Bandwidth

Du lity

We will focus on plots 1-4.

Fourier Transform

Fourier Transform

Fourier Transform

Duality

$S(\tau;\nu)$

$\int S(\tau;\nu)\,d\nu$

$\int S(\tau;\nu)\,d\tau$

Average Power as a function of Delay τ and Broadening Frequency ν

Scattering Function

4.

$S(\nu)$

f_d Spectral Broadening

Doppler Power Spectrum

(fading bandwidth)

1.

$S(\tau)$

T_m Multipath Spread

Multipath Intensity Profile

(time between first & last return)

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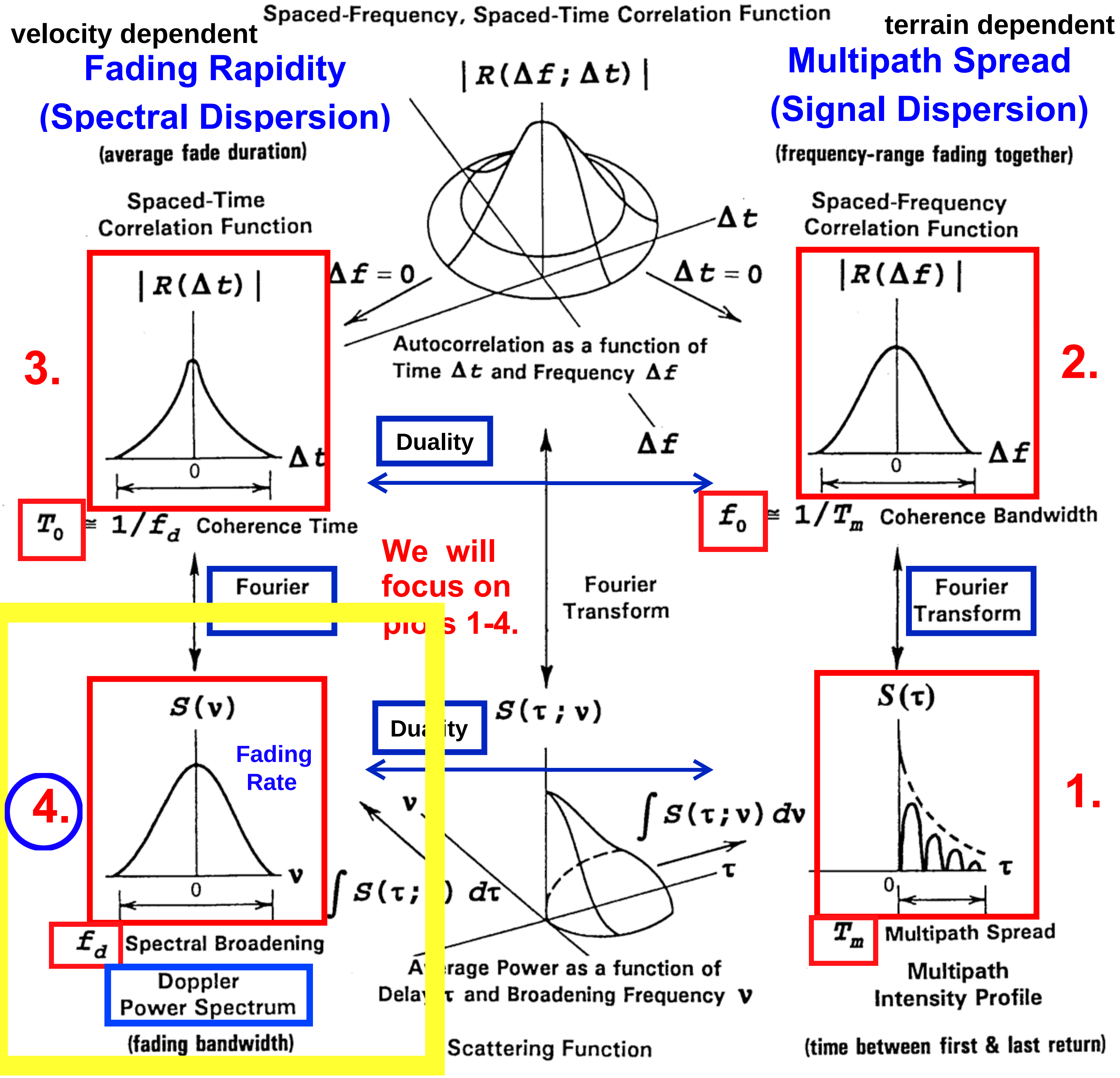
Fading Rate f_d

Relationships Among the Channel Correlation Functions and Power Spectra



Spaced-Frequency, Spaced-Time Correlation Function

$|R(\Delta f;\Delta t)|$

velocity dependent

Fading Rapidity

(Spectral Dispersion)

(average fade duration)

terrain dependent

Multipath Spread

(Signal Dispersion)

(frequency-range fading together)

Spaced-Time Correlation Function

Spaced-Frequency Correlation Function

Autocorrelation as a function of Time Δt and Frequency Δf

3. $|R(\Delta t)|$

$T_0 \cong 1/f_d$ Coherence Time

2. $|R(\Delta f)|$

$f_0 \cong 1/T_m$ Coherence Bandwidth

Duality

Fourier

Fourier Transform

Fourier Transform

We will focus on plots 1-4.

4. $S(\nu)$

Fading Rate

f_d Spectral Broadening

Doppler Power Spectrum

(fading bandwidth)

$S(\tau;\nu)$

Duality

$\int S(\tau;\nu)\,d\tau$

$\int S(\tau;\nu)\,d\nu$

Average Power as a function of Delay τ and Broadening Frequency ν

Scattering Function

1. $S(\tau)$

T_m Multipath Spread

Multipath Intensity Profile

(time between first & last return)

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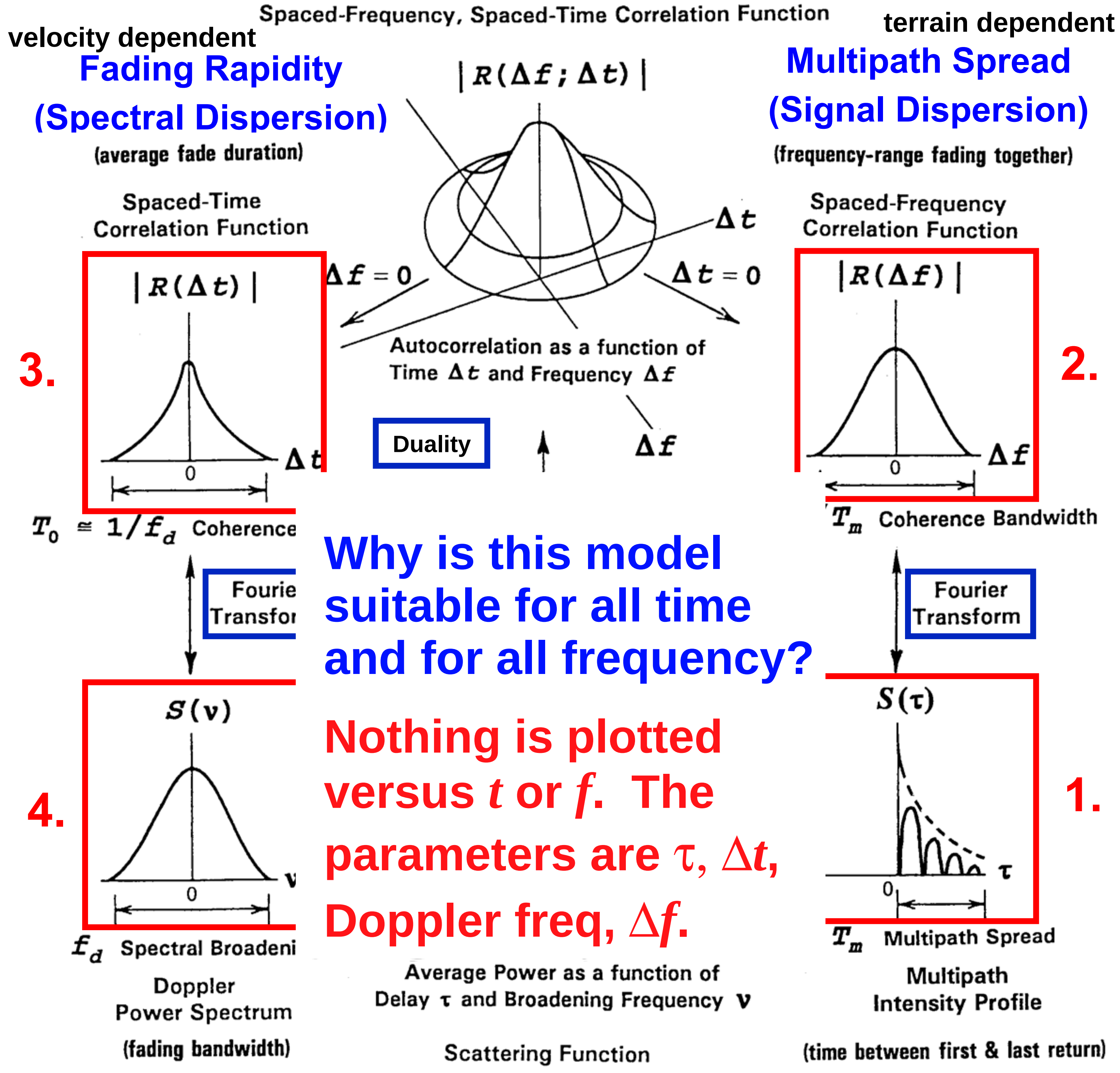
Portrayal of the four key WSSUS functions by Paul Green:

Relationships Among the Channel Correlation Functions and Power Spectra



Spaced-Frequency, Spaced-Time Correlation Function

Autocorrelation as a function of Time Δt and Frequency Δf

velocity dependent

Fading Rapidity

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Spaced-Time Correlation Function

$T_0 \cong 1/f_d$ Coherence

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f_d Spectral Broadeni

Doppler Power Spectrum

(fading bandwidth)

terrain dependent

Multipath Spread

(Signal Dispersion)

(frequency-range fading together)

Spaced-Frequency Correlation Function

T_m Coherence Bandwidth

Fourier Transform

T_m Multipath Spread

Multipath Intensity Profile

(time between first & last return)

Duality

Why is this model suitable for all time and for all frequency?

Nothing is plotted versus *t* or *f*. The parameters are τ, Δt, Doppler freq, Δf.

Average Power as a function of Delay τ and Broadening Frequency ν

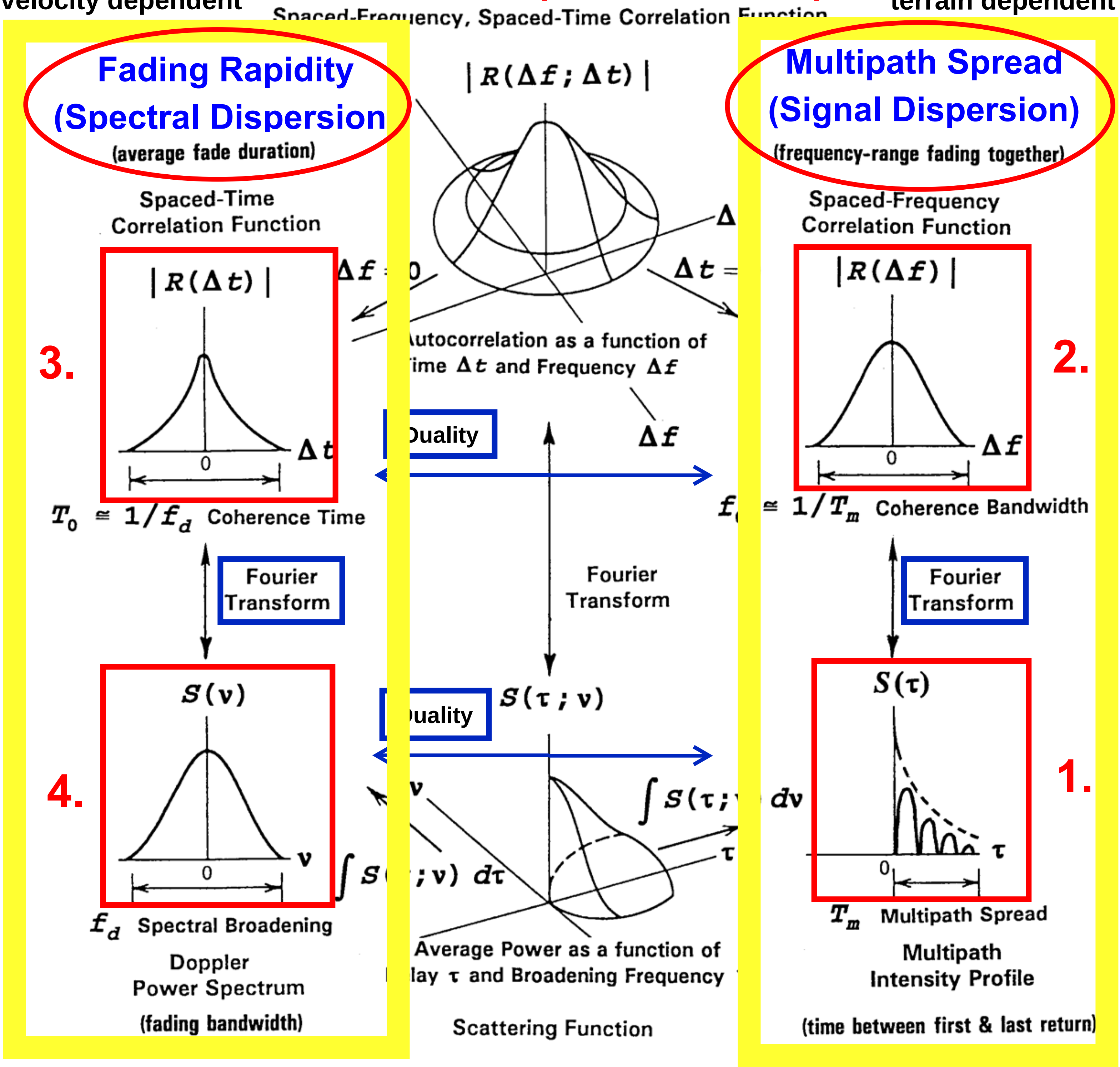
Scattering Function

REF: PROAKIS, J.G., DIGITAL COMMUNICATIONS, McGRAW-HILL BOOK COMPANY, NEW YORK, 1983, Green, P. E., Jr., "Radar Astronomy Measurement Techniques ," MIT Lincoln Laboratory, Tech Report #282, December 1962.

Portrayal of the Four Key WSSUS functions:

Relationships Among the Channel Correlation Functions and Power Spectra

The BIG Picture (Paul Green 1963)



velocity dependent

terrain dependent

Spaced-Frequency, Spaced-Time Correlation Function

Fading Rapidity

(Spectral Dispersion

(average fade duration)

Spaced-Time
Correlation Function

3.

$T_0 \cong 1/f_d$ Coherence Time

Fourier Transform

4.

f_d Spectral Broadening

Doppler
Power Spectrum

(fading bandwidth)

Multipath Spread

(Signal Dispersion)

(frequency-range fading together)

Spaced-Frequency
Correlation Function

2.

$f_c \cong 1/T_m$ Coherence Bandwidth

Fourier Transform

1.

T_m Multipath Spread

Multipath
Intensity Profile

(time between first & last return)

utocorrelation as a function of
ime Δt and Frequency Δf

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Fourier
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Average Power as a function of
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velocity dependent

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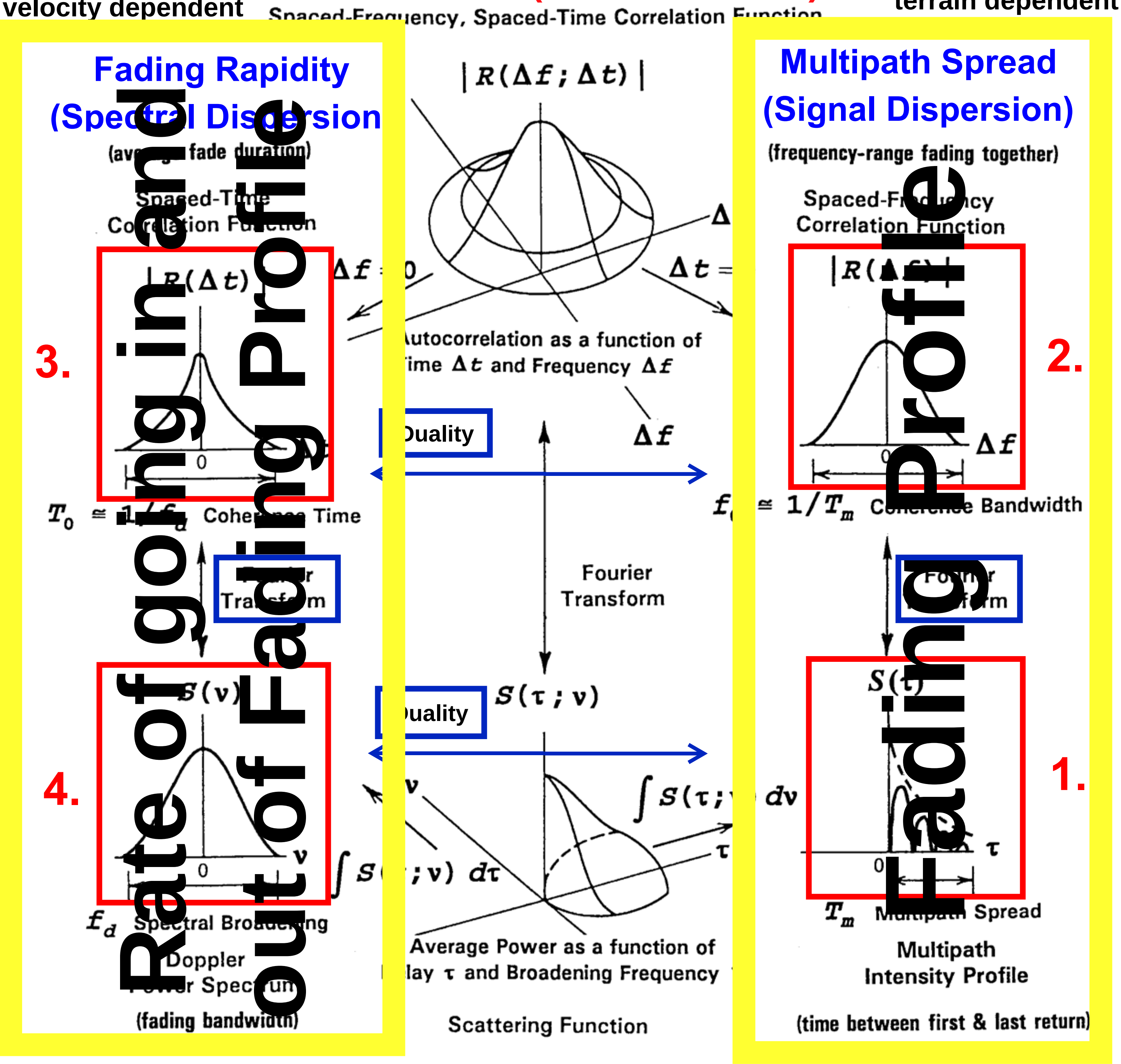

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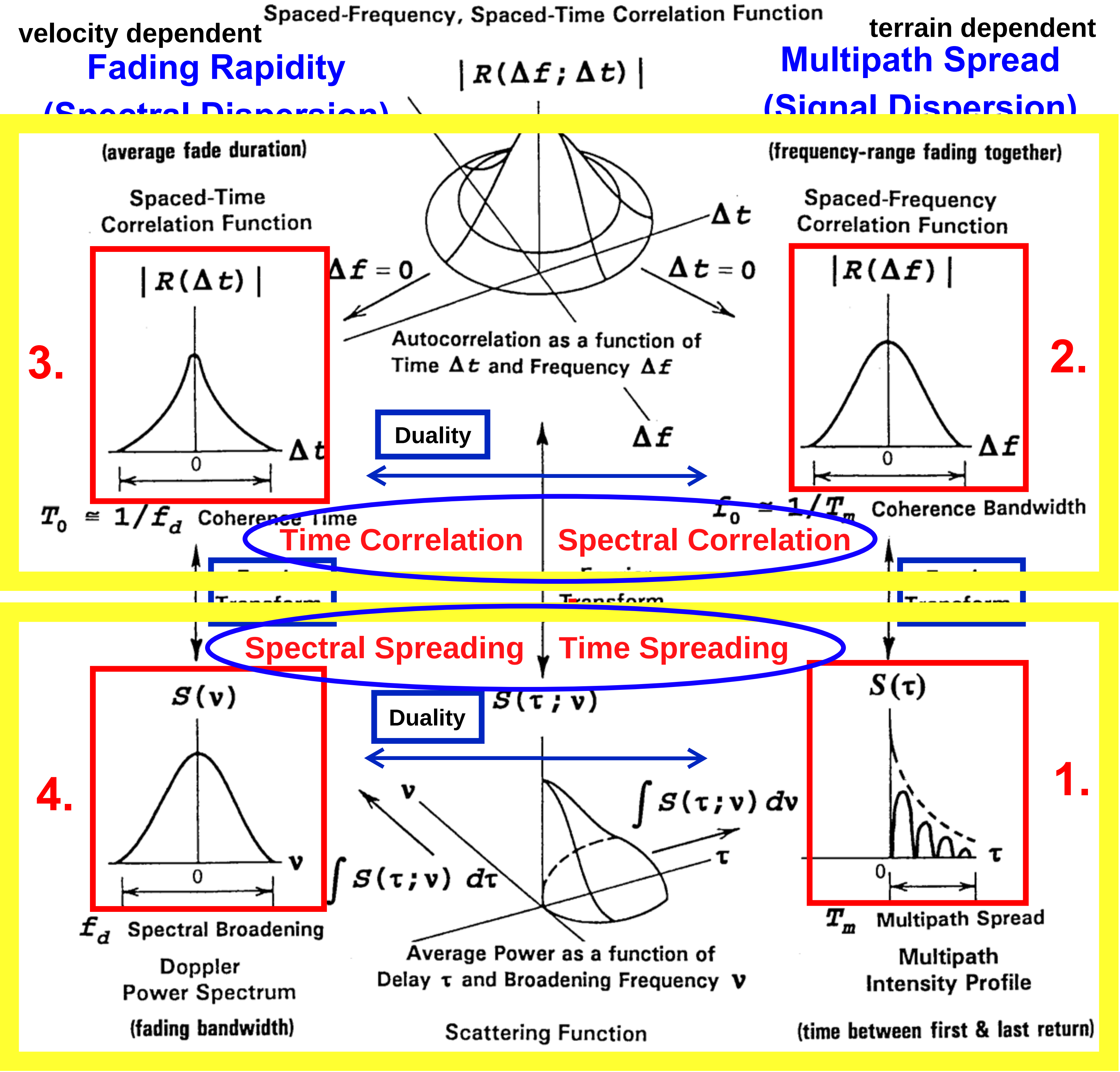


REF: PROAKIS, J.G., DIGITAL COMMUNICATIONS, McGRAW-HILL BOOK COMPANY, NEW YORK, 1983, Green, P. E., Jr., "Radar Astronomy Measurement Techniques ," MIT Lincoln Laboratory, Tech Report #282, December 1962.

Multipath Spread (Signal Dispersion)

2. Spaced-Frequency Correlation Function shows the spectral correlation of received narrow-band signals spaced Δf apart.

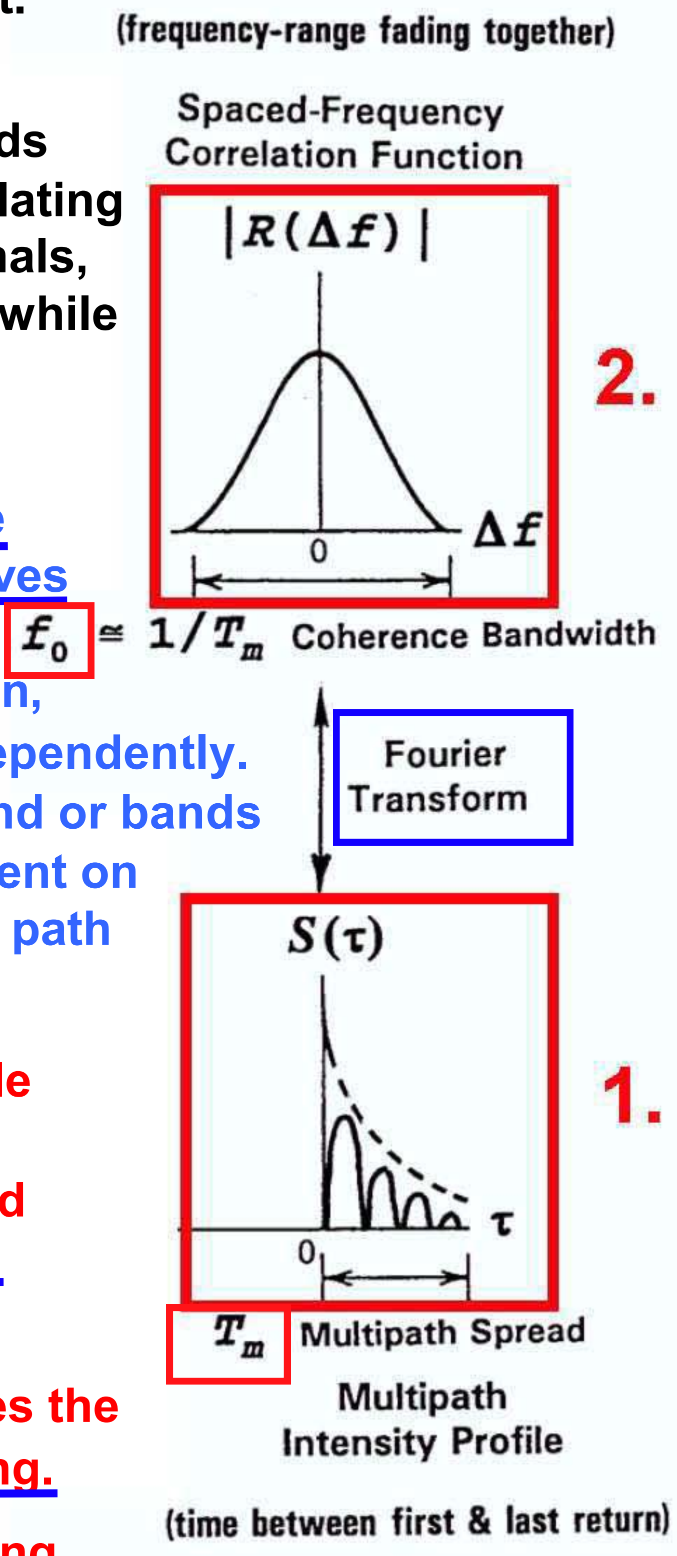
It can be measured by transmitting a pair of sinusoids separated by Δf, cross-correlating their separately received signals, and repeating multiple times while increasing Δf.

Coherence Bandwidth represents the spectral range over which the channel behaves coherently (fading or not fading). Outside of this region, signals will behave quite independently. The positioning of such a band or bands is a random process, dependent on the nature of the propagation path (the terrain). *

1. The Multipath Intensity Profile shows a signal's received average power (main lobe and echoes) as a function of time delay. *

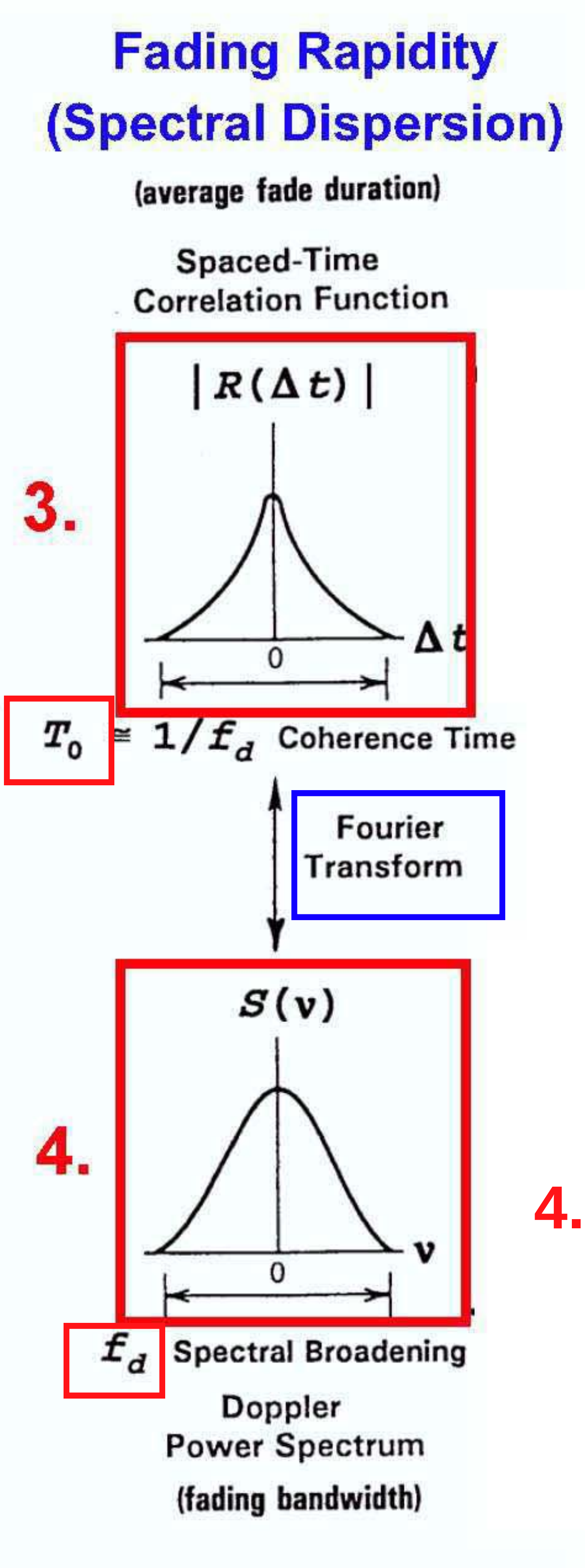
Multipath spread T_m indicates the maximum such time spreading.

σ_τ indicates the rms spreading.



Terrain Dependent

Velocity Dependent



3. **Spaced-Time Correlation Function shows the time correlation of received narrow-band signals spaced Δt apart. It can be measured by transmitting a pair of sinusoids separated by Δt, cross-correlating their separately received signals, and repeating multiple times while increasing Δt.**

Coherence Time represents the time duration during which the channel behaves coherently (fading or not fading). Outside of this duration, signals will behave quite independently. The positioning of such a band or bands is a random time-variant process due to motion (spatial changes). *

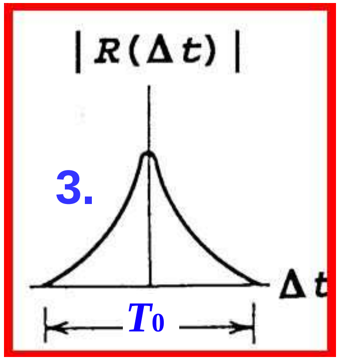
4. **Doppler Power Spectrum (received signal’s intensity as a function of Doppler frequency) shows spectral spreading as a function of speed of channel state changes (fading rapidity). It is also termed Doppler Spreading, or Fading BW, or Fading Rate.** *

The Effects of a Multipath Channel on a Received Signal

Note that there are no "Good" operating regions.
The possibilities are either "Awful" or merely "Bad."

Summary of the Correlation and Power Spectrum Relationships in the context of Fading Behaviors: Frequency-Selective Fading, Flat Fading, Fast Fading, and Slow Fading

Fading Rapidity (Spectral Dispersion) **Multipath Spread (Signal Dispersion)**



Spaced Time Correlation Function

Fast Fading $T_0 < T_s$

Awful ↑

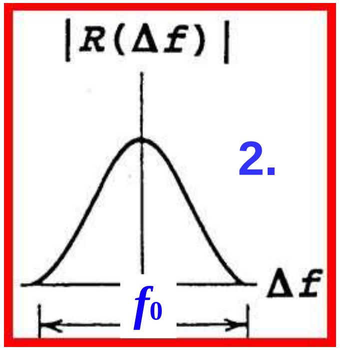
Time Domain — **Boundary** — Frequency Domain

↓ Bad

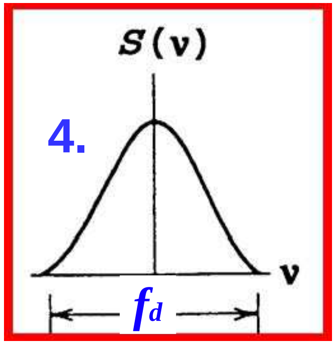
Slow Fading $T_0 > T_s$

Frequency-Selective Fading $f_0 < W$

Flat Fading $f_0 > W$



Spaced Frequency Correlation Function



Fading Bandwidth

Fast Fading $f_d > W$

Awful ↑

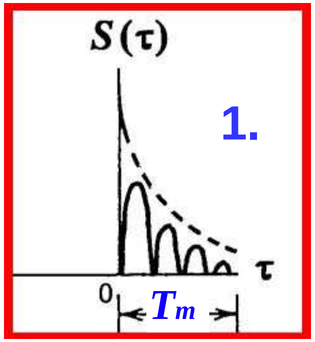
Doppler-Shift Domain — **Boundary** — Time-Delay Domain

↓ Bad

Slow Fading $f_d < W$

Frequency-Selective Fading $T_m > T_s$

Flat Fading $T_m < T_s$



Multipath Spread

Velocity Dependent

signal BW

signal time interval

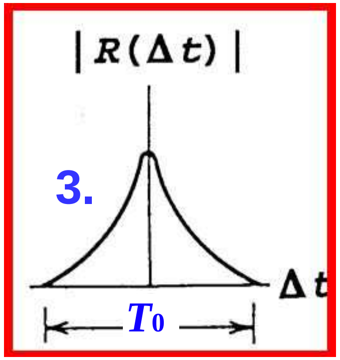
Terrain Dependent

Summary of the Correlation and Power Spectrum Relationships in the context of Fading Behaviors: Frequency-Selective Fading, Flat Fading, Fast Fading, and Slow Fading

Fading Rapidity (Spectral Dispersion)

Multipath Spread (Signal Dispersion)

terrain dependent



Spaced Time Correlation Function

Fast Fading $T_0 < T_s$

Awful

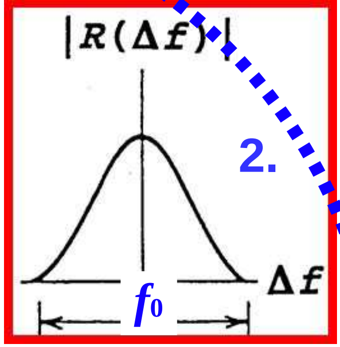
Frequency-Selective Fading $f_0 < W$

Time Domain — **Boundary** — Frequency Domain

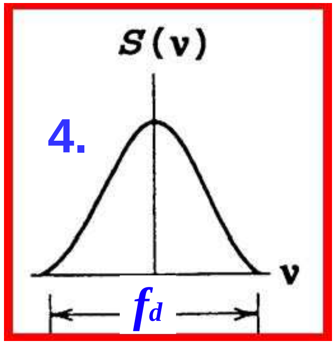
Slow Fading $T_0 > T_s$

Bad

Flat Fading $f_0 > W$



Spaced Frequency Correlation Function



Fading Bandwidth

Fast Fading $f_d > W$

Awful

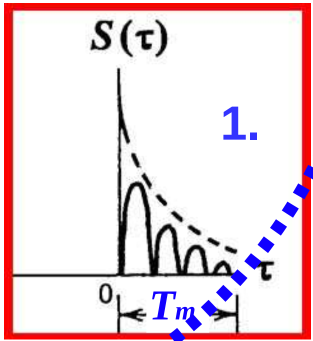
Frequency-Selective Fading $T_m > T_s$

Doppler-Shift Domain — **Boundary** — Time-Delay Domain

Slow Fading $f_d < W$

Bad

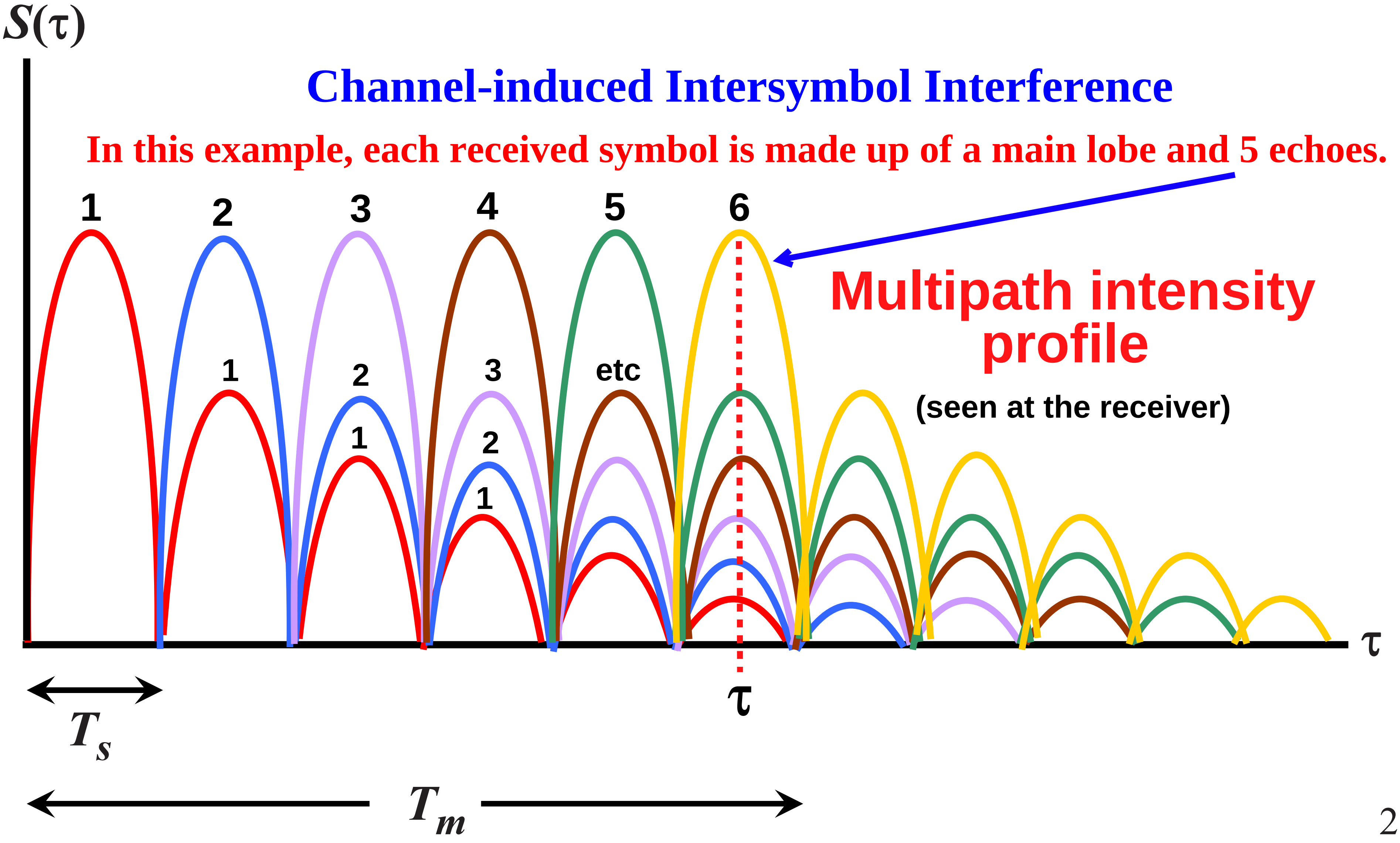
Flat Fading $T_m < T_s$



Multipath Spread

Received Power as a function of Delay Time

Example of Frequency-Selective Fading ($T_m > T_s$)



Channel-induced Intersymbol Interference

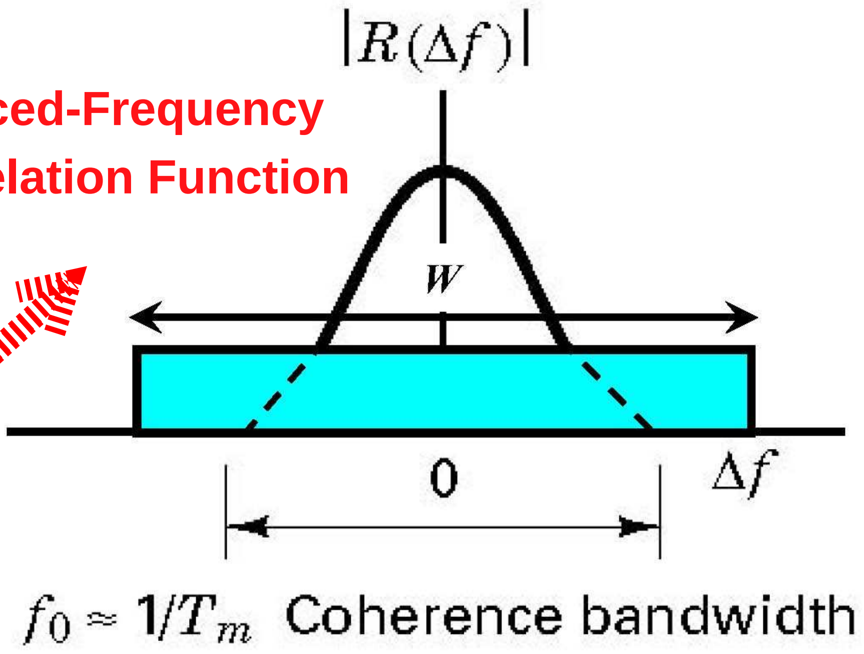
In this example, each received symbol is made up of a main lobe and 5 echoes.

Multipath intensity profile

(seen at the receiver)

Frequency-Selective Fading

Spaced-Frequency Correlation Function



$f_0 \approx 1/T_m$ Coherence bandwidth

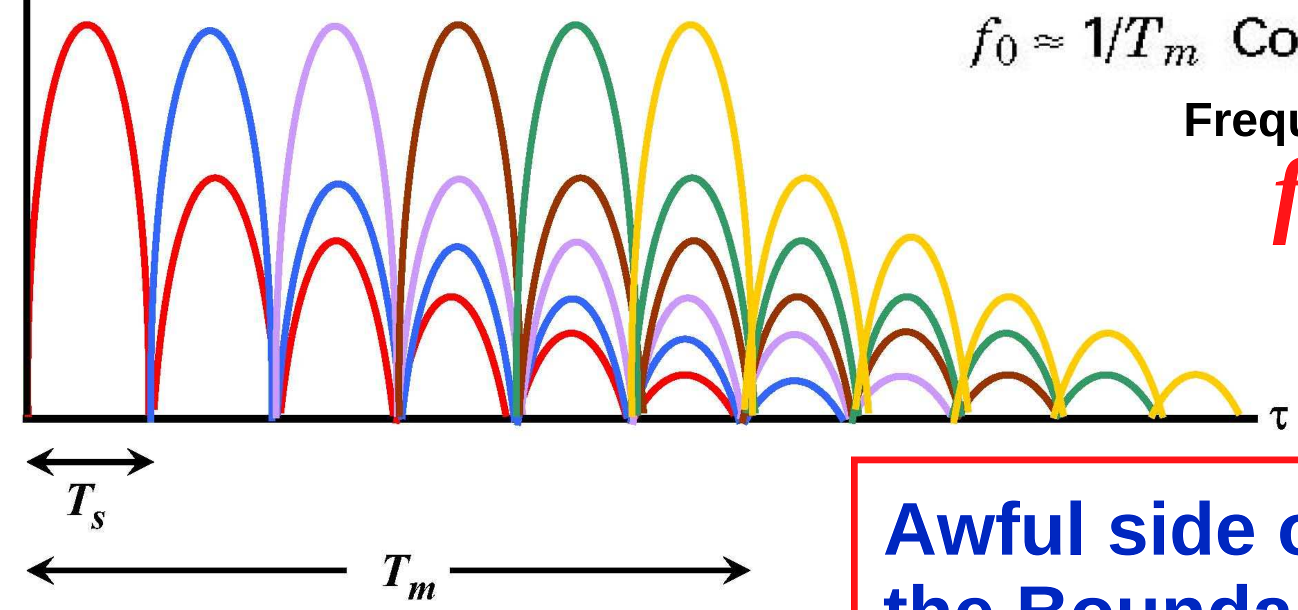
Frequency Domain

$$f_0 < W$$

Fourier Transform pair

Multipath Intensity Profile

$$T_m > T_s$$

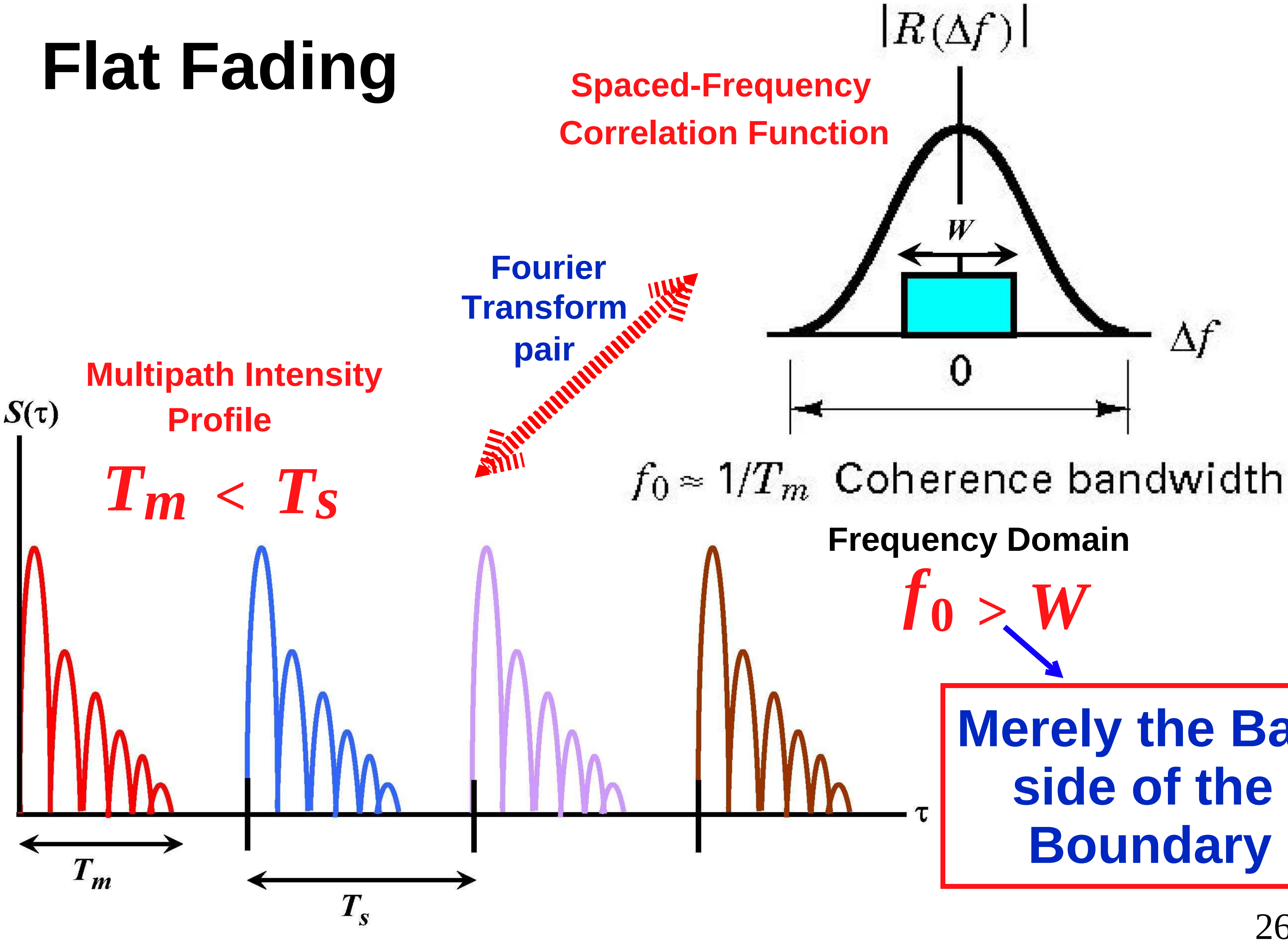


Time-Delay Domain

Awful side of the Boundary

Note that the name Frequency-Selective Fading makes sense as applied to the frequency domain.

Flat Fading



Spaced-Frequency Correlation Function

Fourier Transform pair

Multipath Intensity Profile

$$T_m < T_s$$

$f_0 \approx 1/T_m$ Coherence bandwidth

Frequency Domain

$$f_0 > W$$

Merely the Bad side of the Boundary

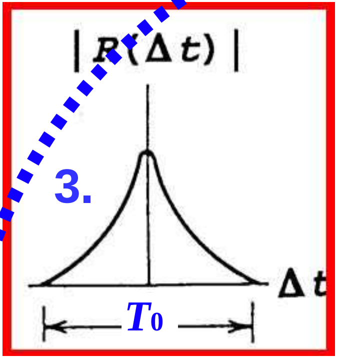
Time-Delay Domain

Summary of the Correlation and Power Spectrum Relationships in the context of Fading Behaviors: Frequency-Selective Fading, Flat Fading, Fast Fading, and Slow Fading

Fading Rapidity (Spectral Dispersion)

velocity dependent

Multipath Spread (Signal Dispersion)



Spaced Time Correlation Function

Fast Fading $T_0 < T_s$

Awful

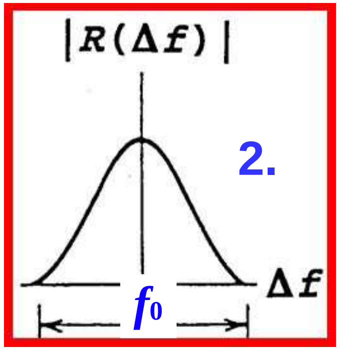
Time Domain — Boundary — Frequency Domain

Bad

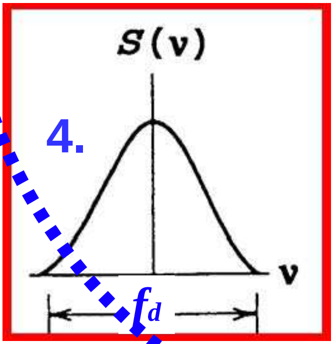
Slow Fading $T_0 > T_s$

Frequency-Selective Fading $f_0 < W$

Flat Fading $f_0 > W$



Spaced Frequency Correlation Function



Fading Bandwidth

Fast Fading $f_d > W$

Awful

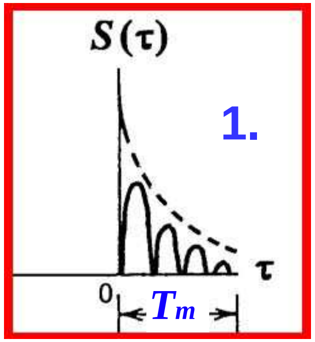
Doppler-Shift Domain — Boundary — Time-Delay Domain

Bad

Slow Fading $f_d < W$

Frequency-Selective Fading $T_m > T_s$

Flat Fading $T_m < T_s$



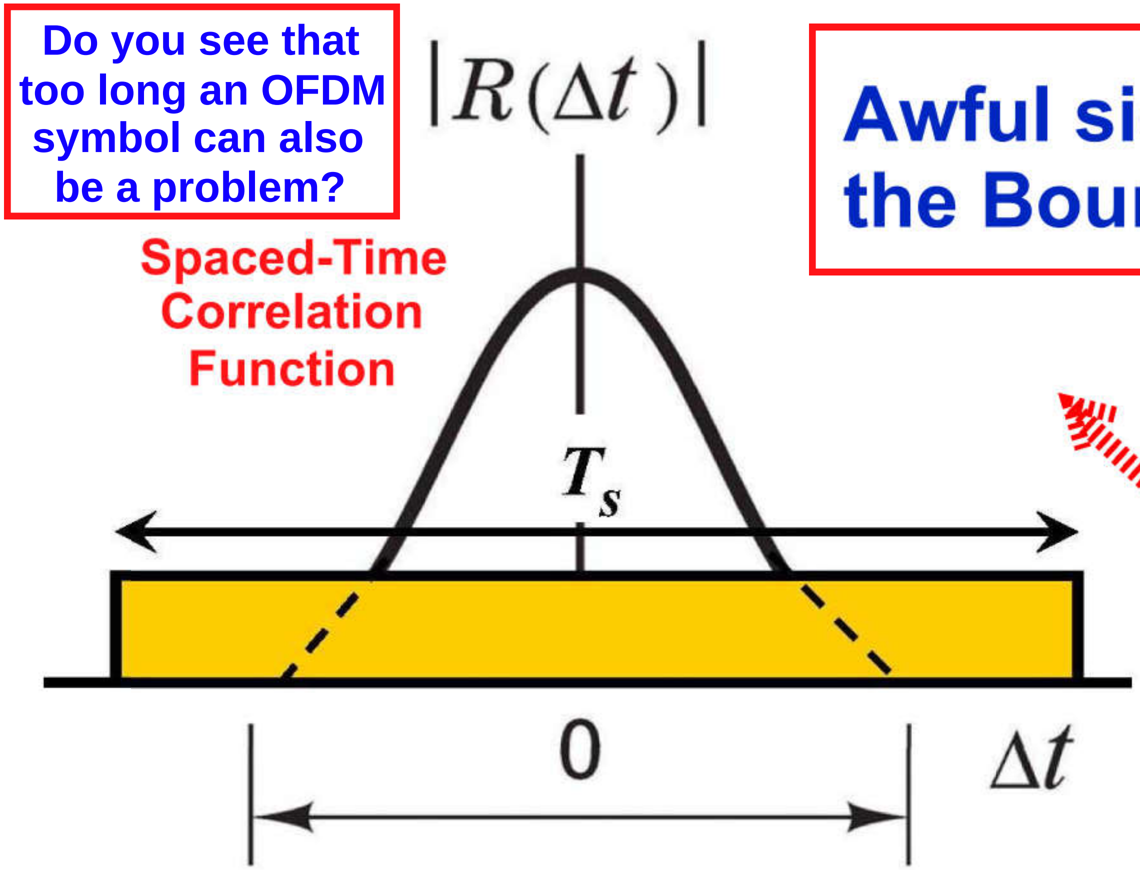
Multipath Spread

Do you see that too long an OFDM symbol can also be a problem?

Awful side of the Boundary

Fast Fading

Spaced-Time Correlation Function



$$T_0 \approx 1/f_d$$ Coherence Time

Average Fading (or not-fading) Duration

$$T_0 < T_s$$

$$\text{or } f_d > 1/T_s$$

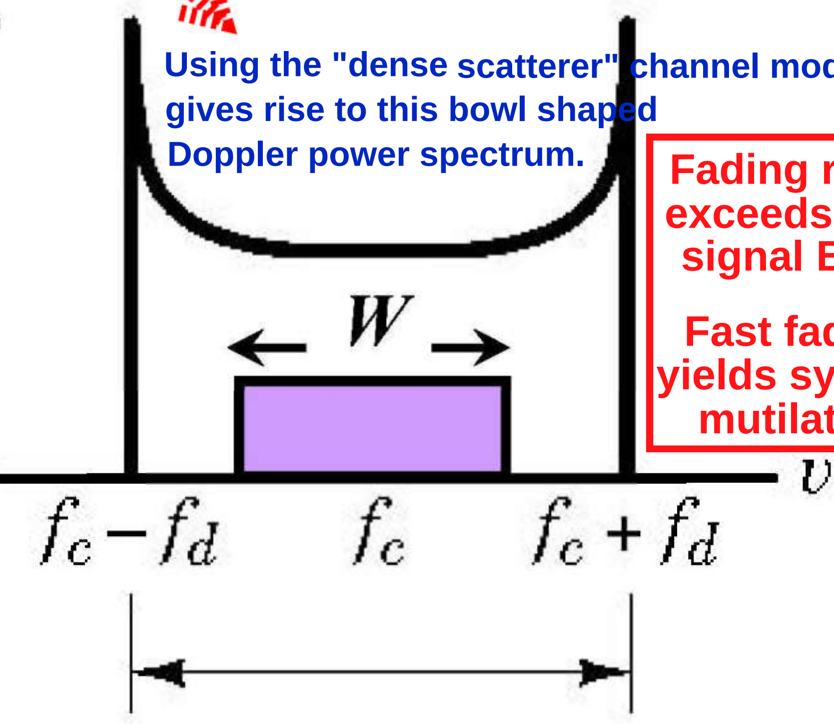
fading BW > signal BW

Fourier Transform pair

Doppler Power Spectrum, or Average Fading Rate

$S(\nu)$ $f_d > W$

Using the "dense scatterer" channel model, gives rise to this bowl shaped Doppler power spectrum.



f_d Spectral broadening

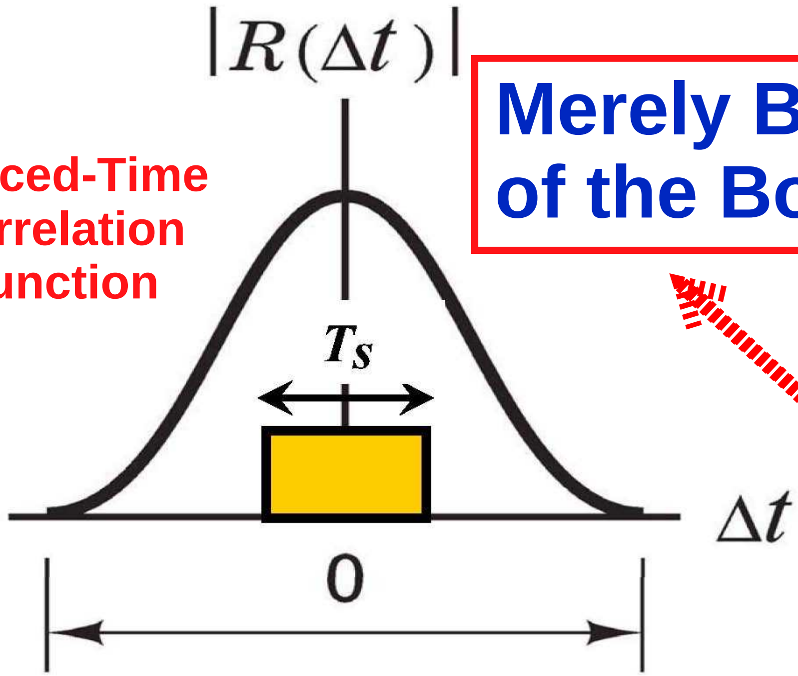
Fading rate exceeds the signal BW.

Fast fading yields symbol mutilation.

Slow Fading

Merely Bad side of the Boundary

Spaced-Time Correlation Function

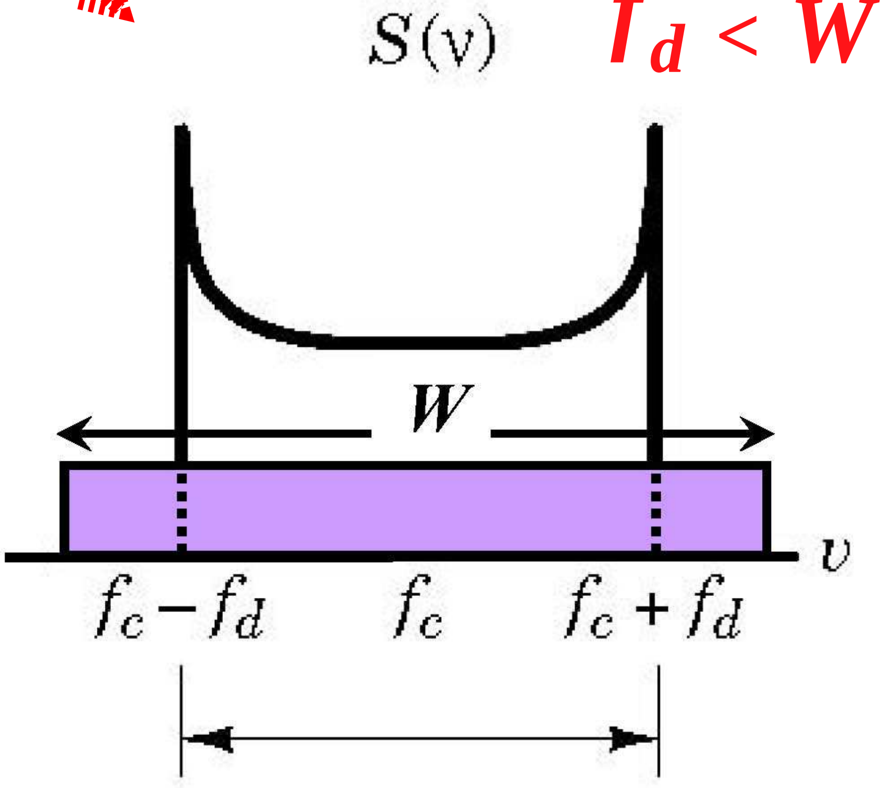


$T_0 \approx 1/f_d$ Coherence Time

Fourier Transform pair

Doppler Power Spectrum

$$f_d < W$$



f_d Spectral broadening

$$T_0 > T_s$$

or $f_d < 1/T_s$

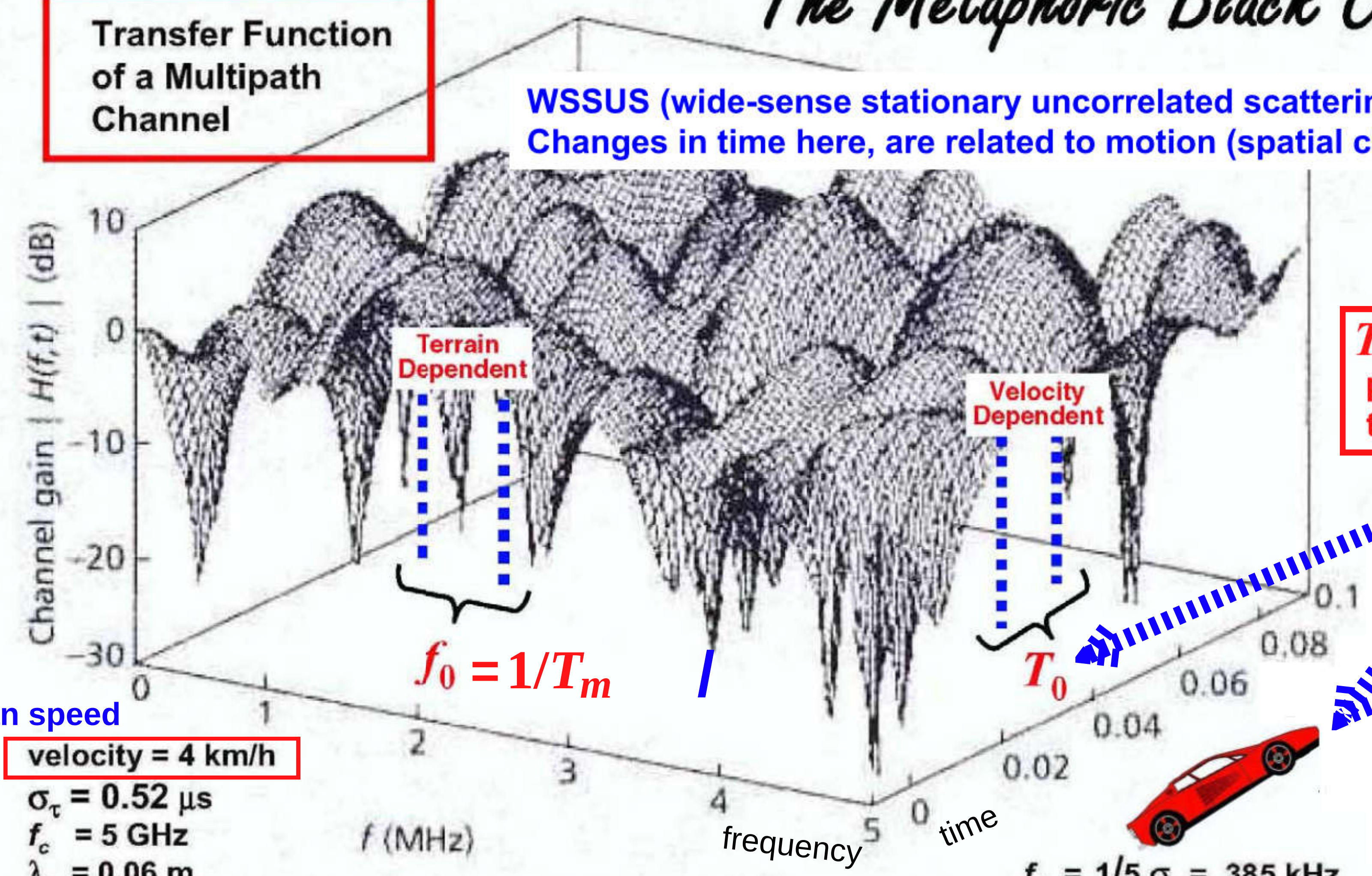
signal BW > fading BW

Channel gain versus the dual functions f_0 and T_0

The Metaphoric Black Cloud

Transfer Function of a Multipath Channel

WSSUS (wide-sense stationary uncorrelated scattering) model.
Changes in time here, are related to motion (spatial changes).



T_0 is inversely proportional to velocity.

pedestrian speed

velocity = 4 km/h
$\sigma_\tau = 0.52\ \mu s$
$f_c = 5$ GHz
$\lambda = 0.06$ m

Ref: F. Adachi, et. al., "Broadband CDMA Techniques," *IEEE Wireless Communications*, vol. 12, no. 2, April 2005. pp. 8-18.

$f_0 = 1/5\,\sigma_\tau = 385$ kHz
$T_0 = 0.5/f_d = 0.5\,\lambda/v$
$= 0.027$ s

f_d = velocity/λ

Multipath spread & Coherence BW are functions of terrain.
Fading rapidity & Coherence time are functions of velocity.

Channel gain versus the dual functions f_0 and T_0

The Metaphoric Black Cloud

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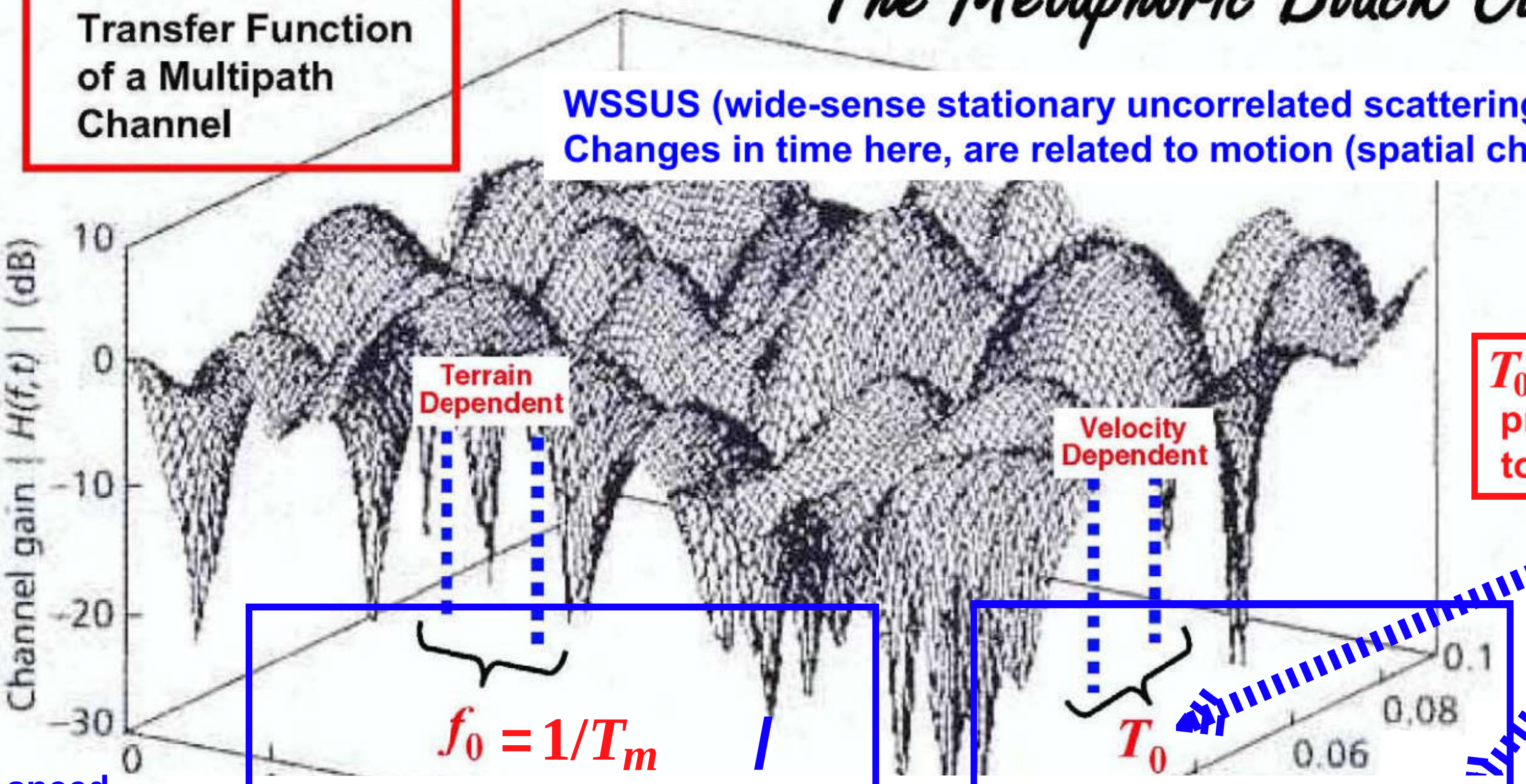

Terrain Dependent

Velocity Dependent

T_0 is inversely proportional to velocity.

$f_0 = 1/T_m$

Range of channel spectral consistency based on the terrain

T_0

Range of channel time consistency based on vehicle velocity

pedestrian speed
velocity =
σ_τ = 0.52
f_c = 5 GH
λ = 0.06 m

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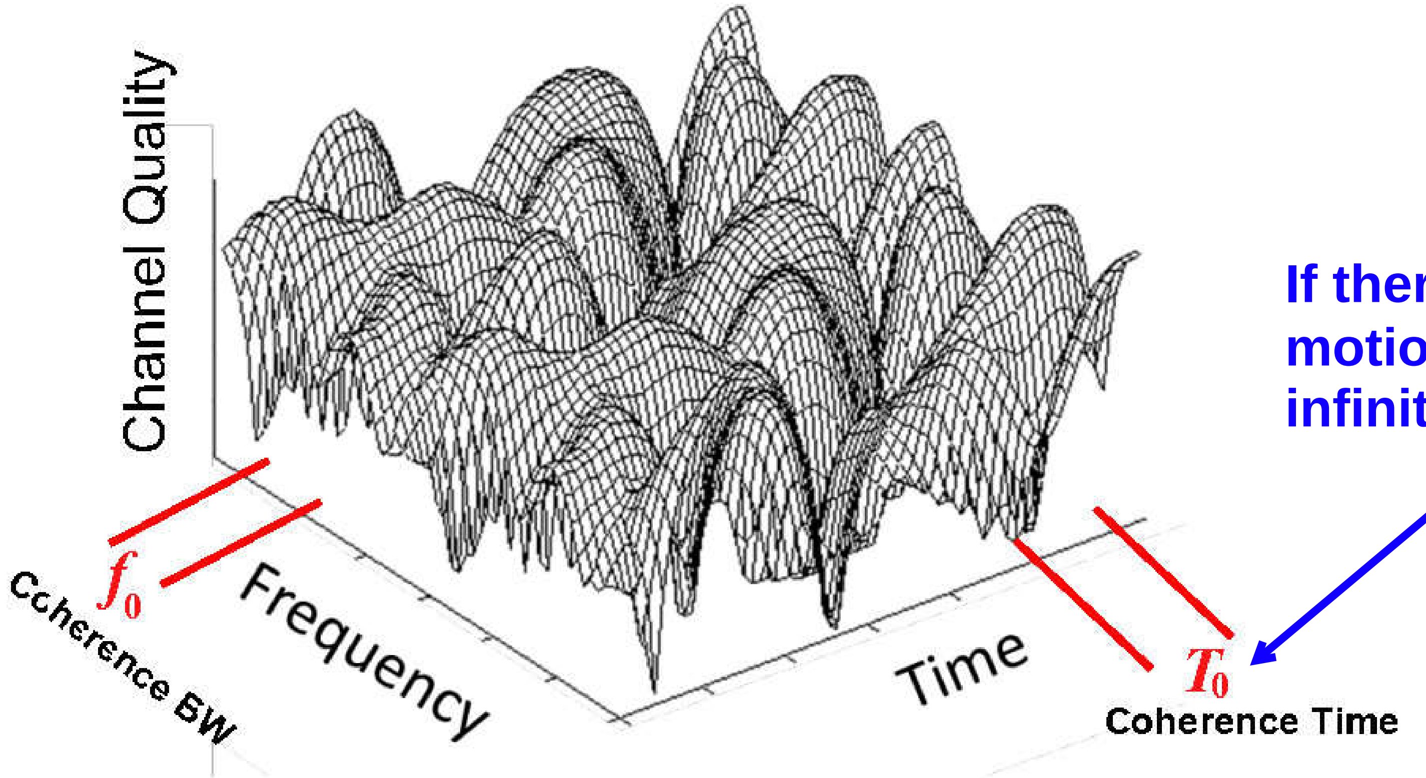
f_d = velocity/λ
$= 1/T_0$

Multipath spread & Coherence BW are functions of terrain.
Fading rapidity & Coherence time are functions of velocity.

Reminder about Coherence BW and Coherence Time



If there is no motion, T_0 = infinity.

Channel frequency responses are stable.

Coherence BW f_0 represents a "consistent" spectral region where the channel behaves coherently (fading or not fading). f_0 is terrain dependent.

Channel time responses are stable.

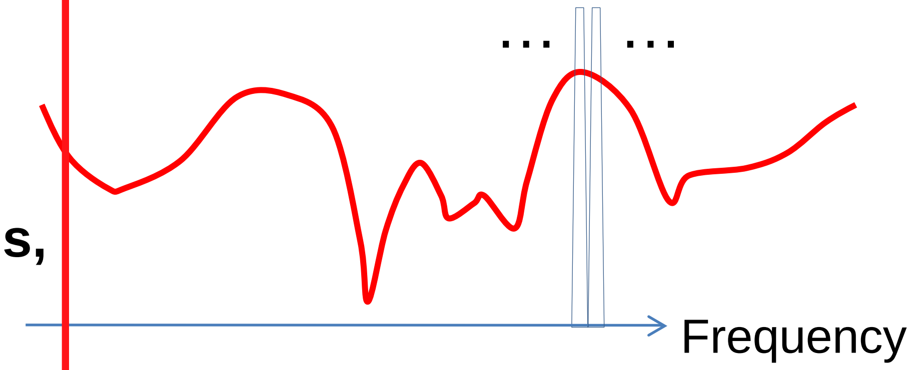
Coherence Time T_0 represents a "consistent" time duration during which the channel behaves coherently. T_0 is velocity dependent.

* In partitioning the data for an OFDM channel, we desire that $W << f_0$ (to preclude frequency-selective fading). And we desire that $T_S << T_0$ (to preclude fast fading).

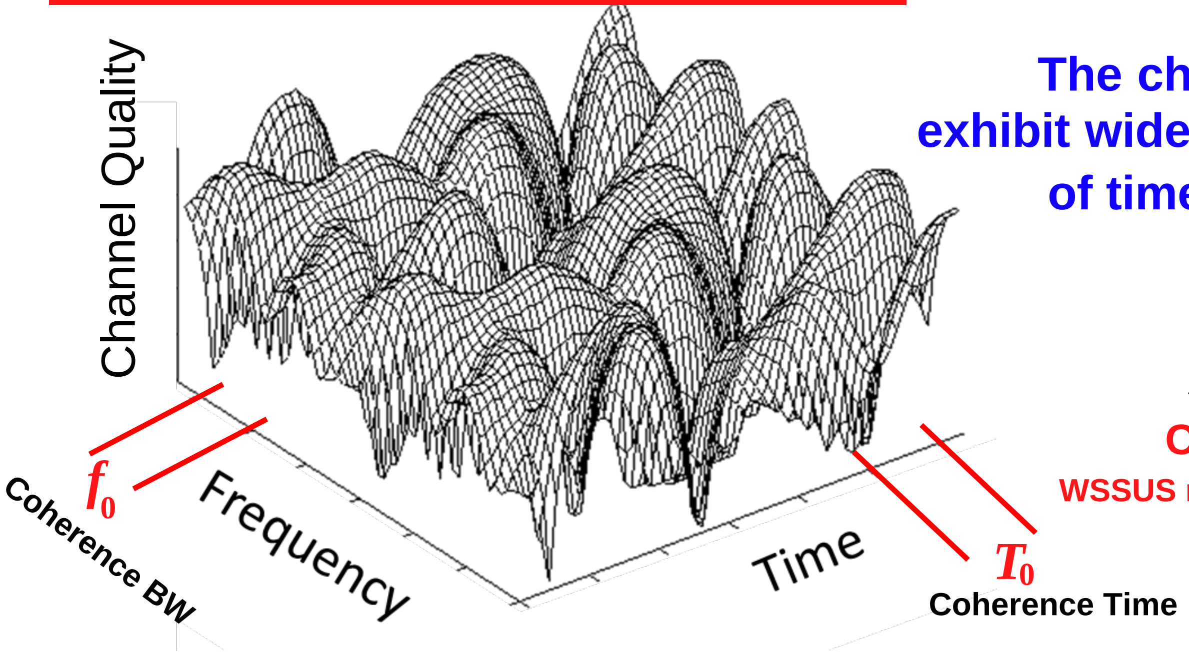
OFDM for Frequency-Selective Fading Channels

Frequency-variable channel appears flat over the narrow band of an OFDM subcarrier.

- OFDM transforms a frequency- and time-variable fading channel into parallel correlated flat-fading channels, eliminating the need for complex equalization.



The channel quality can exhibit wide swings, as a function of time and frequency.



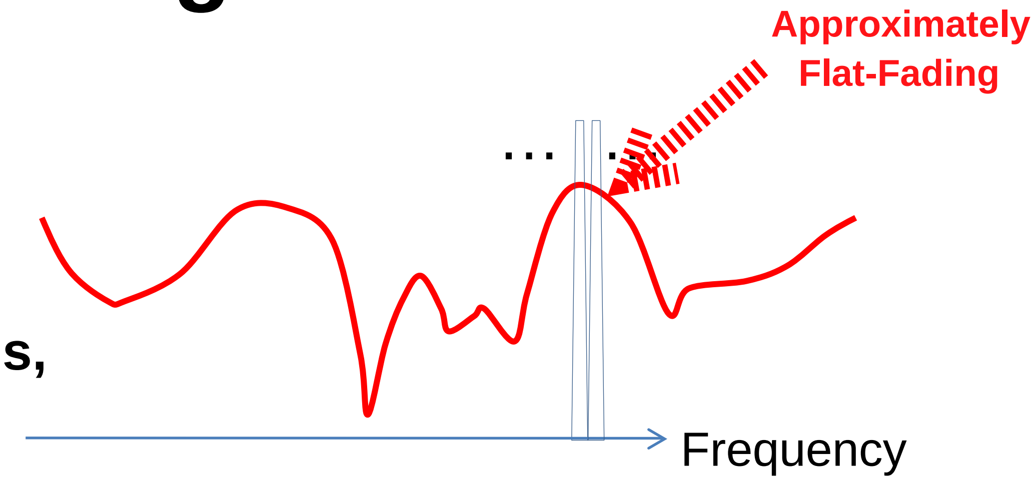
Metaphoric Black Cloud

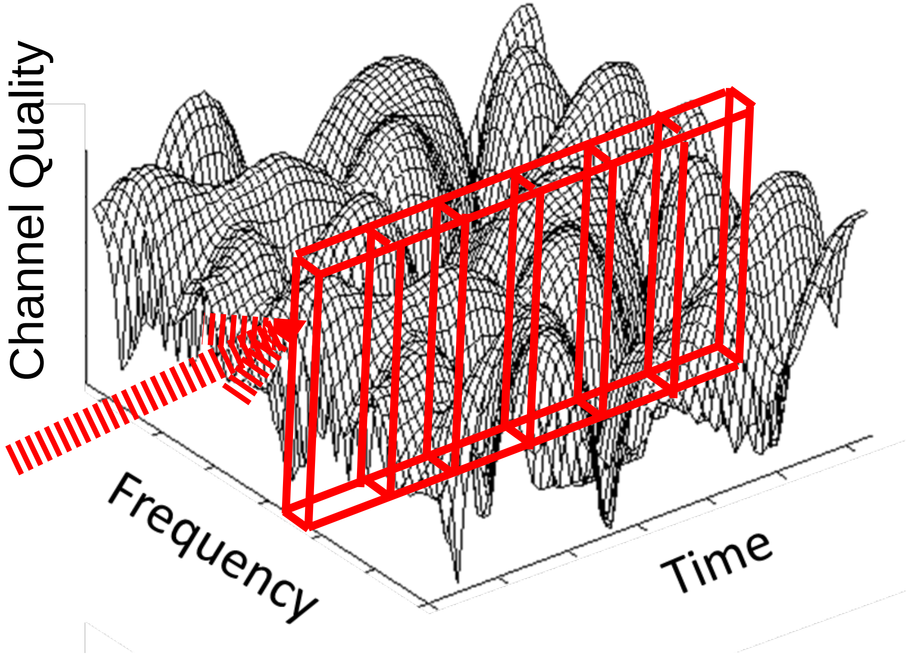
Channel Transfer Function

WSSUS model: Spaced-Time, Spaced Frequency Correlation

OFDM for Frequency-Selective Fading Channels

- **OFDM transforms a frequency- and time-variable fading channel into parallel correlated flat-fading channels, eliminating the need for complex equalization.**





Frequency-variable channel appears flat over the narrow band of an OFDM subcarrier.

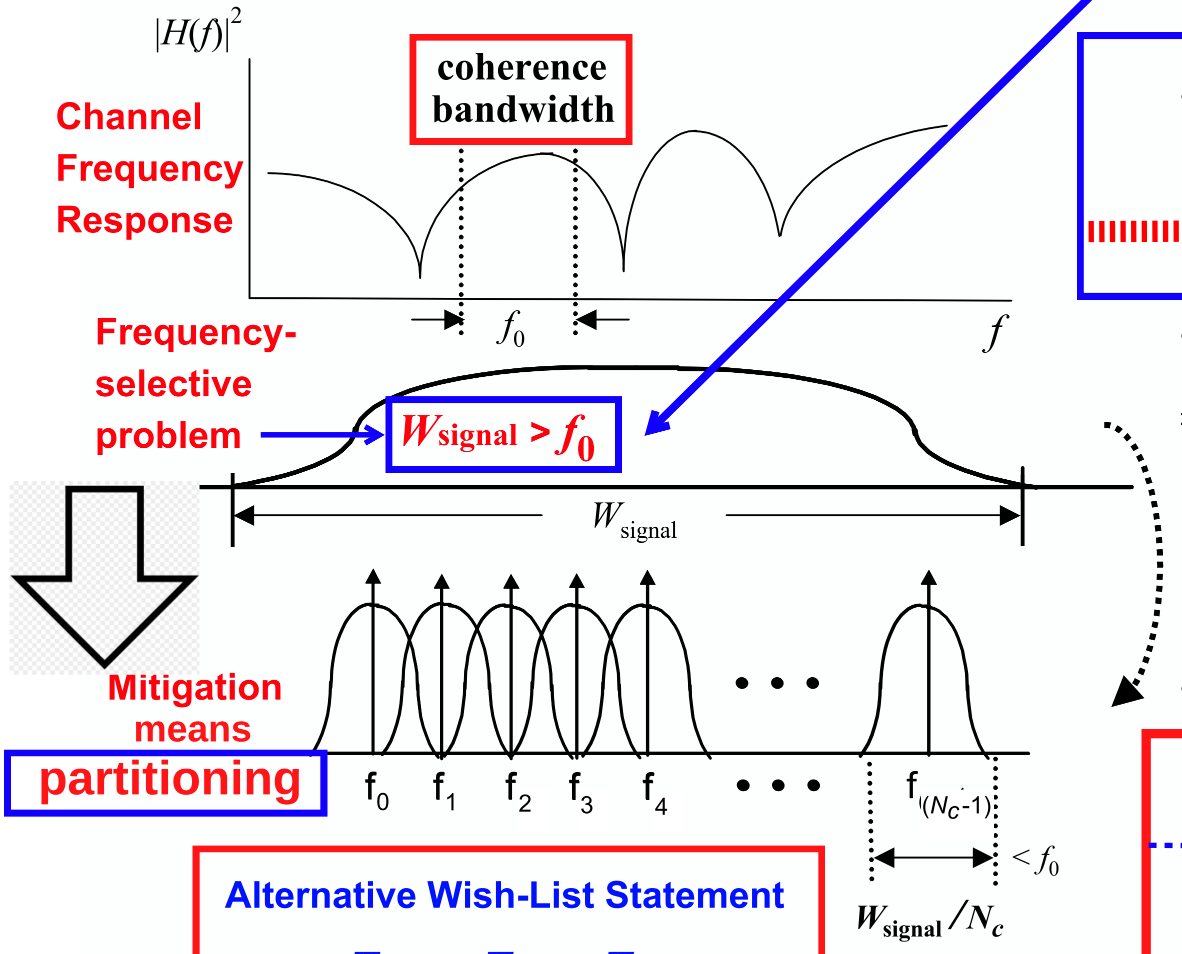
Thanks to Partitioning

Why OFDM?

Why OFDM?

Recall the WSSUS model. We want to avoid frequency-selective fading. Notice our wish-list below. OFDM makes it easy to achieve flat fading.

- Divide-and-Conquer
  - Mitigation for frequency-selective fading environments. Parse the single, high-rate channel into N_c low-rate overlapping, and orthogonal, sub-channels.



partitioning a high-data-rate

- Subdivide W_{signal} by large N_c so that $W_{signal}/N_c << f_0$

  ⇒ We desire flat faded sub-channels

- Large $N_c \Rightarrow$ Large $T_s = N_c/W_{signal}$

$\Rightarrow$ Reduced *relative* ISI when $T_s >> \sigma_\tau$
But, for slow fading, we want
$T_s < T_0$, thus T_0 defines the upper
bound of N_c **Otherwise, pulse mutilation**

- We want that:

	Coherence BW		Symbol rate		Fading Rate
In General	f_0	>	$1/T_s$	>	$1/T_0$
For OFDM	f_0	>	partitioned W_{signal}/N_c	>	$1/T_0$

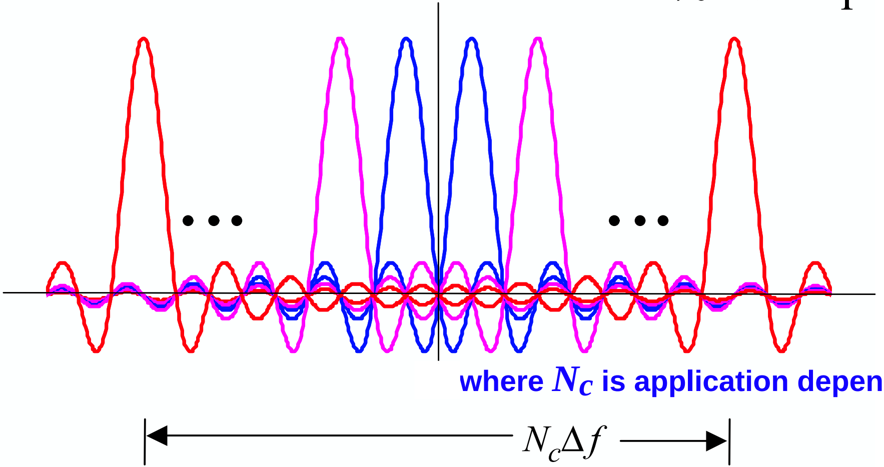
Our "Wish List"

Alternative Wish-List Statement

$$T_m < T_s < T_0$$

where T_s = time duration of the data portion of OFDM symbol

Sketch doesn't show this efficient 50% overlap.



where N_C is application dependent

Approximate transmission bandwidth, W_{signal}

$\boxed{N_c}$ represents the number of potential (candidate) subcarriers, with locations of $k\Delta f$, where k is any positive or negative integer.

- We want that:

Coherence BW **Fading Rate**

In General $f_0 > 1/T_s > 1/T_0$

Symbol Rate

For OFDM $f_0 > W_{\text{signal}}/N_c > 1/T_0$

Our "Wish List"

to preclude frequency-selective & fast fading

where $\boxed{W_{\text{signal}}/N_c = \Delta f} = \dfrac{\text{Total OFDM bandwidth}}{\text{number of candidate subcarriers}}$

subchannel BW

Reminder that our Wish List has 2 Inequalities:

The left-hand side wants the OFDM symbol rate to be less than the channel coherence BW to preclude frequency-selective fading.

The right-hand side wants the OFDM symbol rate to be larger than the channel fading rate to preclude fast fading (symbol mutilation).

$\boxed{N_c}$ represents the number of potential (candidate) subcarriers, with locations of $k\Delta f$, where k is any positive or negative integer.

- We want that:

Coherence BW **Fading Rate**

In General $f_0 > 1/T_s > 1/T_0$

Symbol Rate

For OFDM $f_0 > W_{\text{signal}}/N_c > 1/T_0$

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Coherence BW　　　　Fading Rate

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where $\boxed{W_{signal}/N_c = \Delta f} = \dfrac{\text{Total OFDM bandwidth}}{\text{number of candidate subcarriers}}$

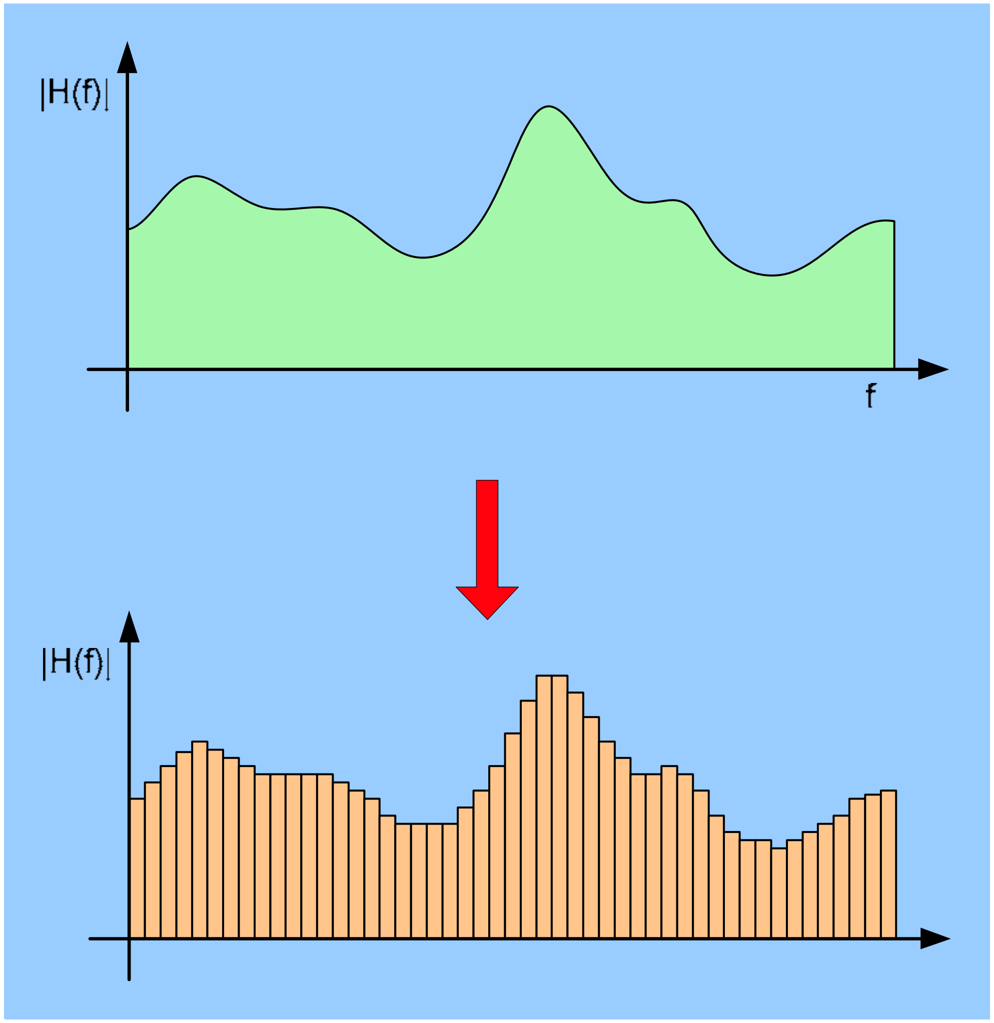
subchannel BW

Channel Frequency Response

OFDM channels fit nicely into our assumed MIMO channel model, narrowband flat-fading channels.

Received phasors can be described with complex-valued gain factors.

Equalization of such channels is accomplished with a simple scaling (in the frequency domain).



A broadband channel divided into many parallel narrowband channels

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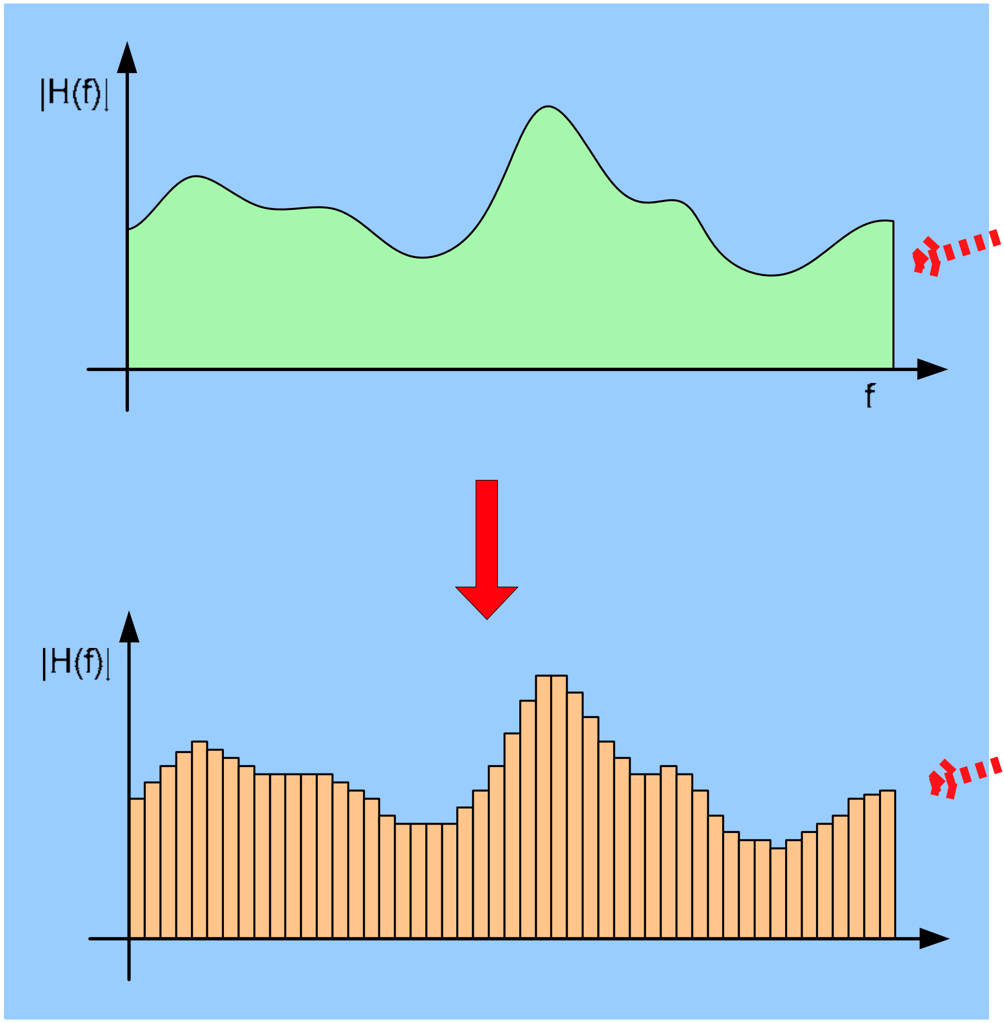

Equalization (inversion of the channel) is difficult here.

OFDM's Major Benefit: By partitioning a high-rate signal into many low-rate subchannels, Equalization is made Easy (simple scaling in the frequency domain).

A broadband channel divided into many parallel narrowband channels

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The ABCs of OFDM
By Dr. Bernard Sklar

Part 1 March 18, 2021
Part 2 March 25, 2021

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- Tricking the Channel.
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- Single-Carrier OFDM (SC-OFDM).

OFDM Implementation Examples

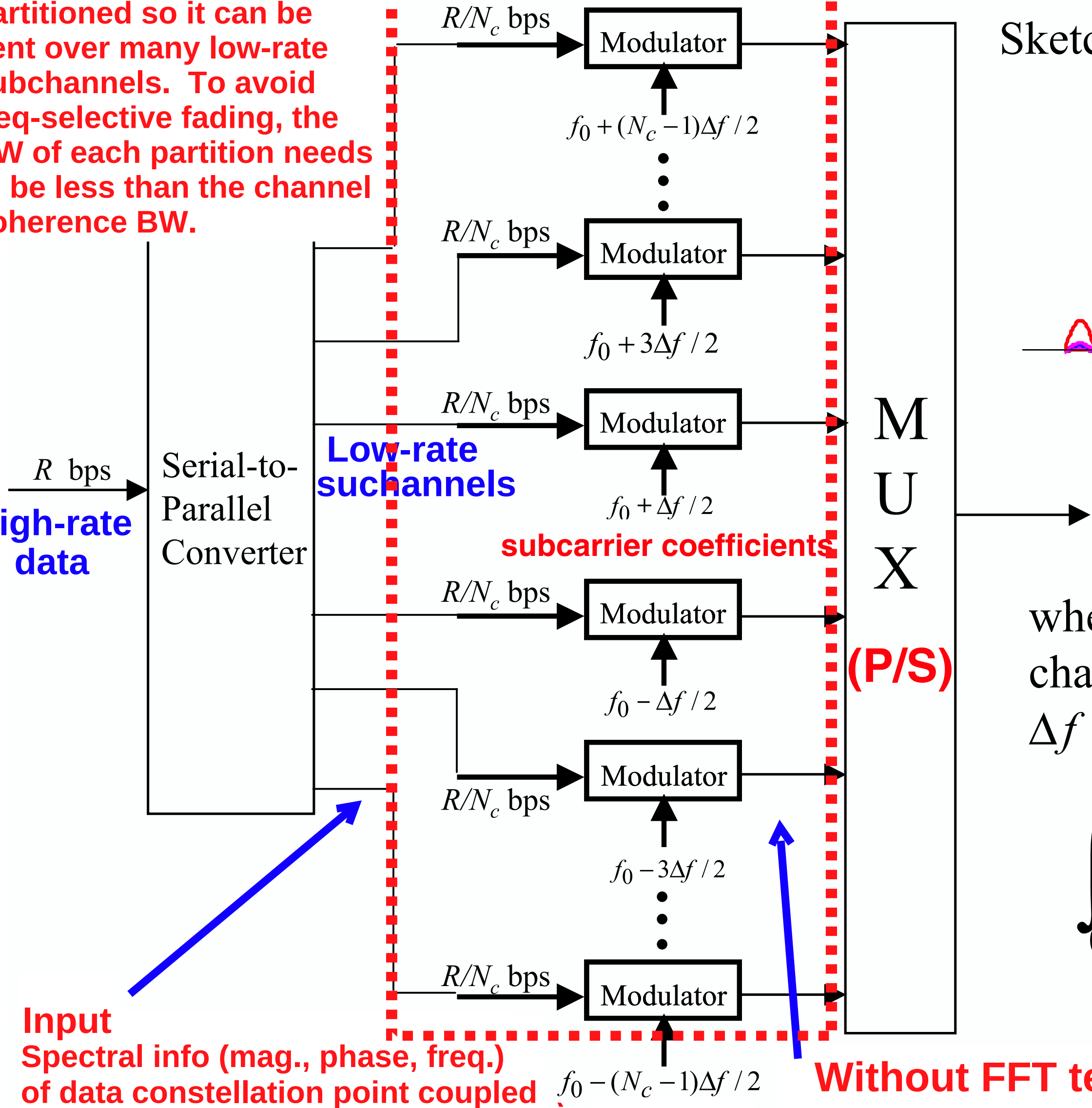
$\Delta f = 1/T_s$

where T_s is the data portion of the OFDM symbol time

What is OFDM?

OFDM is a multi-carrier transmission system, with orthogonal subcarriers that are spaced $\Delta f = (T_s)^{-1}$ Hz apart. Thus the individual spectra overlap each neighbor 50% - Improved BW efficiency.

A high-rate signal is partitioned so it can be sent over many low-rate subchannels. To avoid freq-selective fading, the BW of each partition needs to be less than the channel coherence BW.



High-rate data

Low-rate subchannels

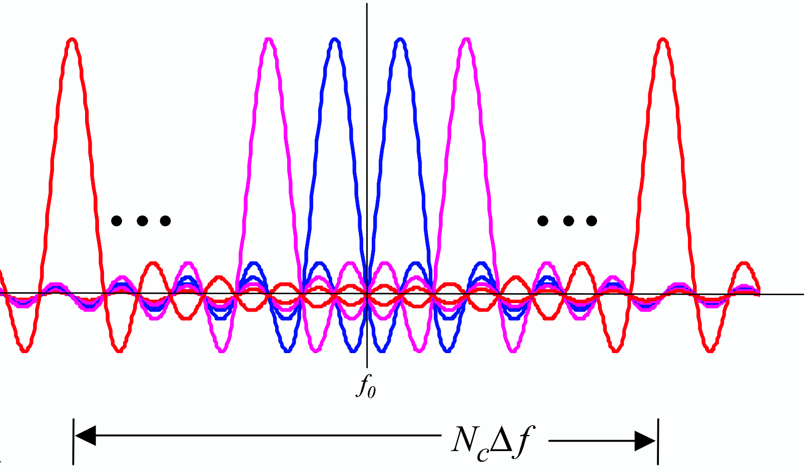
subcarrier coefficients

Input
Spectral info (mag., phase, freq.) of data constellation point coupled to subcarrier – needed to build the output sinusoid.

IDFT

Without FFT technology, each output wire will output a sinusoidal time waveform (the Fourier tranform of the phasor on its paired input wire).

Sketch doesn't show this efficient 50% overlap.



where, orthogonal spectra means that the channel spacing is an integer multiple of Δf Hz, such that

$$\int_0^{T_s} e^{j\omega_m t} \cdot e^{j\omega_n t}\, dt = \begin{cases} 1, & \text{for } m = n \\ 0, & \text{otherwise} \end{cases}$$

DFT anf IDFT are processes.
FFT and IFFT are algorithms.

$\Delta f = 1/T_s$

where T_s is the data portion of the OFDM symbol time

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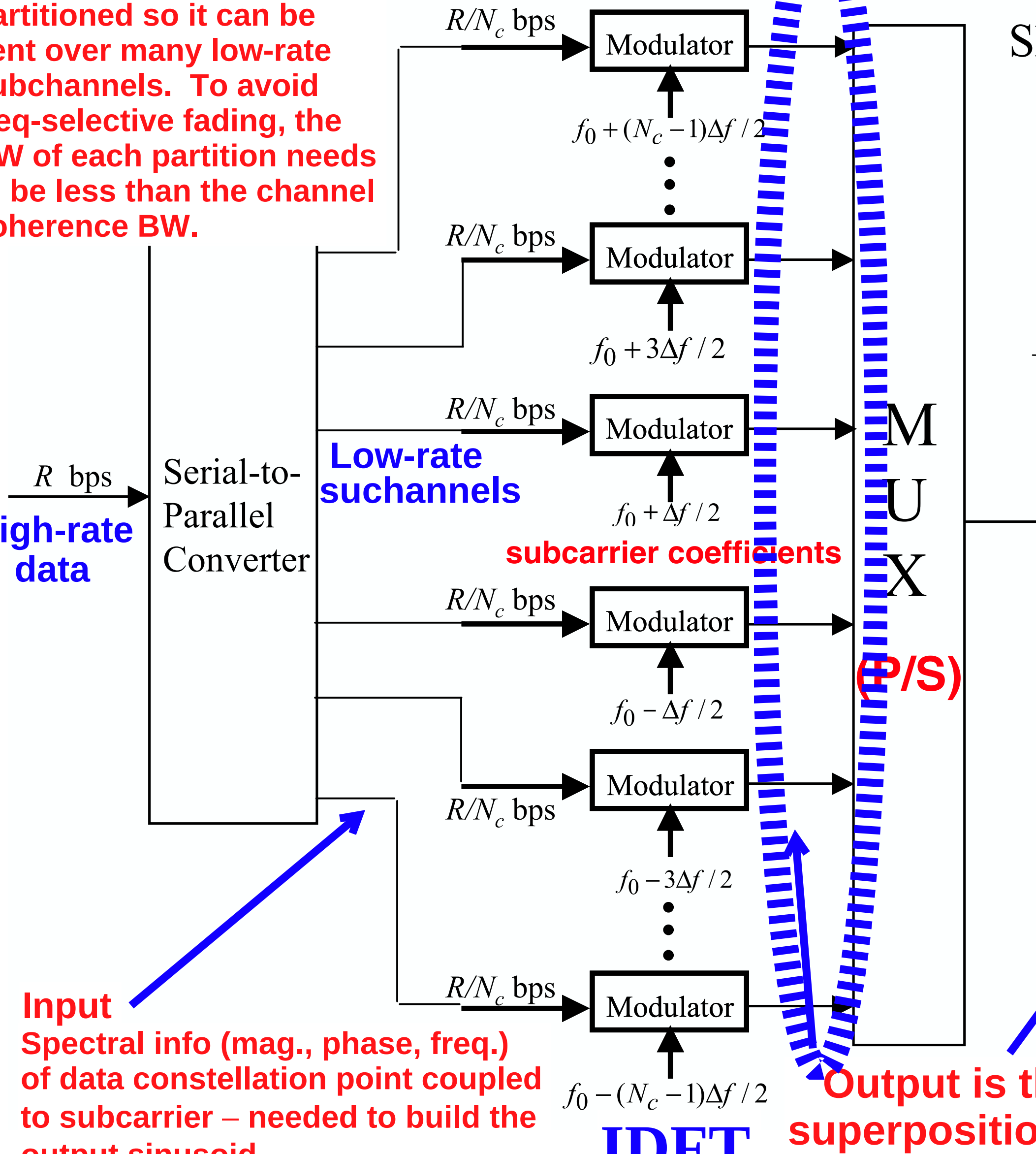

High-rate data

Low-rate suchannels

subcarrier coefficients

(P/S)

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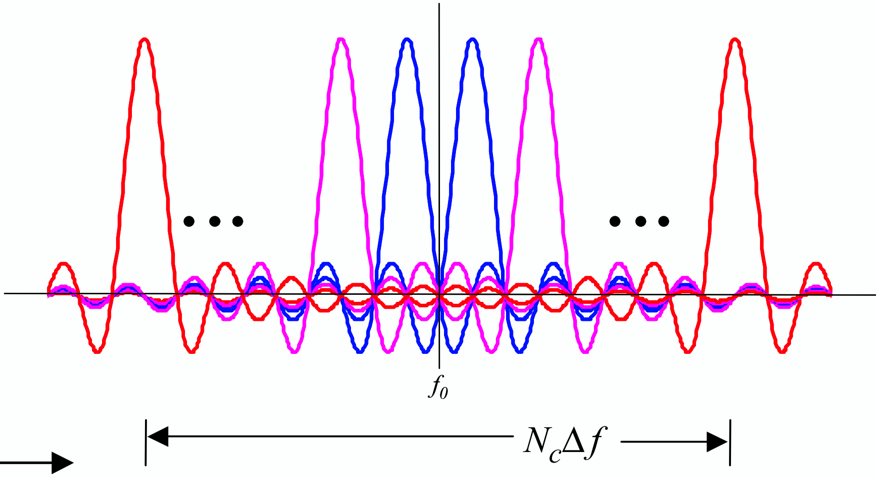


where, orthogonal spectra means that the channel spacing is an integer multiple of Δf Hz, such that

Thanks to FFT operation, each wire out of the IDFT, carries the SAME super-position of gated sinusoids.

Each IDFT output wire presents a successive moment-in-time of the same gated sinusoids.

Input

Spectral info (mag., phase, freq.) of data constellation point coupled to subcarrier – needed to build the output sinusoid.

IDFT

Output is the superposition of gated sinusoids

✱ Effect of Lengthening the Symbol Time (low-rate subchannels)

Example of Modem & Channel Parameter Values

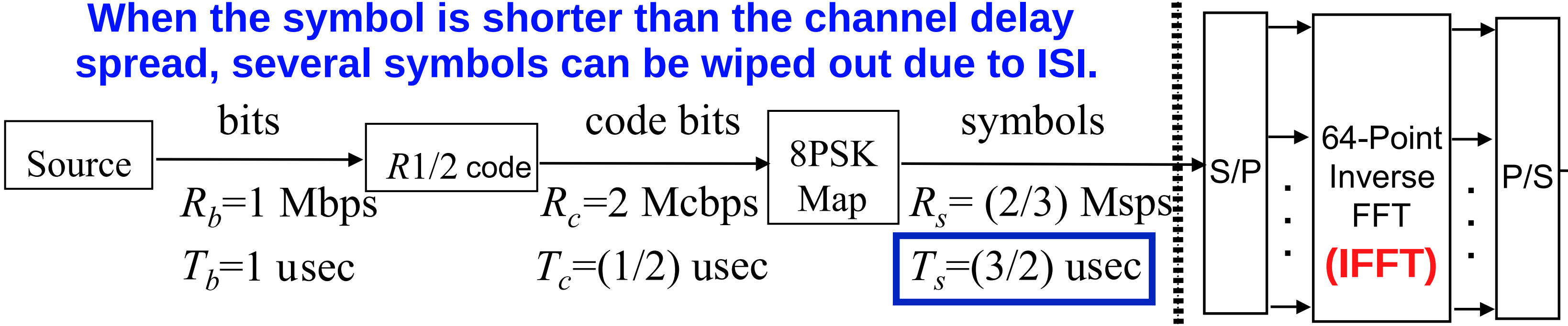
What do we want? **We want the channel delay spread to be much smaller than T_s**

- Channel Parameter Values

delay (secs) / T_S = delay (symbols)

  - $\boxed{\sigma_\tau = 3 \text{ usec}}$, and $R = 1$ Mbit/sec $\Rightarrow \boxed{\sigma_\tau / T_s = \sigma_\tau R_s = 2 \text{ symbols}}$ $\qquad \sigma_\tau = 2T_s$
  - $T_m \approx 5\sigma_\tau \approx 10\,T_s$ **where σ_τ is the rms channel delay spread**

When the symbol is shorter than the channel delay spread, several symbols can be wiped out due to ISI.



With OFDM, symbol is 64 times longer here.

64 x 1.5 usec

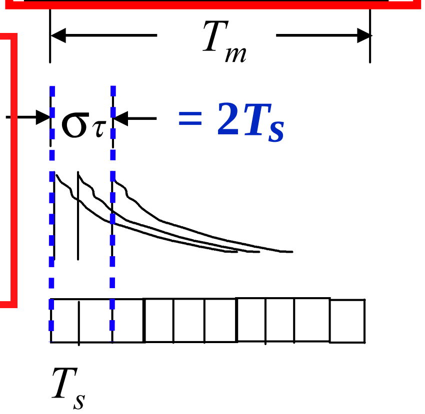
$$\boxed{T_{\text{OFDM}} = 64\ T_s = 96 \text{ usec}}$$

$$R_{\text{OFDM}} = 1/T_{\text{OFDM}}$$

$$\sigma_\tau = 2T_s = 3 \text{ usec}$$

Here, channel-induced ISI can cause smearing over several symbol times.

Without OFDM



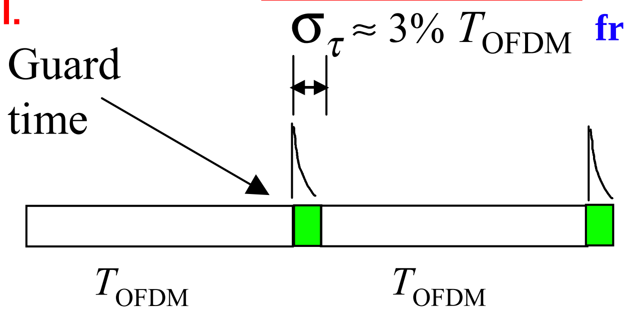
Pre-OFDM Symbols, T_s

For each subchannel, the time duration of an OFDM symbol will be longer than that of the original symbol.

ISI $\longrightarrow$ Mitigation

With OFDM

Symbol has been made longer, from 1.5 us to 96 us.



Post-OFDM Symbols, T_{OFDM}

OFDM Goal:
Lengthen the symbol time to mitigate channel-smearing.

With OFDM, the effect of channel induced ISI is small.

Relationship Between Channel Coherence Bandwidth f_0 and rms Multipath Spread σ_τ

An exact relationship between coherence bandwidth and rms delay spread is a function of specific channel impulse responses and applied signals. In general, accurate multipath channel models must be used in the design of specific modems for wireless applications [1].

The relationships below are all "ball park estimates."

$$\boxed{f_0 \approx \frac{1}{T_m}}$$

where T_m is the max multipath spread **Rule of Thumb**

50% is a popular estimate for coherence BW

$$f_0(90\%) \approx \frac{1}{50\sigma_\tau} \quad [1]$$

Frequency interval over which channel's freq transfer function has correlation of at least 0.9 amplitude of $|R(\Delta f)|$

$$f_0(50\%) \approx \frac{1}{5\sigma_\tau} \quad [2]$$

Frequency interval over which channel's freq transfer function has correlation of at least 0.5 amplitude of $|R(\Delta f)|$

Hence,

$$T_m \approx 5\sigma_\tau$$

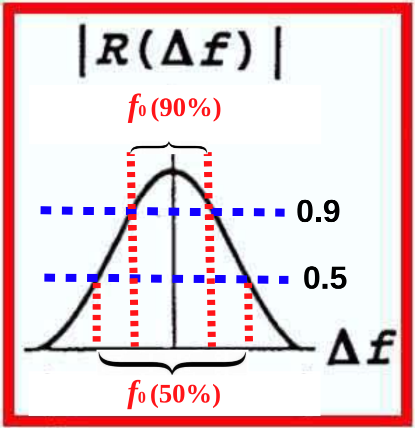
where σ_τ is the rms multipath delay spread

1. Lee, W.C.Y., Mobile Cellular Communications Systems, McGraw Hill, 1989.
2. Rappaport, T.S., Wireless Communications, Prentice Hall, 2002.

$f_0(90\%)$

and

$f_0(50\%)$



Spaced-Frequency Correlation Function

$f_0 \cong 1/T_m$ Coherence Bandwidth

The larger the correlation, the narrower the correlation BW

Multipath Spread (Signal Dispersion)

(frequency-range fading together)

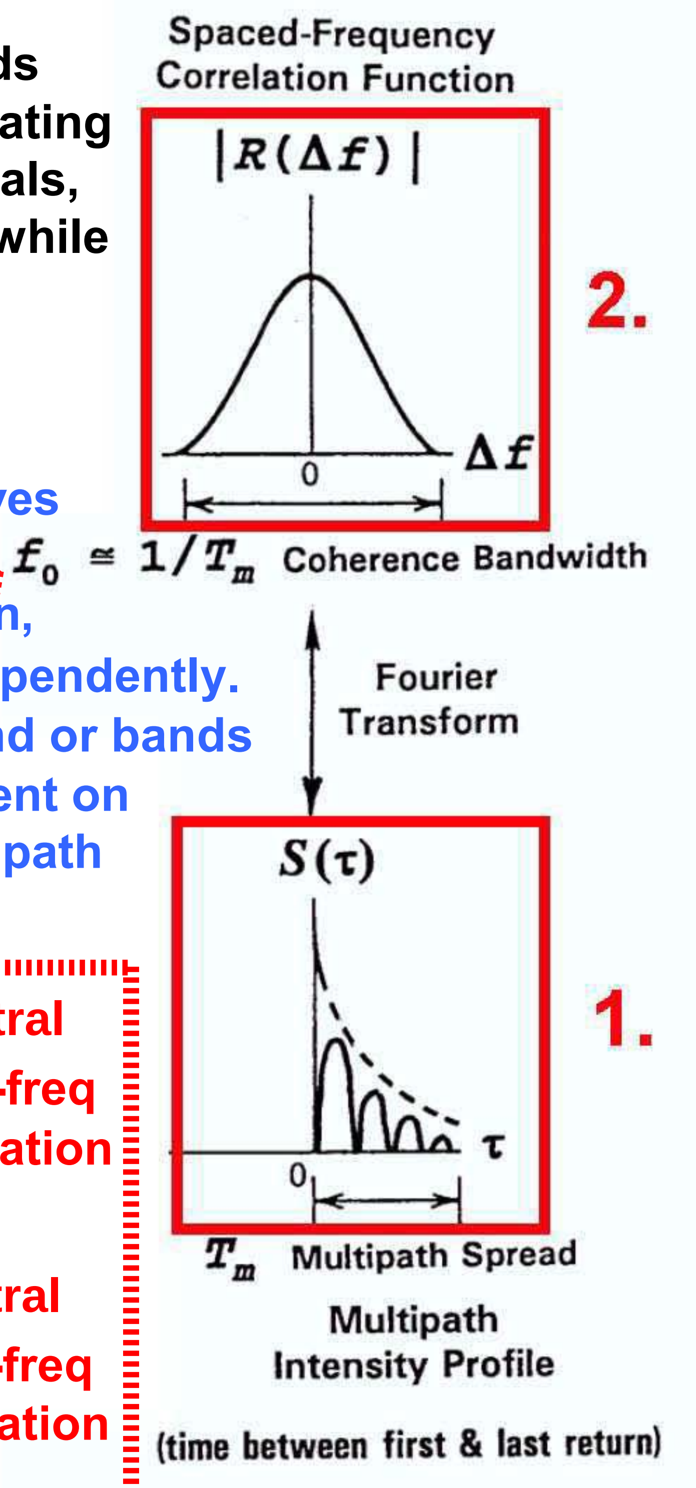
Spaced-Frequency Correlation Function shows the spectral correlation of received narrow-band signals spaced Δf apart.

It can be measured by transmitting a pair of sinusoids separated by Δf, cross-correlating their separately received signals, and repeating multiple times while increasing Δf.

Coherence Bandwidth represents the spectral range over which the channel behaves coherently (fading or not fading). Outside of this region, signals will behave quite independently. The positioning of such a band or bands is a random process, dependent on the nature of the propagation path (the terrain).

f_0 (90%) is defined as the spectral interval over which the spaced-freq correlation fuction has a correlation of at least 0.9.

f_0 (50%) is defined as the spectral interval over which the spaced-freq correlation fuction has a correlation of at least 0.5.



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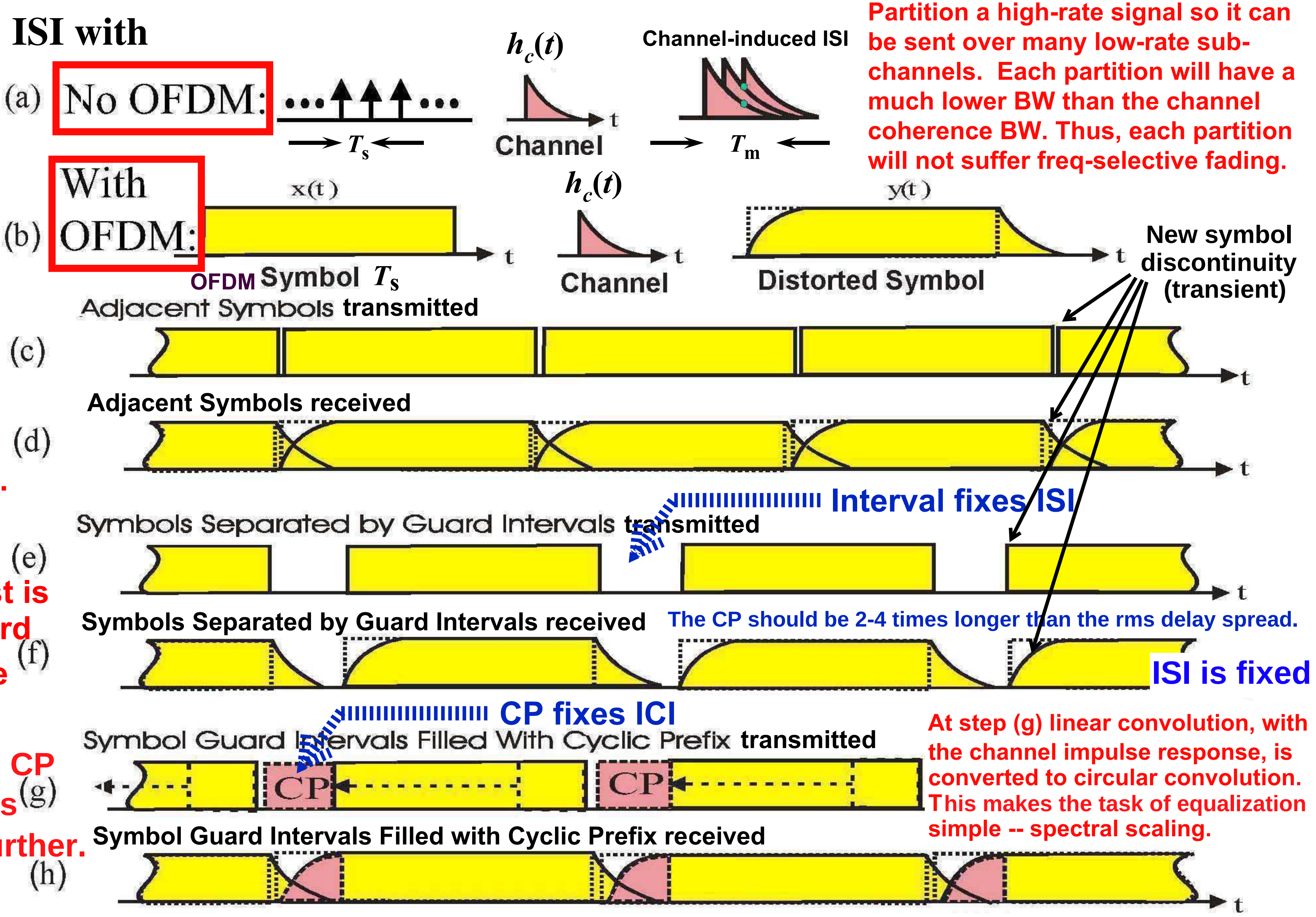
The Importance of the Cyclic Prefix (CP)

Plotting periodic outputs on a unit circle.

Tricking the Channel by Converting Linear Convolution to Circular Convolution.

Nothing in nature prepares us for this trick. How can we connect the front end of a launched signal to its back end?

The trick (with the CP) is one of the reasons we refer to OFDM properties as "elegant."



Partition a high-rate signal so it can be sent over many low-rate sub-channels. Each partition will have a much lower BW than the channel coherence BW. Thus, each partition will not suffer freq-selective fading.

We can fix this.

Cost is guard time

The CP should be 2-4 times longer than the rms delay spread.

The CP fixes it further.

At step (g) linear convolution, with the channel impulse response, is converted to circular convolution. This makes the task of equalization simple -- spectral scaling.

* CP benefits: Mitigates signal dispersion. Helps maintain orthogonality by preserving constant envelope.

CP helps achieve ssr (no transients).

And simplifies equalization

Transforming Linear Convolution to Circular Convolution

- To convert convolution from linear to circular, the OFDM standard IEEE-802.11 chose to append a cyclic prefix (CP) to the transmitted waveshape, formed by copying a segment from the end of the time signal to the beginning of the signal.
- The back end of the appended cyclic prefix is continuous with the front end of the OFDM signal because the signal's length is a multiple of its sinusoidal basis functions (integer no. of cycles per gated sinusoid).

MOVING THE DISCONTINUITIES

End of cycle at back end

Start of cycle at front end (old transient)

continuous edge

transient at new starting edge

- The cyclic-prefix-end matches the signal-front. There will no longer be a transient at the original time signal's starting edge. The transient now resides at the new starting edge of the cyclic prefix which will be tossed.

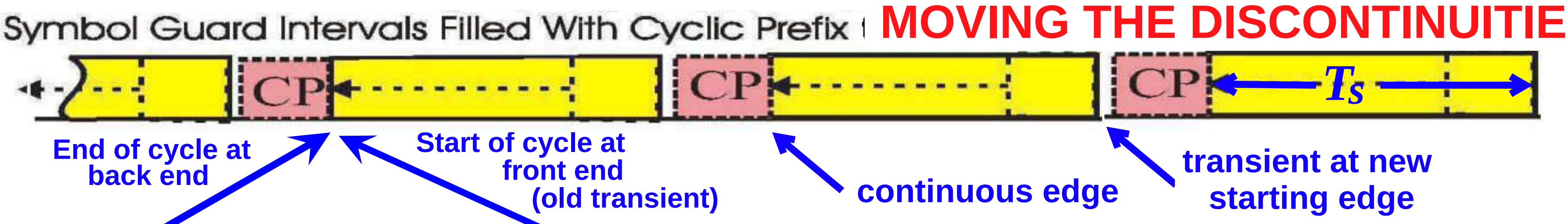
*During convolution, as the channel impulse response slides from the cyclic prefix into the signal interval it has the appearance of leaving the signal's back end, while entering the front end, without discontinuities. Thus the linear convolution *appears* to be circular.

- We complete the process by discarding the CPs, after which there will be an integer number of cycles per symbol time, and all of the orthogonality rules will be saisfied.

Transforming Linear Convolution to Circular Convolution

This is our tool for tricking the channel.

by rearranging the past and the present



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- Integer number of cycles per symbol interval
- Hence back-end of CP = front-end of symbol
- Continuous edge between added CP and old starting edge
- Transient at new starting edge

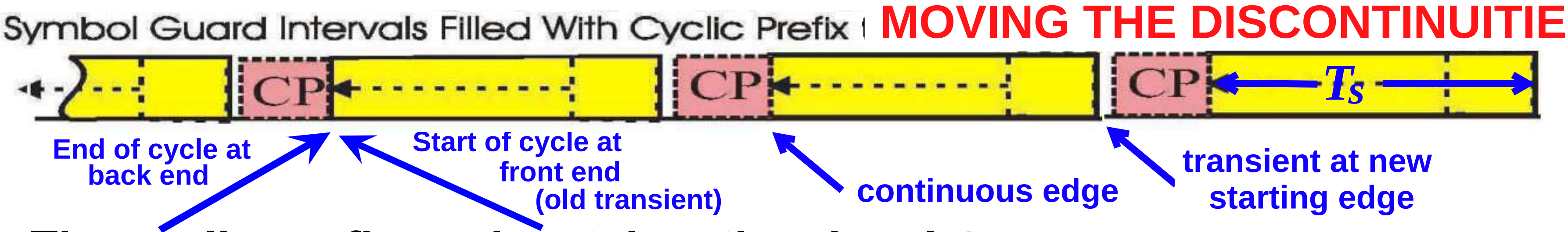
Transforming Linear Convolution to Circular Convolution

If you remember only one slide, from this briefing, let it be this one.

This is our tool for tricking the channel.

by rearranging the past and the future

MOVING THE DISCONTINUITIES



- **The cyclic-prefix-end matches the signal-front. There will no longer be a transient at the original time signal's starting edge. The transient now resides at the new starting edge of the cyclic prefix** **which will be tossed.**

- **Integer number of cycles per symbol interval**
- **Hence back-end of CP = front-end of symbol**
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- **Transient at new starting edge**

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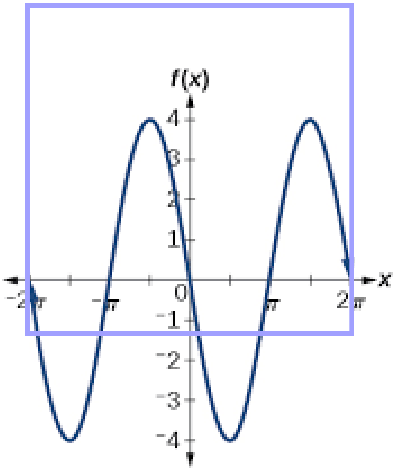
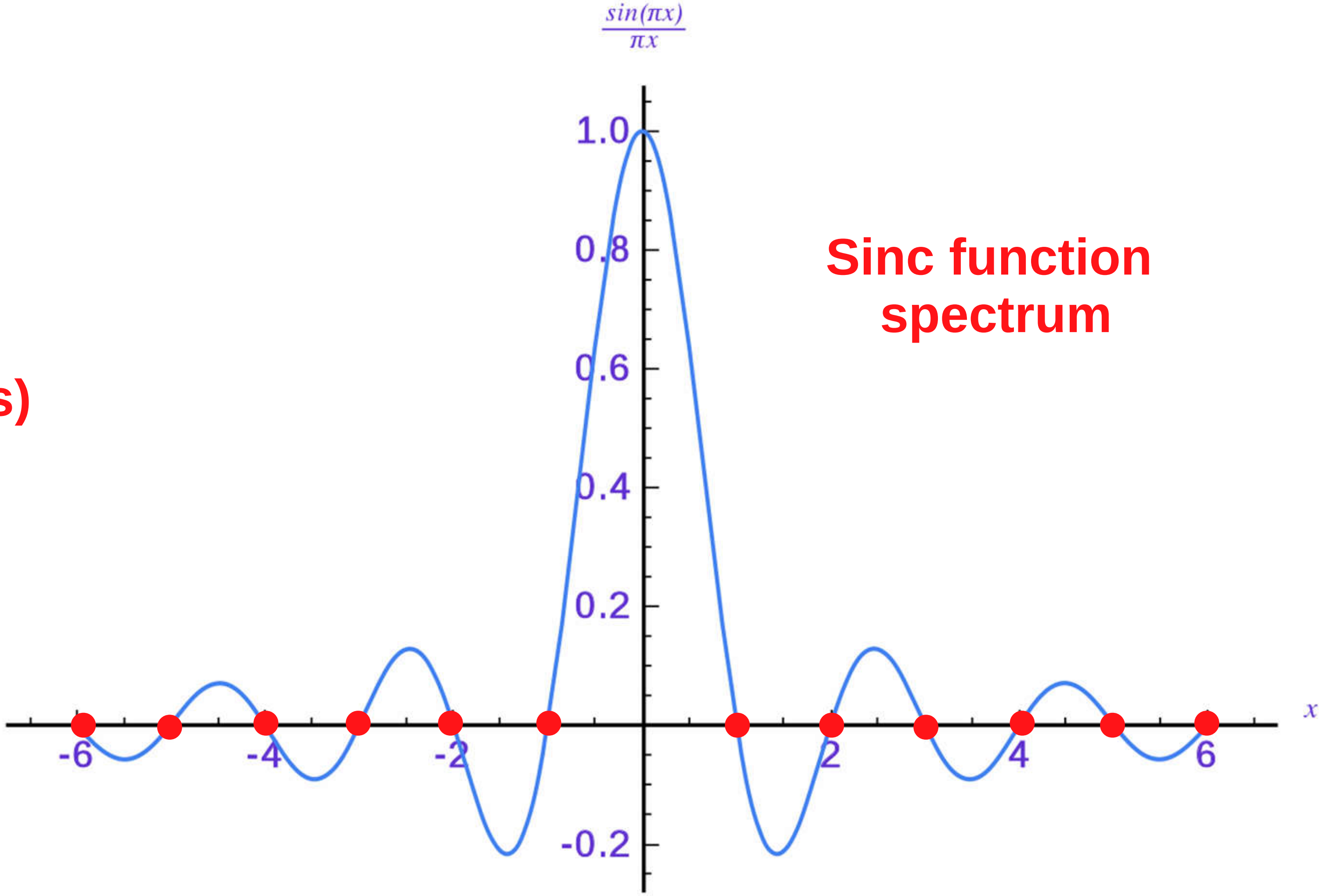
Periodic Outputs on a Unit Circle

The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

Gated Sinusoid (one sinusoid with an integer number of cycles)



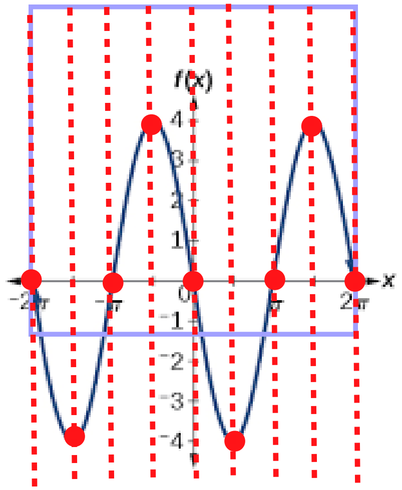


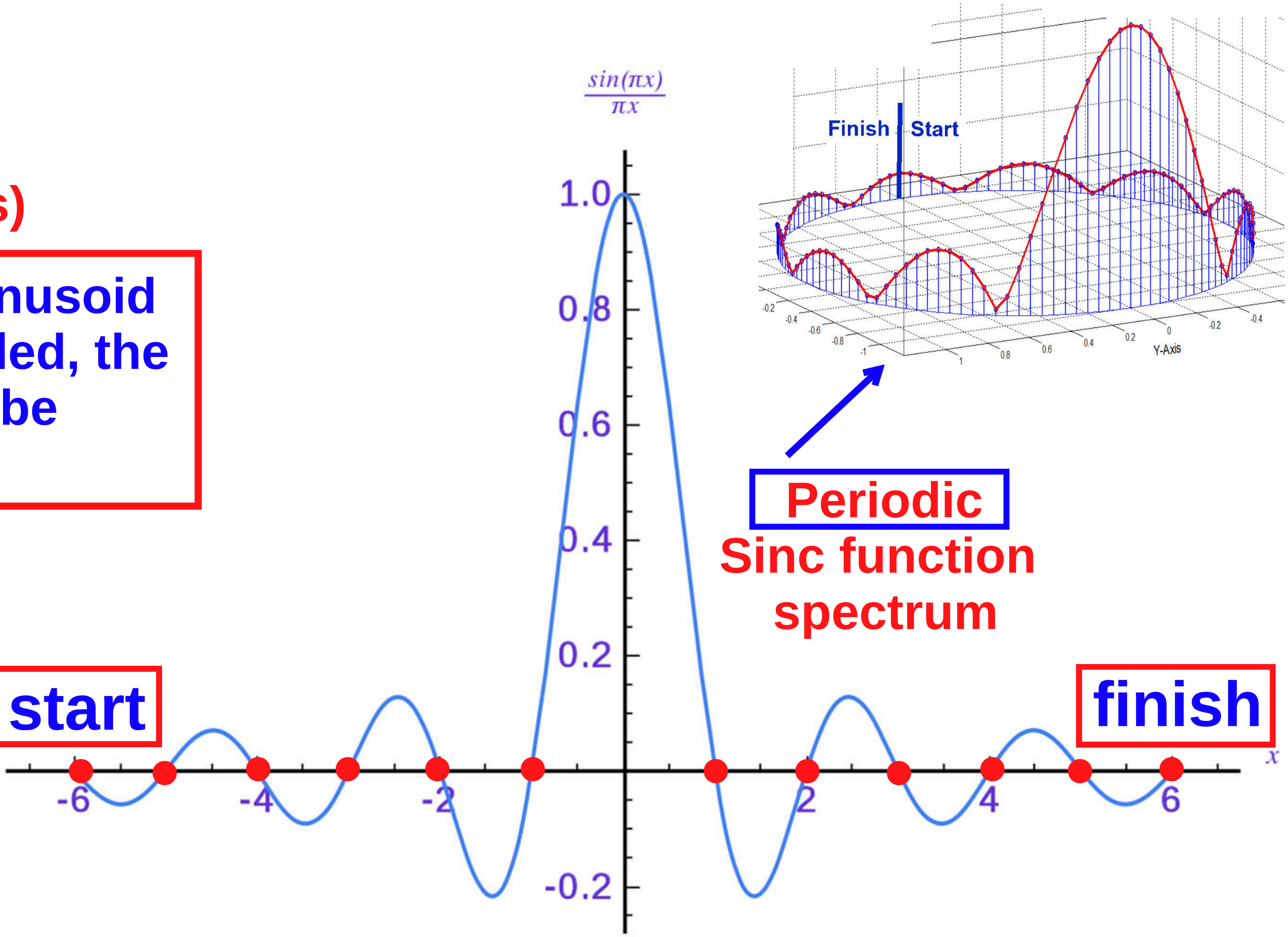
Sinc function spectrum

The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

**Gated Sinusoid
(one sinusoid
with an integer
number of cycles)**

And if the gated sinusoid is discretely sampled, the transform will be periodic.





start

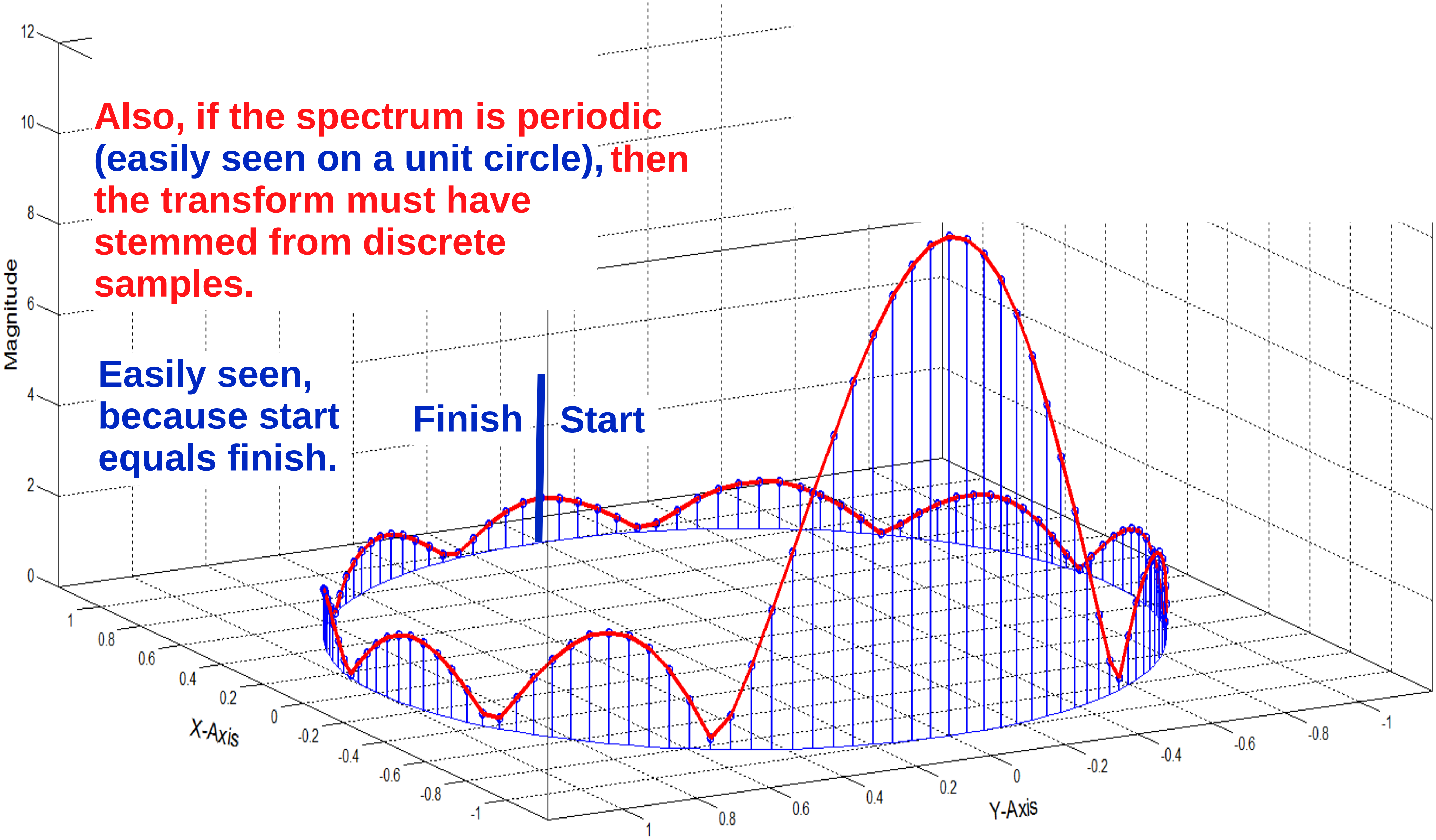
finish

**Periodic
Sinc function
spectrum**

Let's plot this periodic spectrum as a power signal on a unit circle.

SIN(X)/X ON UNIT CIRCLE

The DFT of a discretely sampled time sequence yields a continuous periodic spectrum.



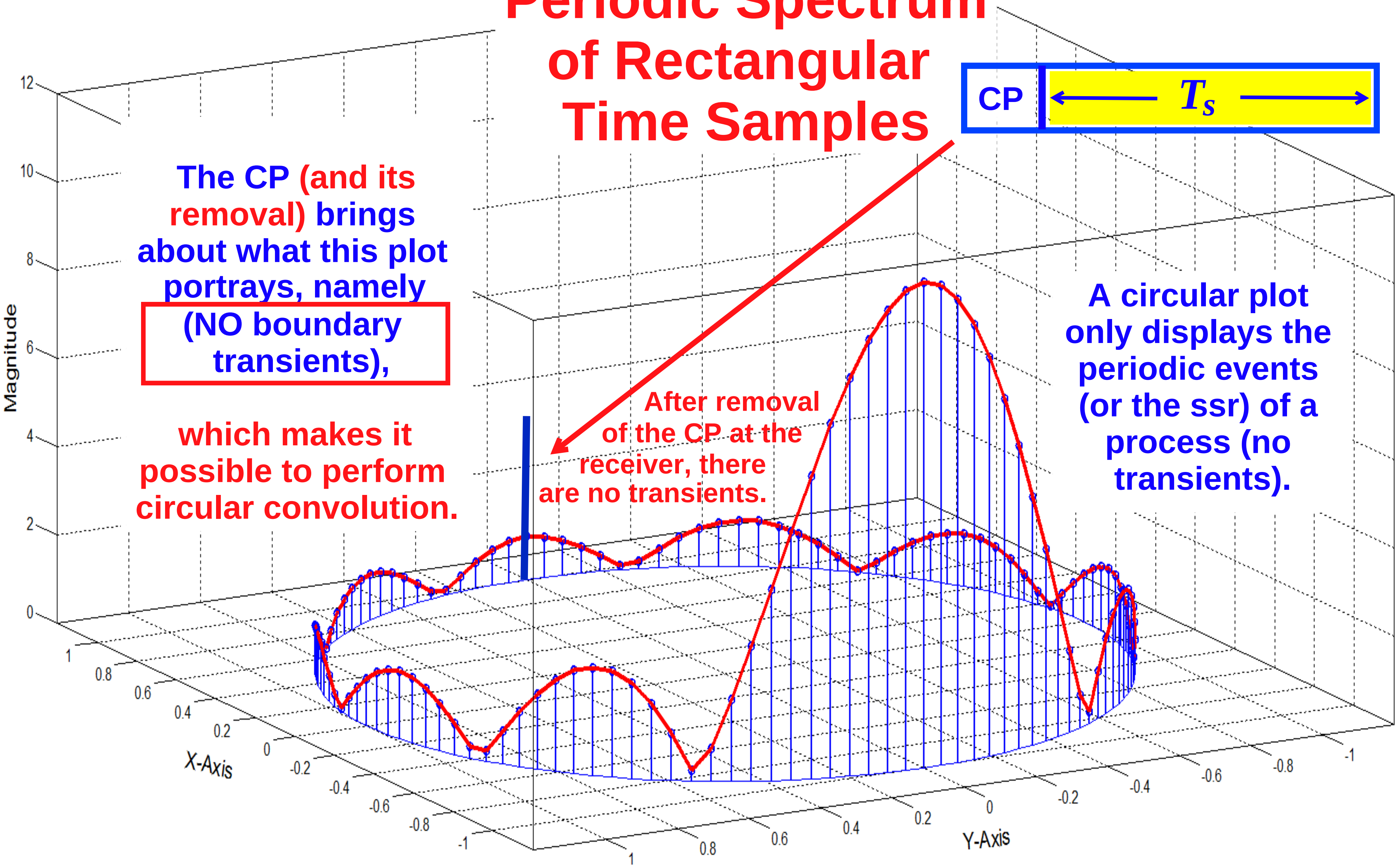
Also, if the spectrum is periodic (easily seen on a unit circle), then the transform must have stemmed from discrete samples.

Easily seen, because start equals finish.

Finish Start

SIN(X)/X ON UNIT CIRCLE

Periodic Spectrum of Rectangular Time Samples



The CP (and its removal) brings about what this plot portrays, namely (NO boundary transients),

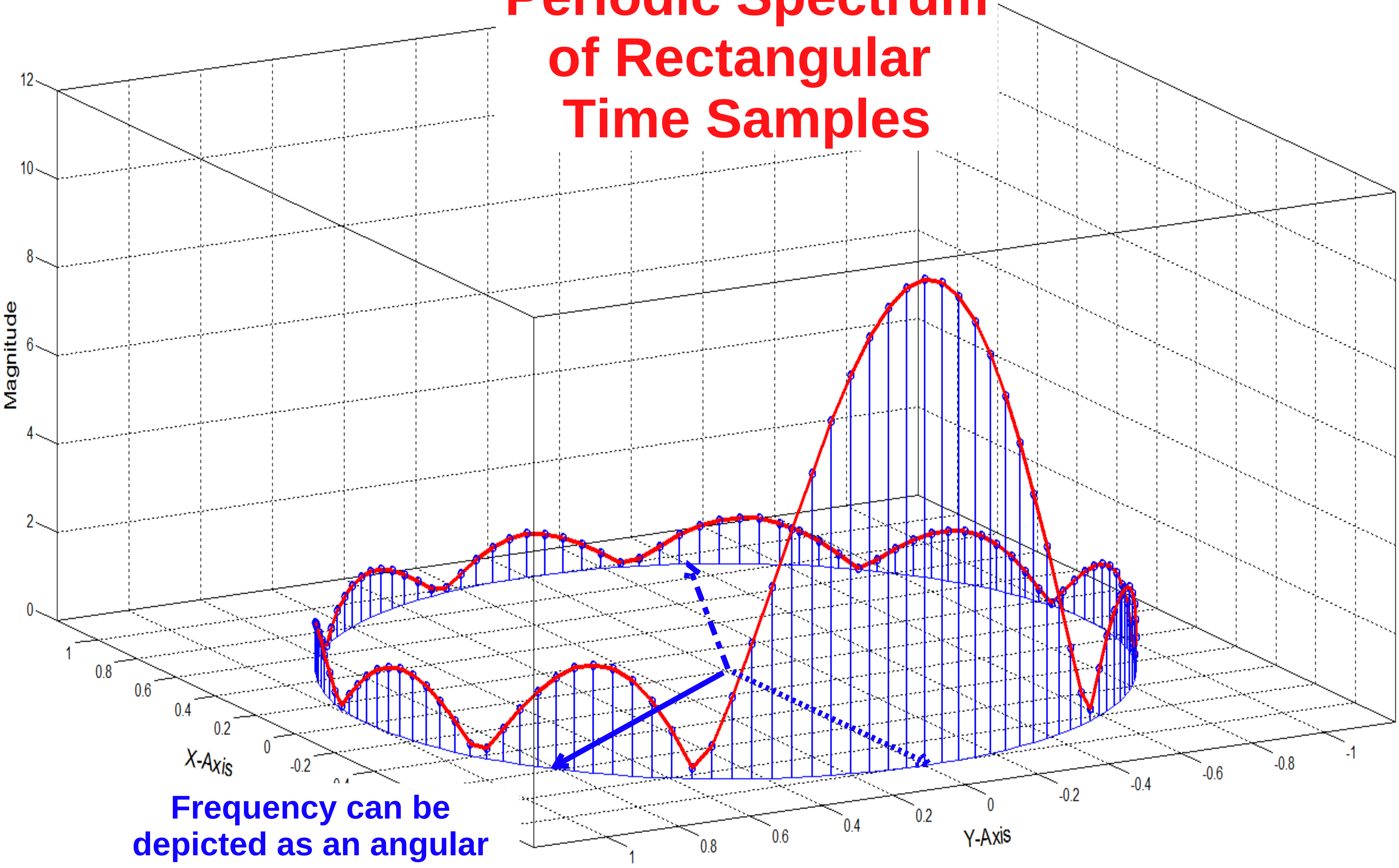
which makes it possible to perform circular convolution.

After removal of the CP at the receiver, there are no transients.

A circular plot only displays the periodic events (or the ssr) of a process (no transients).

Plotting the spectrum on a unit circle helps us visualize (as we go round-and-round the circle) that the spectrum is periodic.

SIN(X)/X ON UNIT CIRCLE

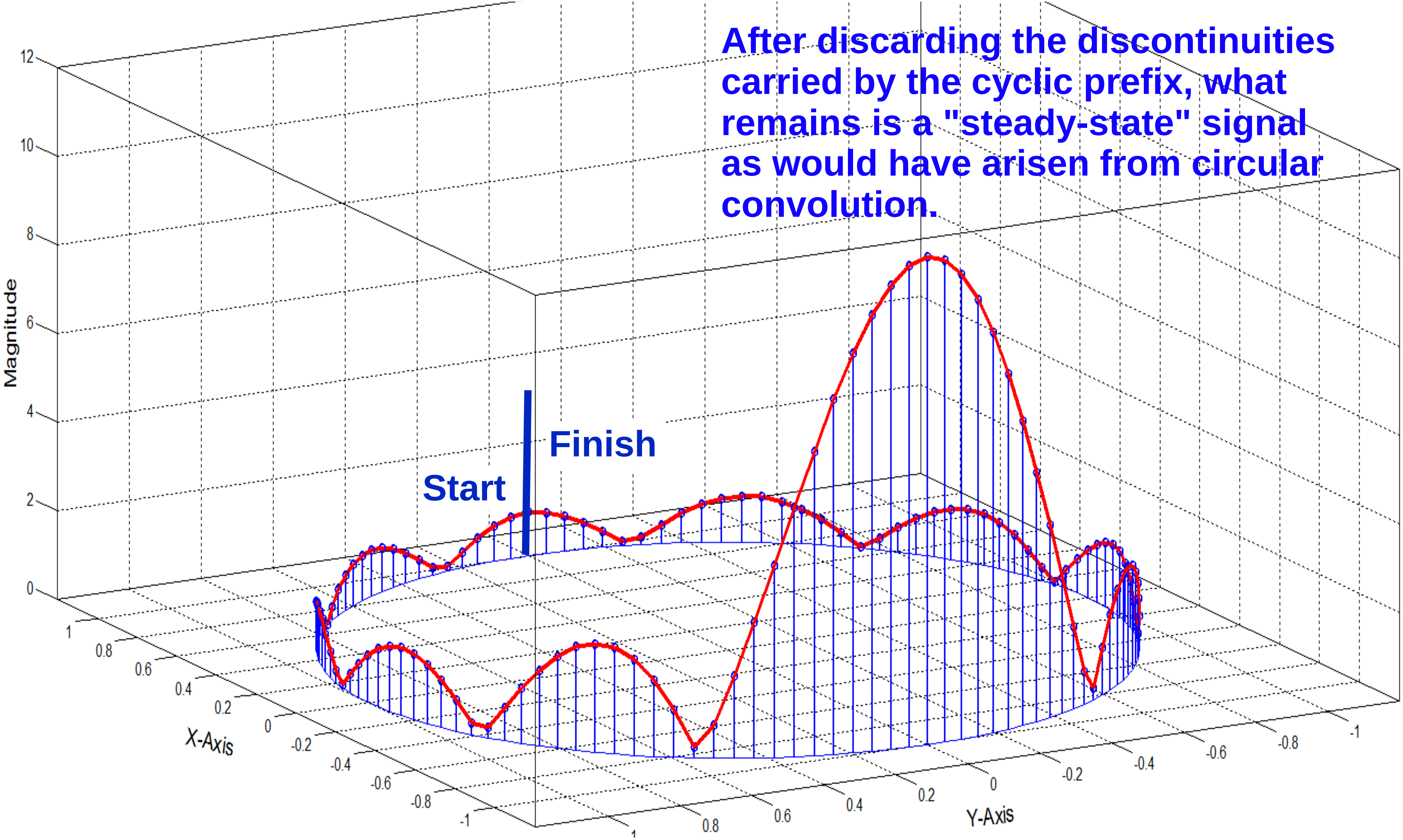


Periodic Spectrum of Rectangular Time Samples

Frequency can be depicted as an angular coordinate on the circle (radians per sample).

SIN(X)/X ON UNIT CIRCLE

Periodic Spectrum at the Receiver



After discarding the discontinuities carried by the cyclic prefix, what remains is a "steady-state" signal as would have arisen from circular convolution.

The steady-state response (ssr) has essentially gotten rid of all the On-Off transients.

Summarizing

The Cyclic Prefix in OFDM Modifies Linear Convolution so that it Appears to be Circular Convolution

- A property of the Fourier Transform:

  Spectral multiplication of continuous signals $X(f)H(f)$ corresponds to linear convolution $x(t)*h(t)$ in time.

- A property of the Discrete Fourier Transform (DFT):

  Spectral multiplication of sampled signals $X(k)H(k)$ corresponds to circular convolution $x(n)\circledast h(n)$ in time (sampling the transform makes the time signal periodic, and sampling the time signal makes the transform periodic).

- When Using DFTs for implementing OFDM systems:

  A continuous waveform, linearly convolved with the channel impulse response, is modified so that it appears to be circularly convolved with the channel impulse response. This makes the task of equalization simple – spectral scaling during the DFT.

*

The DFT forms a sampled-data spectrum. Samples in the frequency domain correspond to periodicity in the time domain. Any periodic function on a time-line is nicely portrayed as one copy of the function plotted on a unit circle (start and finish are the same point). This makes linear convolution appear to be circular.

- For reconstructing the correct OFDM subcarriers at the receiver:

We need to maintain signal orthogonality. This is accomplished by

1. preserving signal length
2. preserving constant envelope
3. preserving an integer number of cycles per gated sinusoid

Maintining important orthogonal characteristics

- Preserving Length

The use of linear convolution with an *N*-point DFT would create a lengthened output. But, by making the signal (with a CP) appear circularly convolved, the original signal length is preserved.

- Preserving Constant Envelope

Convolving a signal with the channel impulse response causes a transient at the start and end of the symbol. Any such transient causes envelope variations. The CP absorbs the starting transient of the current symbol and the stopping transient of the previous symbol. By discarding the CP in the guard interval (the overlapping transient), we thereby preserve a constant envelope for each gated-sinusoid symbol.

- Preserving an Integer Number of Cycles

Discarding the CP guard interval also preserves the integer number of cycles of each symbol (the way it was originally created).

- **For reconstructing the correct OFDM subcarriers at the receiver:**

  We need to maintain signal orthogonality. This is accomplished by
  1. preserving signal length
  2. preserving constant envelope
  3. preserving an integer number of cycles per gated sinusoid

- **Preserving Length**

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  output. But, by making the signal (with
  original signal length is preserved.

- **Preserving Constant Envelope**

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  start and end of the symbol. Any such t
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  transient of the previous symbol. By dis
  (the overlapping transient), we thereby
  gated-sinusoid symbol.

- **Preserving an Integer Number of Cycle**

  Discarding the CP guard interval also p
  cycles of each symbol (the way it was or

If these 3 rules are observed, OFDM signals will experience NO losses in orthogonality.

That Cyclic Prefix is Amazing !!!

IEEE Virtual Presentation
The ABCs of OFDM
By Dr. Bernard Sklar

Part 1 March 18, 2021
Part 2 March 25, 2021

Abstract: The main benefit of OFDM is its ability to cope with Severe multipath channel conditions without needing Complex Equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a High-data-rate signal into Smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

- The Big Picture: Time/Frequency Relationships.
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- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

OFDM Waveform Synthesis

OFDM Modem Block Diagram

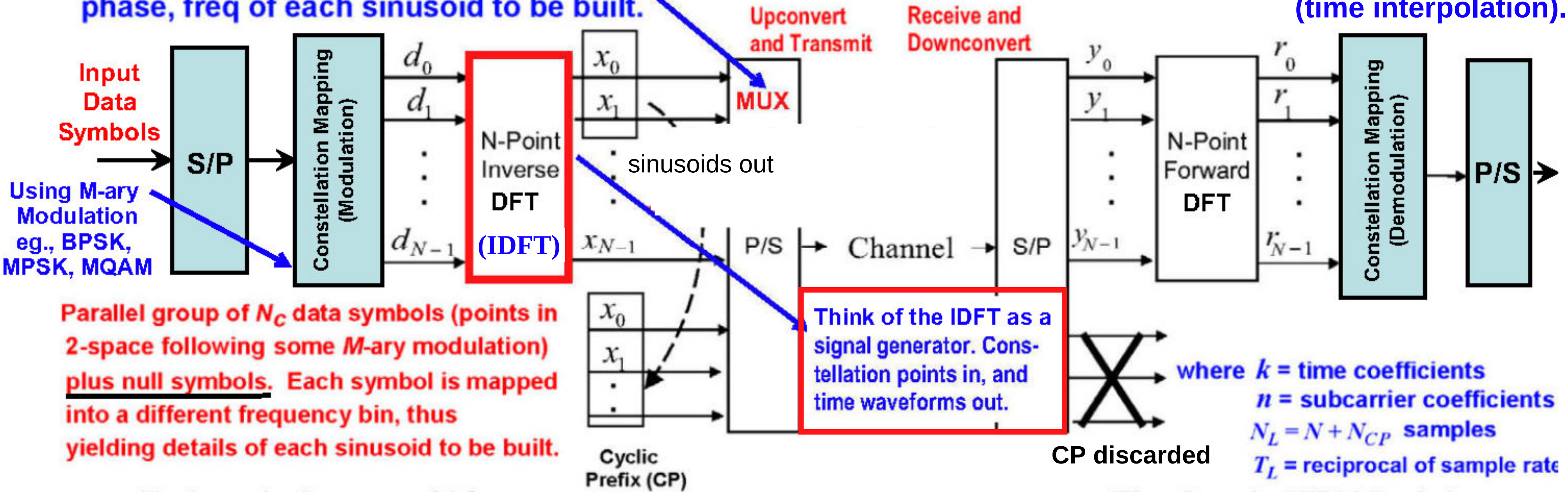
- An OFDM symbol is made up of a sum of N terms (N_C modulated orthogonal carriers plus null bins). Each k^{th} sample of a symbol can be represented as:

$$x_k = \sum_{n=0}^{N-1} d_n e^{j\frac{2\pi}{N}nk} \qquad n, k = 0,1,2,\ldots, N-1$$

where some of the d_n values are zero

N is made larger than Nc by zero-padding Nc in the frequency domain, which raises the output sample rate (time interpolation).

IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.

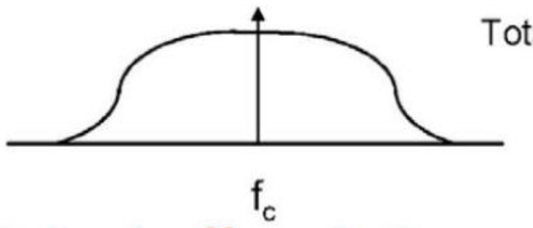


Think of the IDFT as a signal generator. Constellation points in, and time waveforms out.

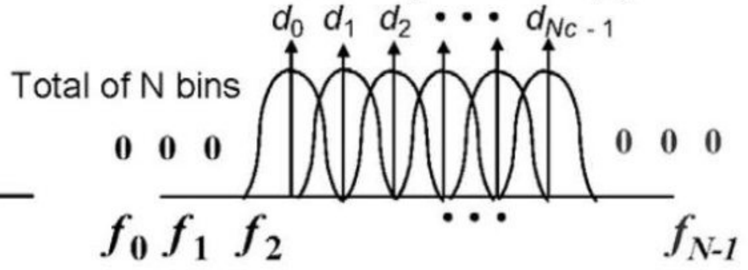
Parallel group of N_C data symbols (points in 2-space following some *M*-ary modulation) plus null symbols. Each symbol is mapped into a different frequency bin, thus yielding details of each sinusoid to be built.

where k = time coefficients
n = subcarrier coefficients
$N_L = N + N_{CP}$ samples
T_L = reciprocal of sample rate

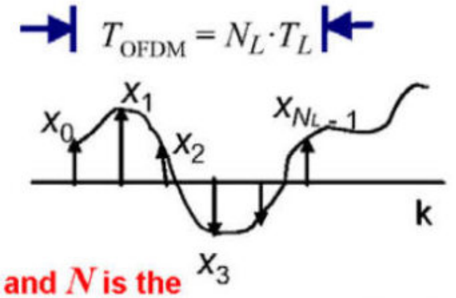
Single-carrier Spectrum of $\{d\}$



Multi-carrier Spectrum of $\{x\}$



Time-Domain OFDM Symbol



Don't confuse Nc with N. Nc represents the data (constellation points) or subcarriers, and N is the transform size. For building real analog filters, we use zero extensions (null bins) to form the transform such that $N > N_C$.

OFDM Modem Block Diagram

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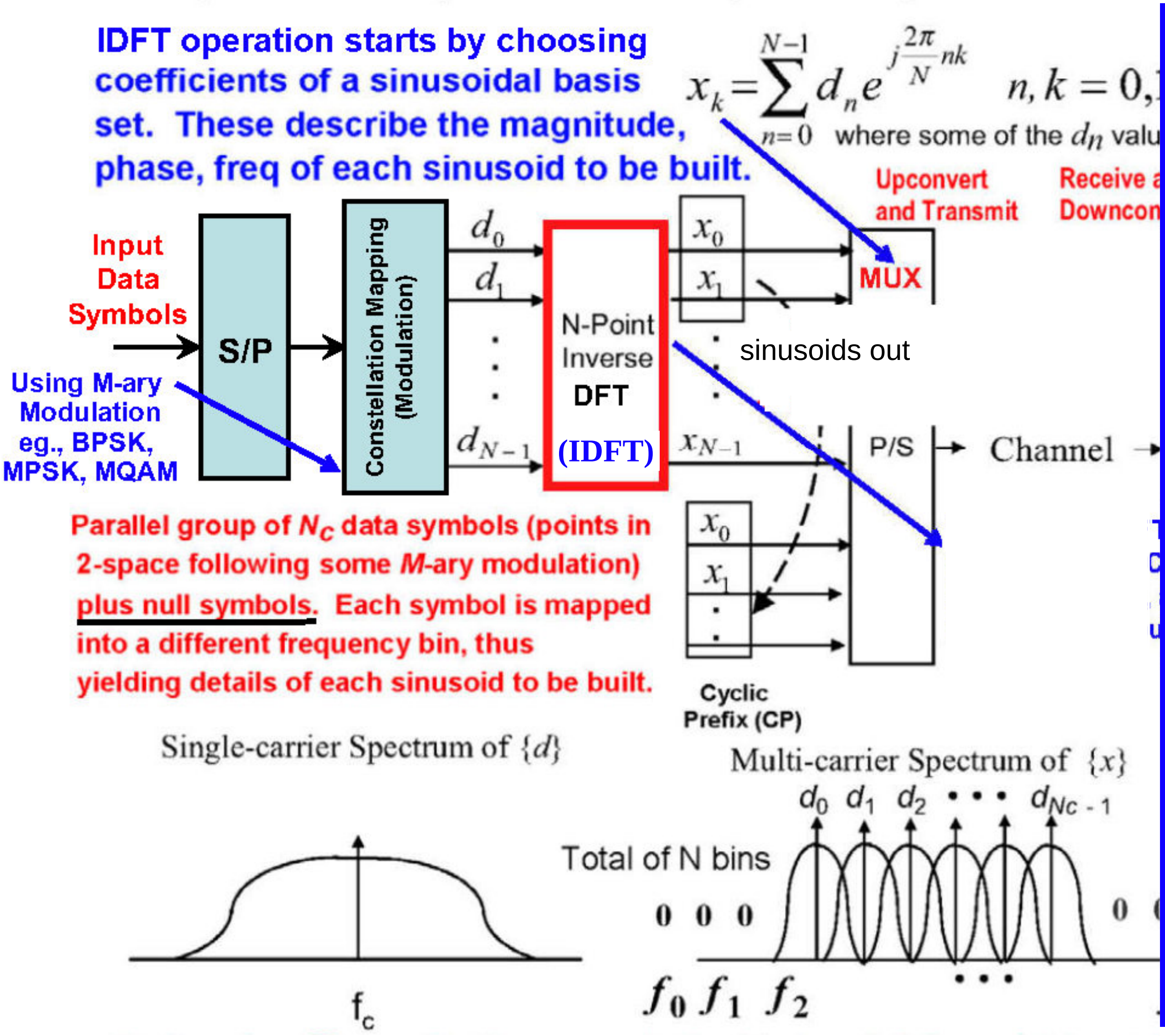
$$x_k = \sum_{n=0}^{N-1} d_n e^{j\frac{2\pi}{N}nk} \qquad n, k = 0,$$

where some of the d_n valu



IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.

Using M-ary Modulation eg., BPSK, MPSK, MQAM

Parallel group of N_C data symbols (points in 2-space following some *M*-ary modulation) plus null symbols. Each symbol is mapped into a different frequency bin, thus yielding details of each sinusoid to be built.

This slide shows the OFDM signal processing if we didn't have an IFFT available. Then, the N output wires of the IDFT would (as the schematic suggests) generate samples of N different tones, which means N coherent oscillator/modulators (very costly processing). Some of the N tones will have zero amplitudes, leaving N_C enabled tones.

By using an IFFT, the actual processor will output on each of its N output wires the superposition of all the enabled N_C tones. Each wire holds a successive sample of the same superposition.

FFT and IFFT are computational algorithms that reduce time & complexity. Operations needed for DFT are $O(N^2)$, but for FFT they are only $O(N \log N)$.

With this modern implementation, it is only possible to see the sum of the N_C tones, but not any one of them alone. We can only see them alone after detection at the receiver.

OFDM Modem Block Diagram

- An OFDM symbol is made up of a sum of N terms (N_C modulated orthogonal carriers plus null bins). Each k^{th} sample of a symbol can be represented as:

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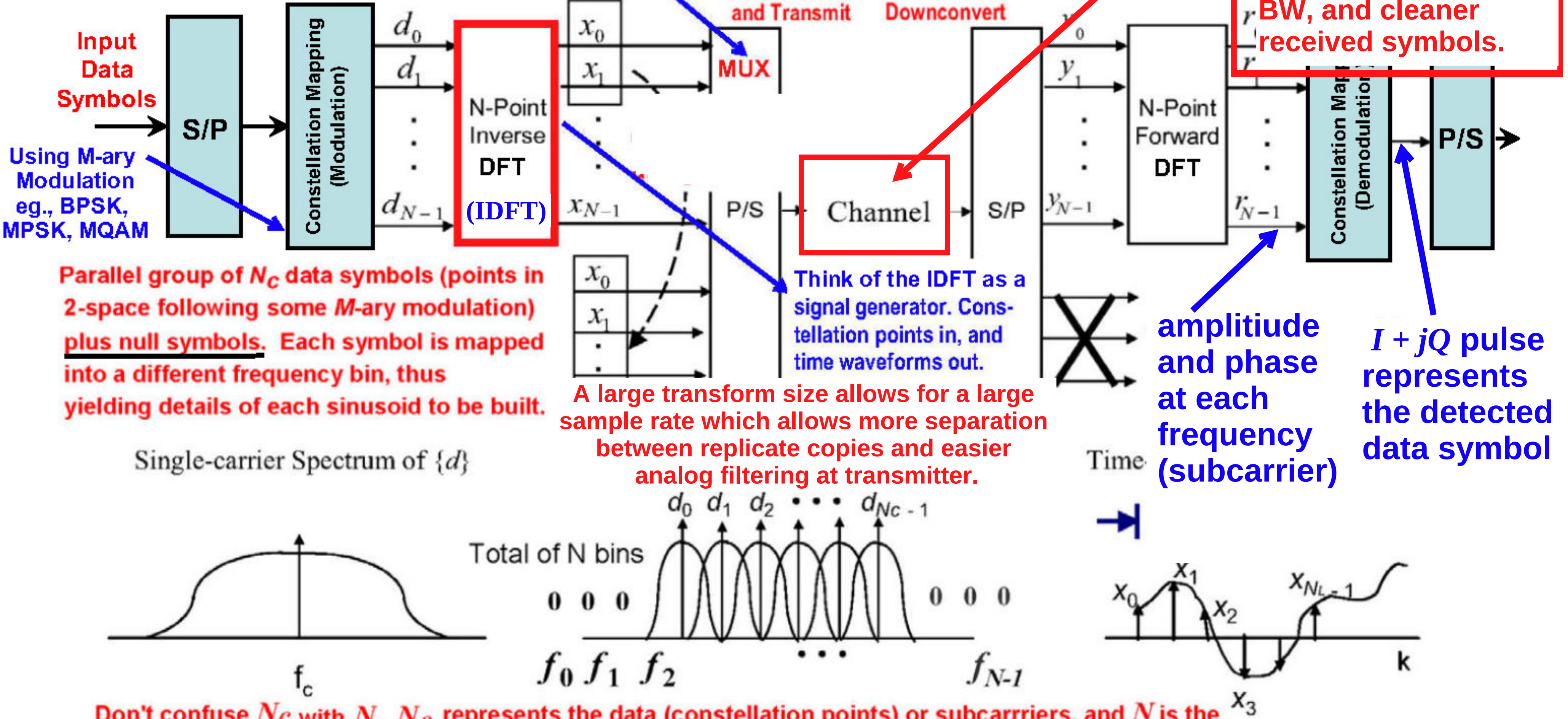
where some of the d_n values are zero

IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.

We do NOT send samples. We send waveforms. A large transform size allows for larger receiver transitional BW, and cleaner received symbols.



Using M-ary Modulation eg., BPSK, MPSK, MQAM

Parallel group of N_C data symbols (points in 2-space following some *M*-ary modulation) plus null symbols. Each symbol is mapped into a different frequency bin, thus yielding details of each sinusoid to be built.

Think of the IDFT as a signal generator. Constellation points in, and time waveforms out.

A large transform size allows for a large sample rate which allows more separation between replicate copies and easier analog filtering at transmitter.

amplitiude and phase at each frequency (subcarrier)

***I + jQ* pulse represents the detected data symbol**

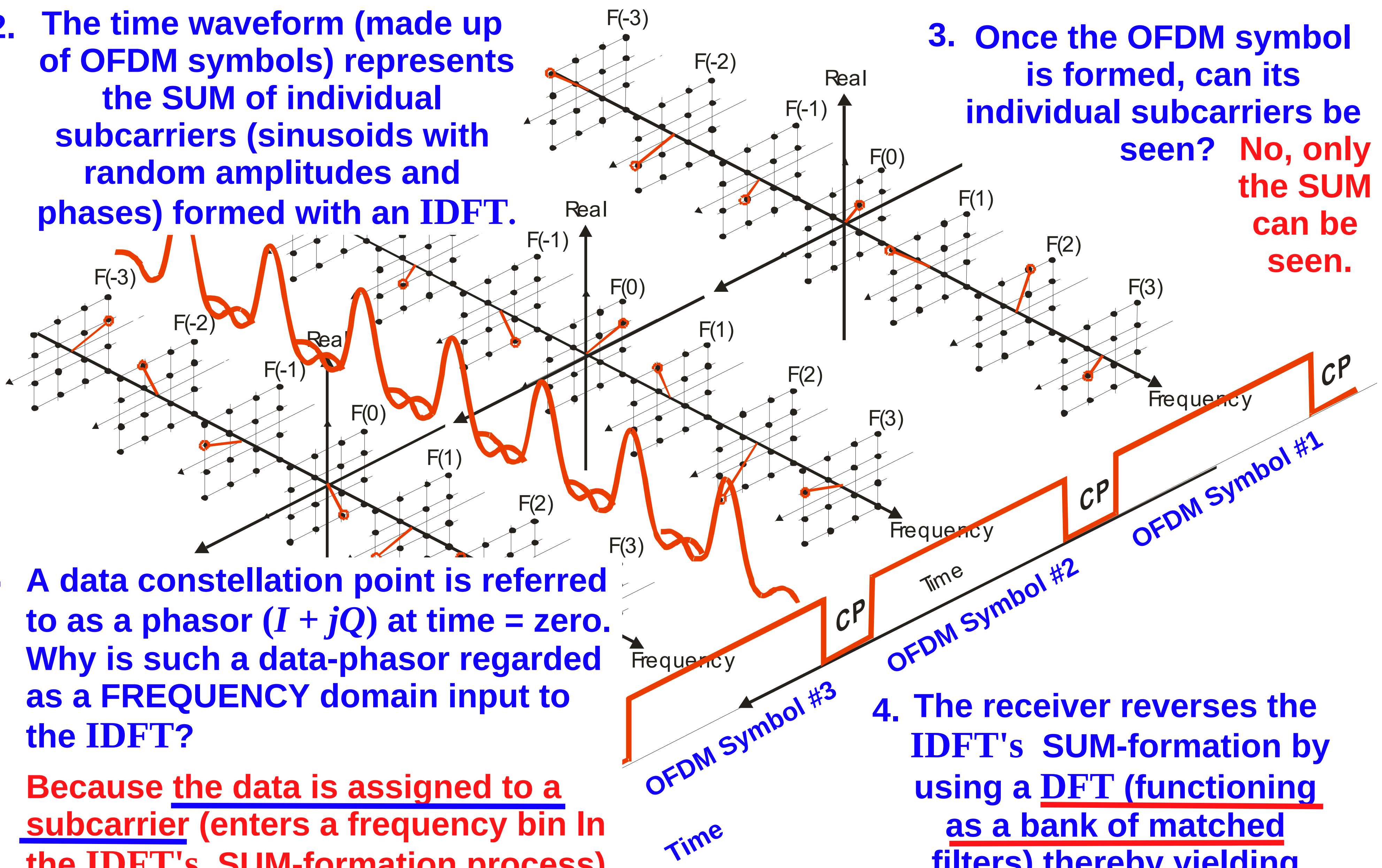
Don't confuse N_C with N. N_C represents the data (constellation points) or subcarriers, and N is the transform size. For building real analog filters, we use zero extensions (null bins) to form the transform such that $N > N_C$.

More Details and a Refresher:

OFDM Time/Frequency Relationships

2. The time waveform (made up of OFDM symbols) represents the SUM of individual subcarriers (sinusoids with random amplitudes and phases) formed with an IDFT.

3. Once the OFDM symbol is formed, can its individual subcarriers be seen? No, only the SUM can be seen.

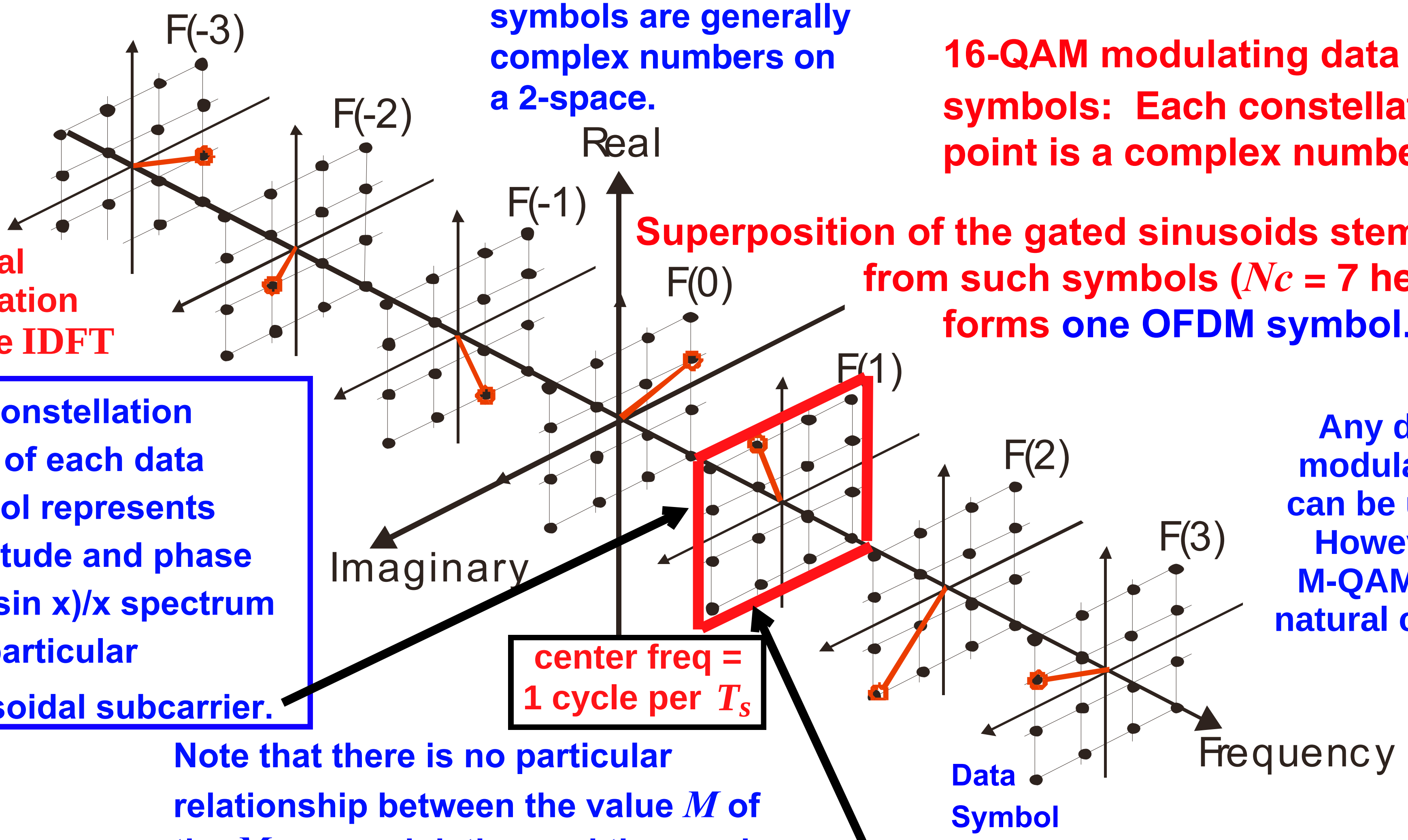


1. A data constellation point is referred to as a phasor ($I + jQ$) at time = zero. Why is such a data-phasor regarded as a FREQUENCY domain input to the IDFT?

Because the data is assigned to a subcarrier (enters a frequency bin In the IDFT's SUM-formation process) which dictates how fast the assigned input data-phasor spins.

4. The receiver reverses the IDFT's SUM-formation by using a DFT (functioning as a bank of matched filters) thereby yielding each individual subcarrier and it's data detection.

Constellation Points Distributed Over Frequency Index



As seen here, data symbols are generally complex numbers on a 2-space.

16-QAM modulating data symbols: Each constellation point is a complex number.

Superposition of the gated sinusoids stemming from such symbols (N_c = 7 here) forms one OFDM symbol.

Spectral information into the IDFT

The constellation point of each data symbol represents amplitude and phase of a (sin x)/x spectrum of a particular sinusoidal subcarrier.

Any data modulation can be used. However, M-QAM is a natural choice.

center freq = 1 cycle per T_s

Note that there is no particular relationship between the value M of the M-ary modulation and the number of subcarriers.

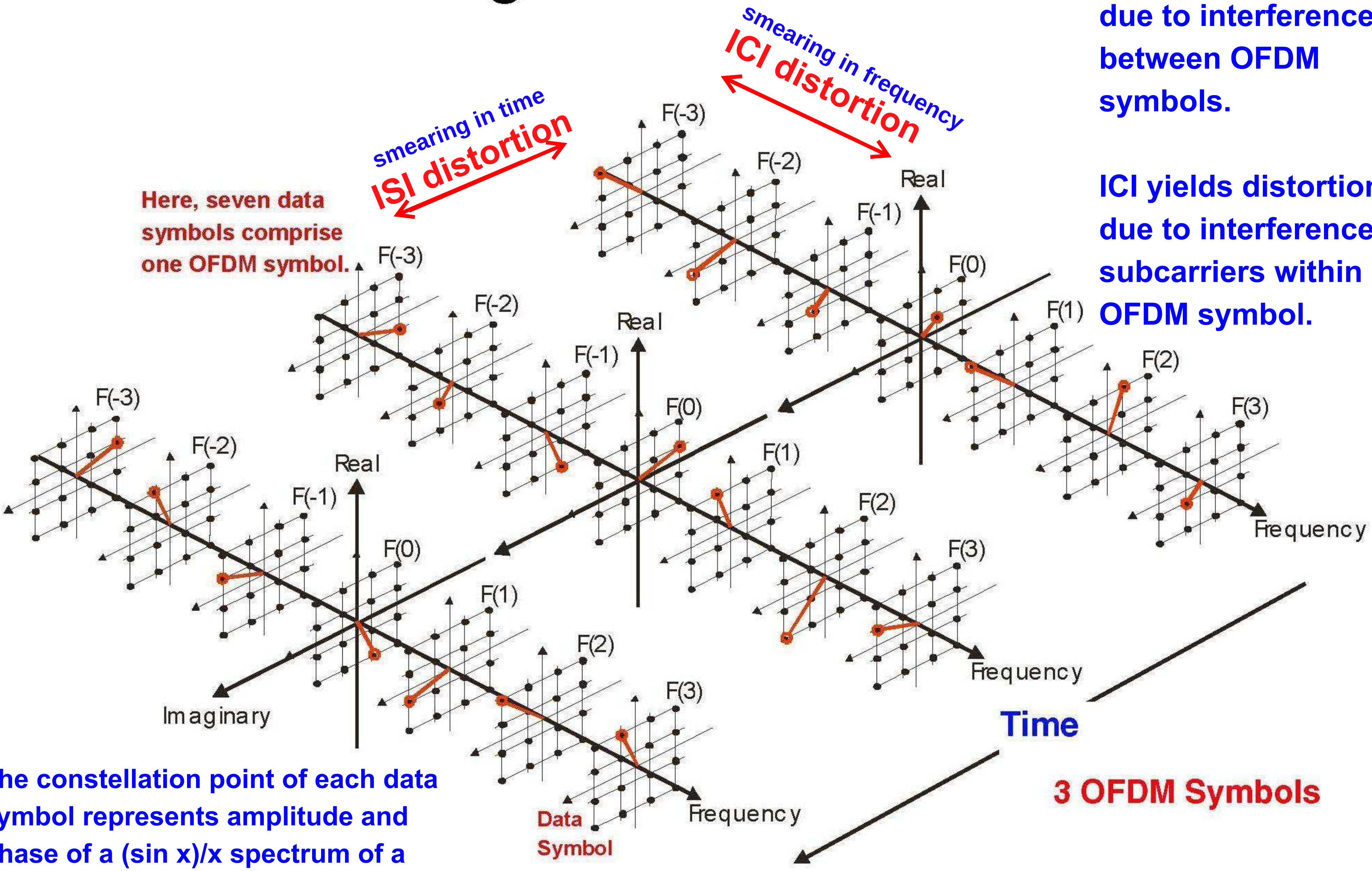
Each data symbol is assigned to a particular subcarrier frequency.

Sequential Spectra Showing Constellation Points

Difference between ISI and ICI

ISI yields distortion due to interference between OFDM symbols.

ICI yields distortion due to interference of subcarriers within an OFDM symbol.



Here, seven data symbols comprise one OFDM symbol.

The constellation point of each data symbol represents amplitude and phase of a (sin x)/x spectrum of a particular sinusoidal subcarrier.

59. In the early "battle" for the best codes (convolutioal vs. Reed-Solomon), what are the arguments for each, and why did convolutional win out?

60. In mobile channels, how does the terrain affect fading? How does the mobile-velocity affect it?

61. What is the advantage of circular-convolution versus linear-convolution? How do we trick the channel into performing circular convolution?

62. In OFDM, what is the mitigation technique for precluding ISI? For precluding ICI?

63. Baseband OFDM symbols are typically made up of independent data at positive and negative spectral locations. How is this effected, and how is a real transmission-signal formed?

64. For maintaining orthogonality among the subcarriers in OFDM, the tone spacing was chosen to be $1/T_S$. Why wasn't it chosen to be $1/T_{\text{OFDM}}$? (Sklar ADC notes, section 3)

65. How can SC-OFDM still be resistant to multipath when the data symbols are so short? Hint: The time duration of a data pulse is longer than its main lobe.

66. Early skeptics about MIMO, claimed that it violated Shannon's capacity theorem. Why is that not the case?

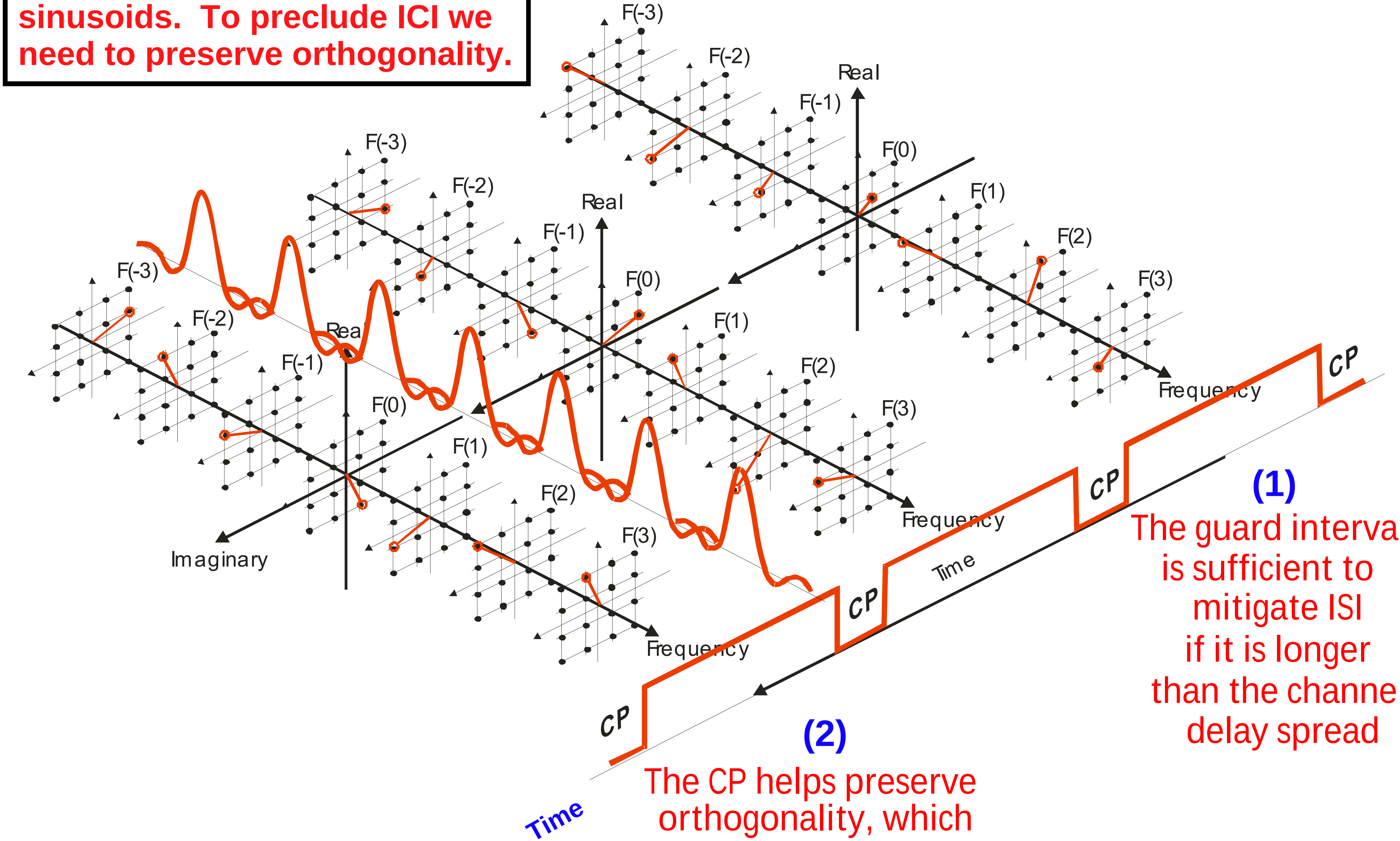
67. Why won't MIMO work in a multipath-free environment?

68. Often, the signal-processing operations "DFT and IDFT" are called out as "FFT and IFFT," when one means the mathematical transformation. Why is this NOT precise?

69. What are the Key Control Loops needed for system Synchronization? (fred harris, "Let's Assume the System is Synchronized.")

70. How do you shape a time waveform to meet system spectral-confinement requirements? Hint: symbol rate, sample rate, window type, filter length, transition BW, out-of-band attenuation.

OFDM manipulates orthogonal sinusoids. To preclude ICI we need to preserve orthogonality.



(1)

The guard interval is sufficient to mitigate ISI if it is longer than the channel delay spread

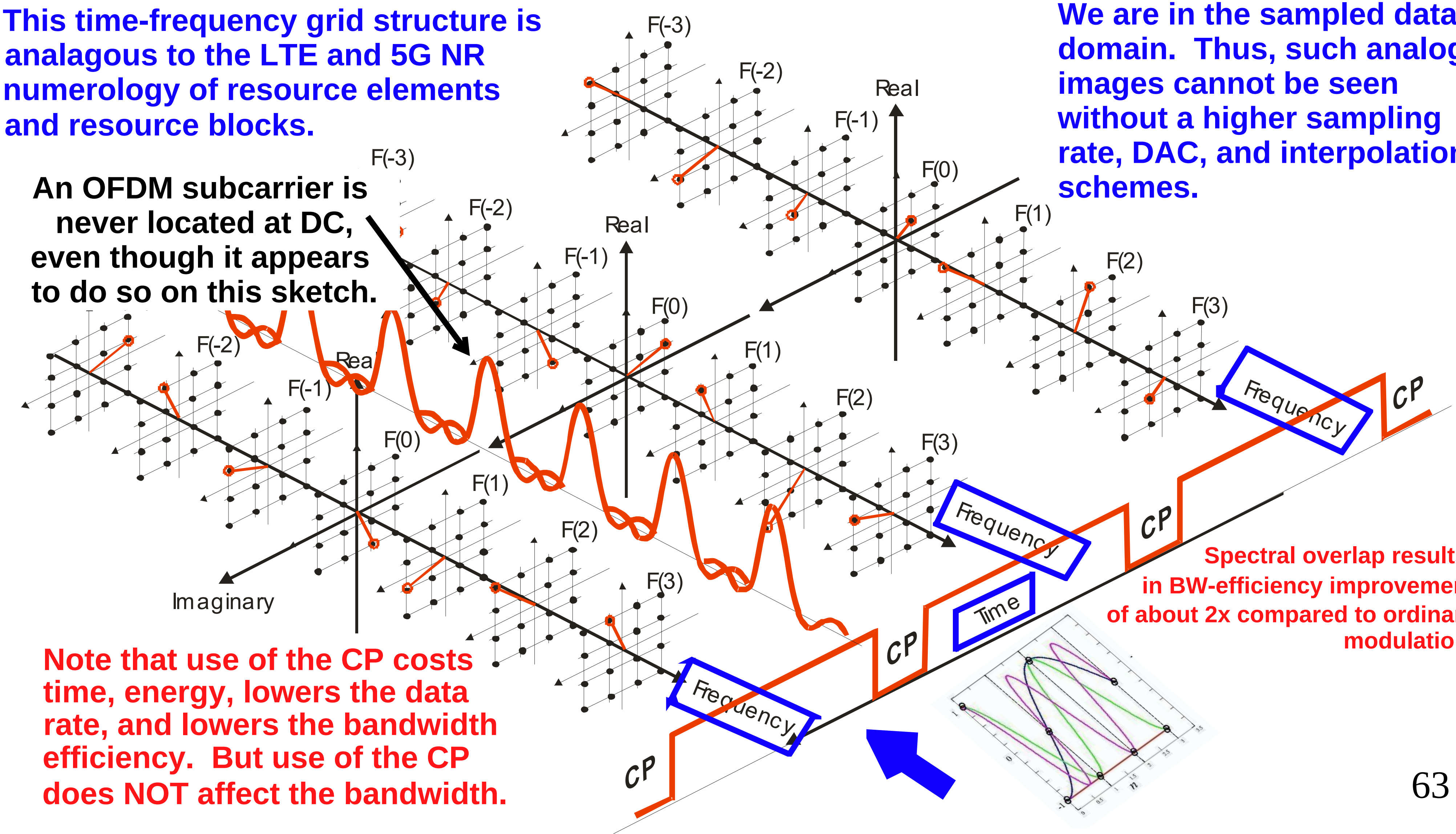
(2)

The CP helps preserve orthogonality, which mitigates ICI

This time-frequency grid structure is analagous to the LTE and 5G NR numerology of resource elements and resource blocks.

We are in the sampled data domain. Thus, such analog images cannot be seen without a higher sampling rate, DAC, and interpolation schemes.

An OFDM subcarrier is never located at DC, even though it appears to do so on this sketch.



Spectral overlap results in BW-efficiency improvement of about 2x compared to ordinary modulation.

Note that use of the CP costs time, energy, lowers the data rate, and lowers the bandwidth efficiency. But use of the CP does NOT affect the bandwidth.

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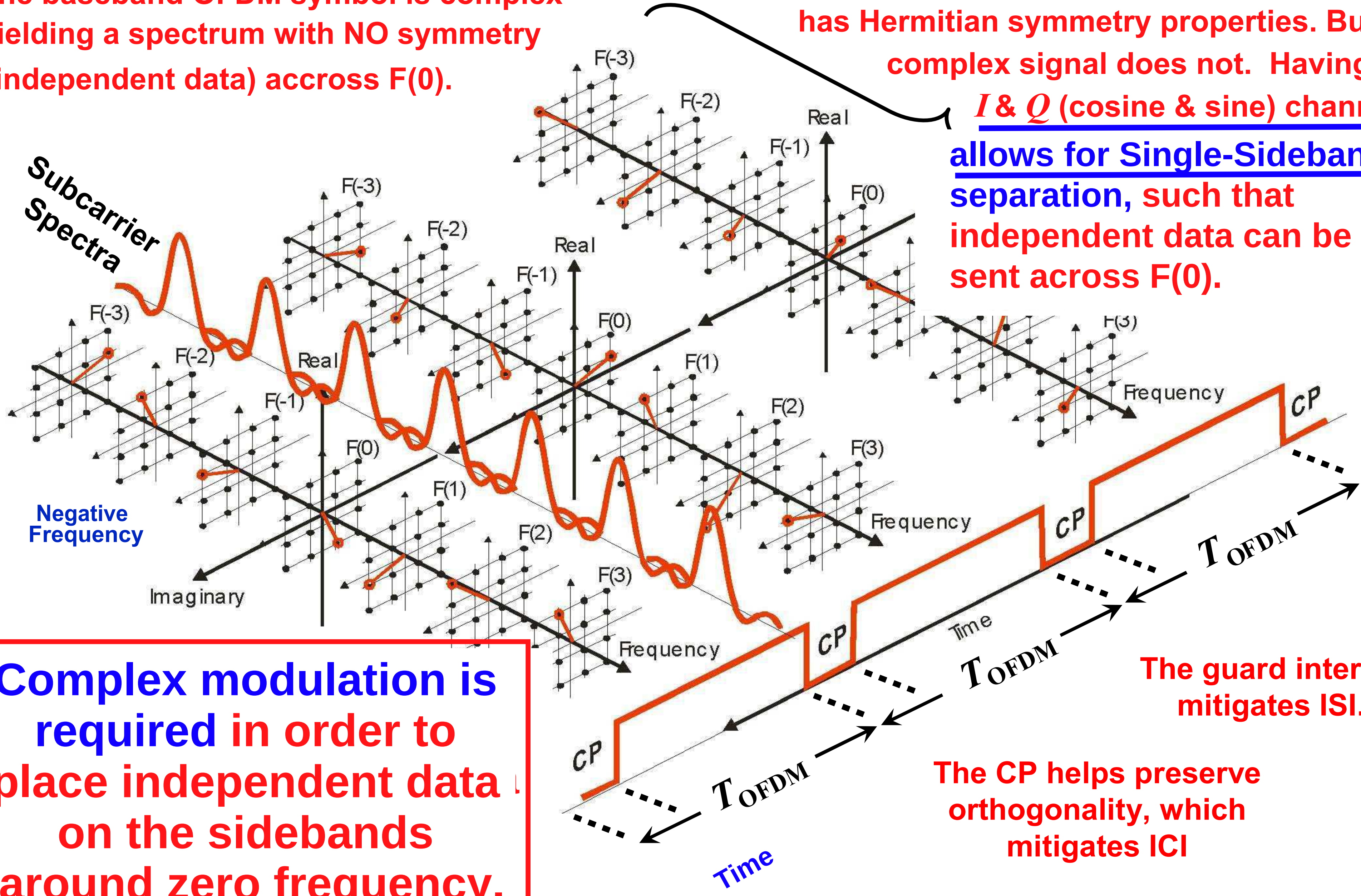
Data Constellation Points Distributed over Time-Frequency Indices

1\. The baseband OFDM symbol is complex yielding a spectrum with NO symmetry (independent data) accross F(0).

3\. Modulation with such complex signals is not unique to OFDM. The transform of any real baseband signal has Hermitian symmetry properties. But a complex signal does not. Having *I* & *Q* (cosine & sine) channels allows for Single-Sideband separation, such that independent data can be sent across F(0).



2\.

Complex modulation is required in order to place independent data on the sidebands around zero frequency.

The guard interval mitigates ISI.

The CP helps preserve orthogonality, which mitigates ICI

Data Constellation Points Distributed over Time-Frequency Indices

The baseband OFDM symbol is complex yielding a spectrum with NO symmetry **(independent data)** accross F(0).

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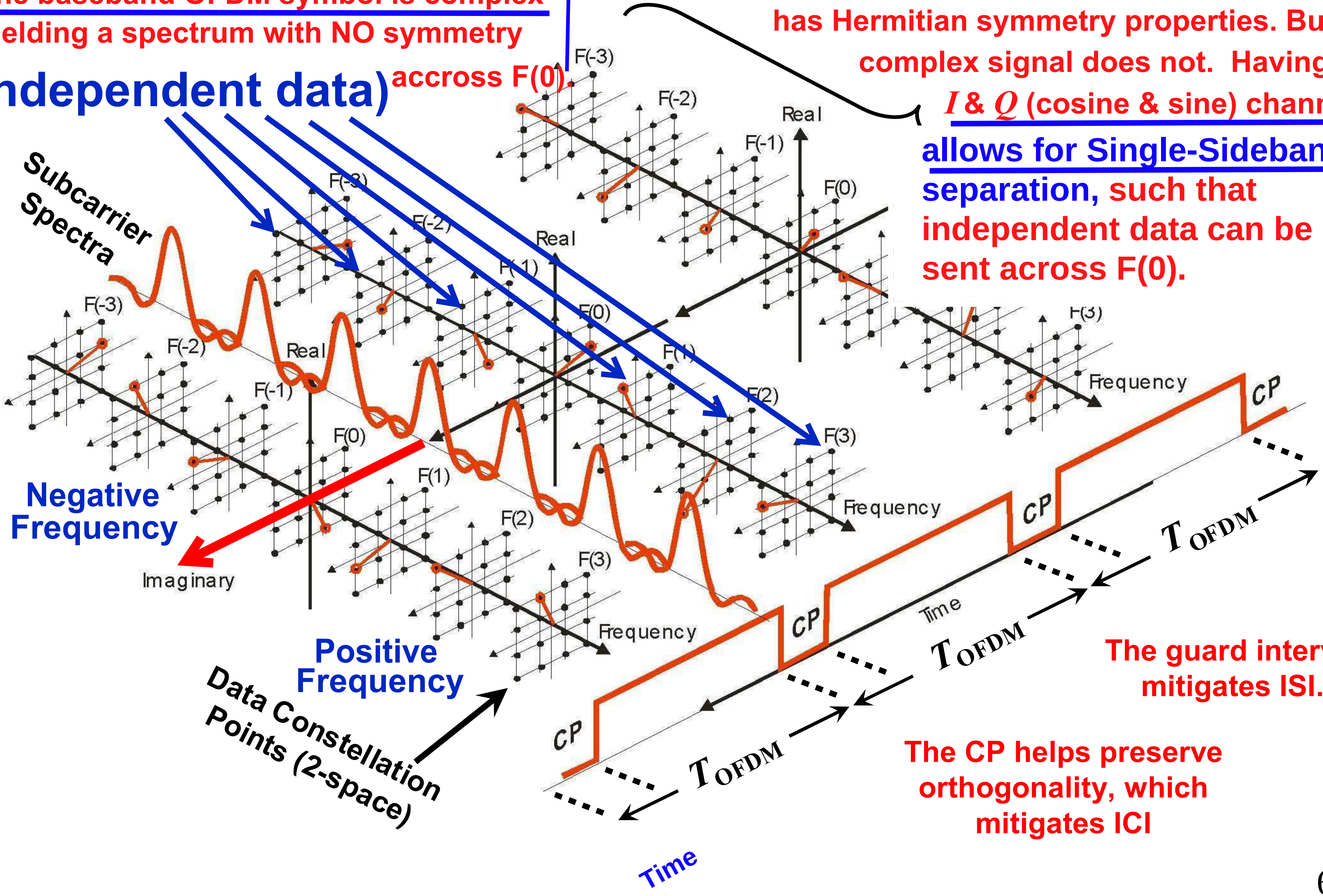

Subcarrier Spectra

Negative Frequency

Positive Frequency

Data Constellation Points (2-space)

Time

The guard interval mitigates ISI.

The CP helps preserve orthogonality, which mitigates ICI

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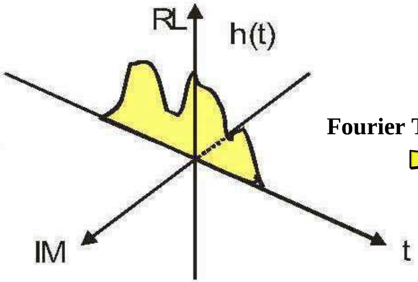
Real and Imaginary Signals

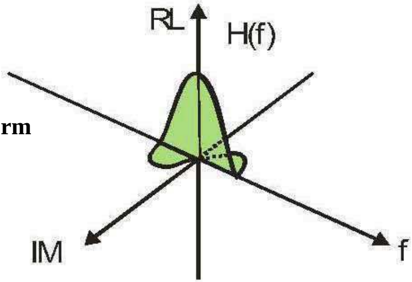
and Hermitian Symmetry

Real Time Signal



Fourier Transform

Complex Spectrum
with even & odd symmetry



Similarly, an Imaginary Time signal has a Complex Spectrum
with even & odd symmetry.

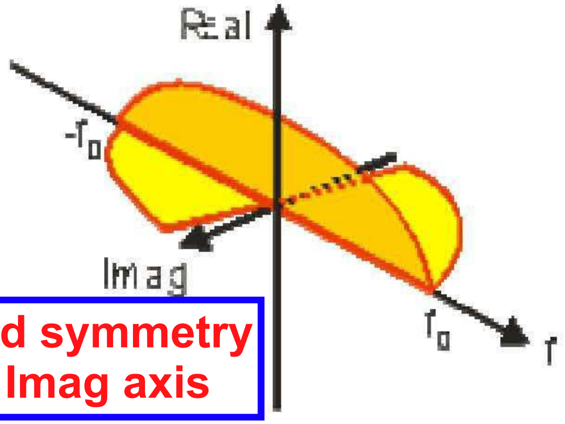
Spectra of Real and Imaginary Signals:

Spectrum of each is Complex.

Spectrum of each displays Hermitian Symmetry

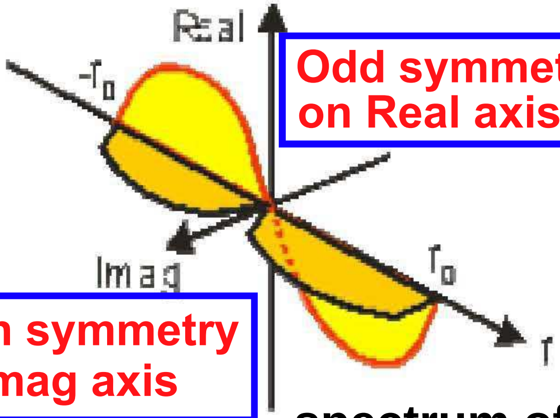
Real signal means there is NO j term

Even symmetry on Real axis



Odd symmetry on Imag axis

spectrum of Real Signal:



Odd symmetry on Real axis

Even symmetry on Imag axis

spectrum of Imaginary Signal:

Hermitian Transform Properties

- Real signals are typically made up of both cosine and sine components. Hence, the Fourier transform of a real signal is generally complex, and shows cosines on the real axis and sines on the imaginary axis.

- In the frequency domain, the spectrum of a real signal manifests even symmetry on the real axis, and odd symmetry on the imaginary axis (known as Hermitian transform properties). Even and/or odd symmetry in one domain corresponds to the same symmetry properties in the other domain.

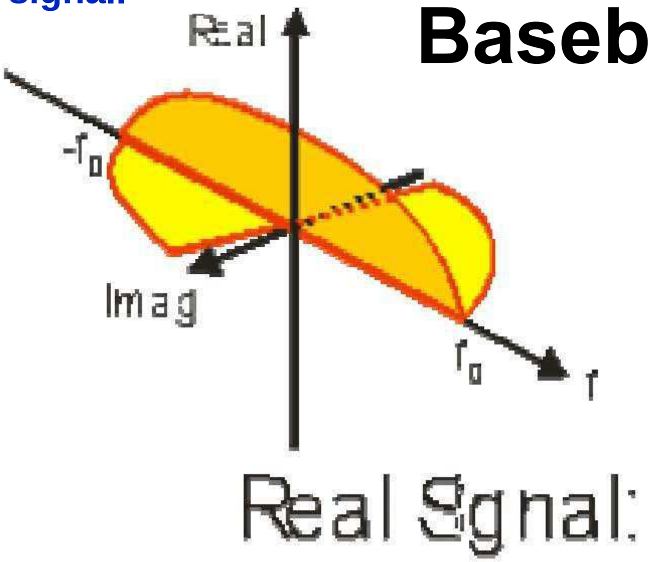
- Upper figure shows the spectrum of a real signal having such Hermitian (even/odd) properties.

- Lower figure shows the spectrum of an imaginary signal (j times a real signal), having anti-Hermitian properties (odd symmetry on the real axis, and even symmetry on the imaginary axis).

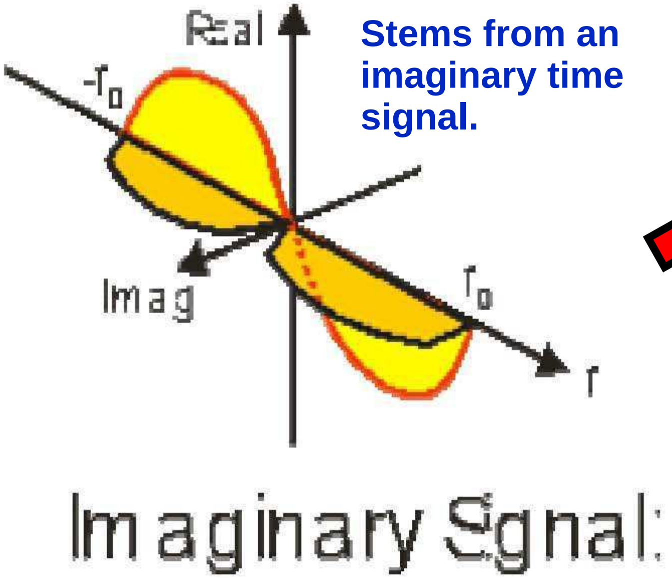
Complex Baseband signals have NO spectral symmetry.

Stems from a real time signal.

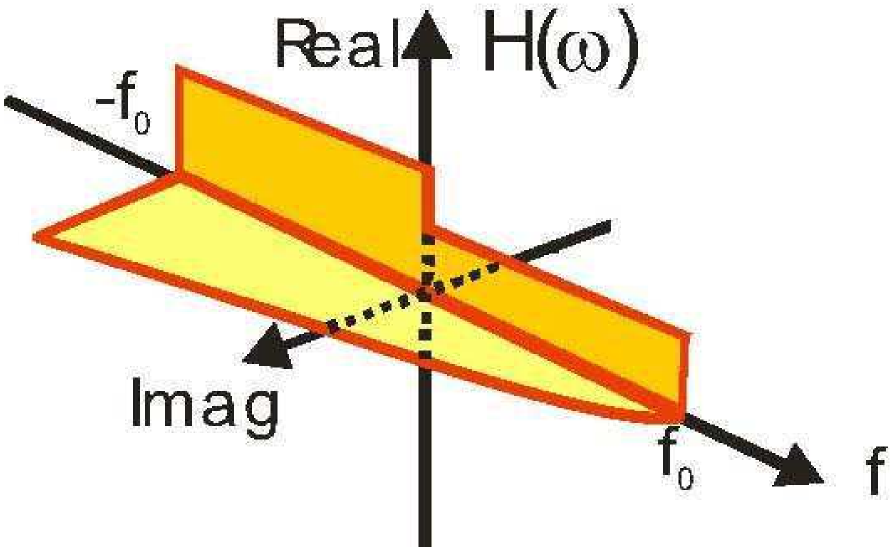
Non-Hermitian Spectrum of a Complex Baseband Signal stemming from a Real and an Imaginary Signal



Stems from an imaginary time signal.



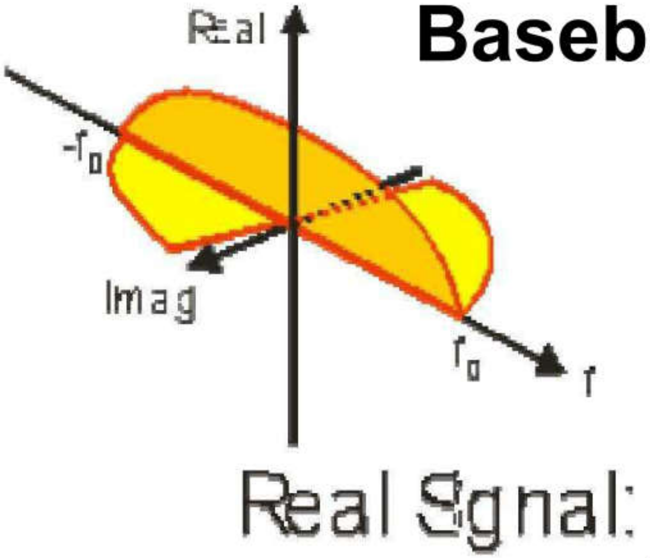
Adding the complex spectra of real and imaginary time signals yields no spectral symmetry whatever.

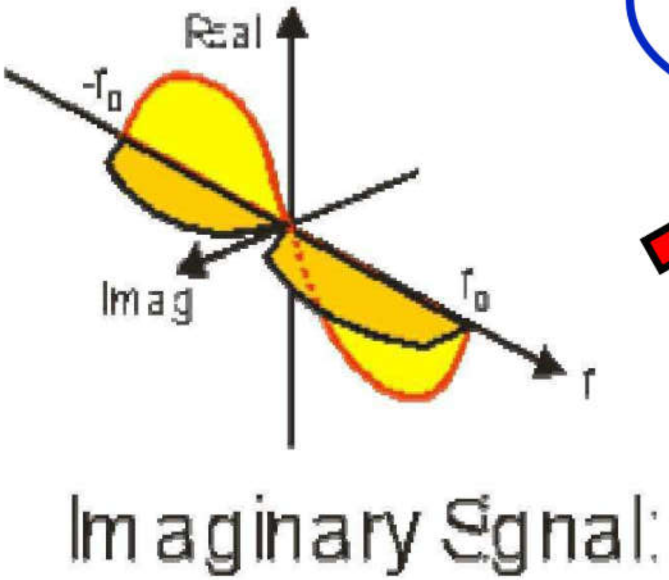


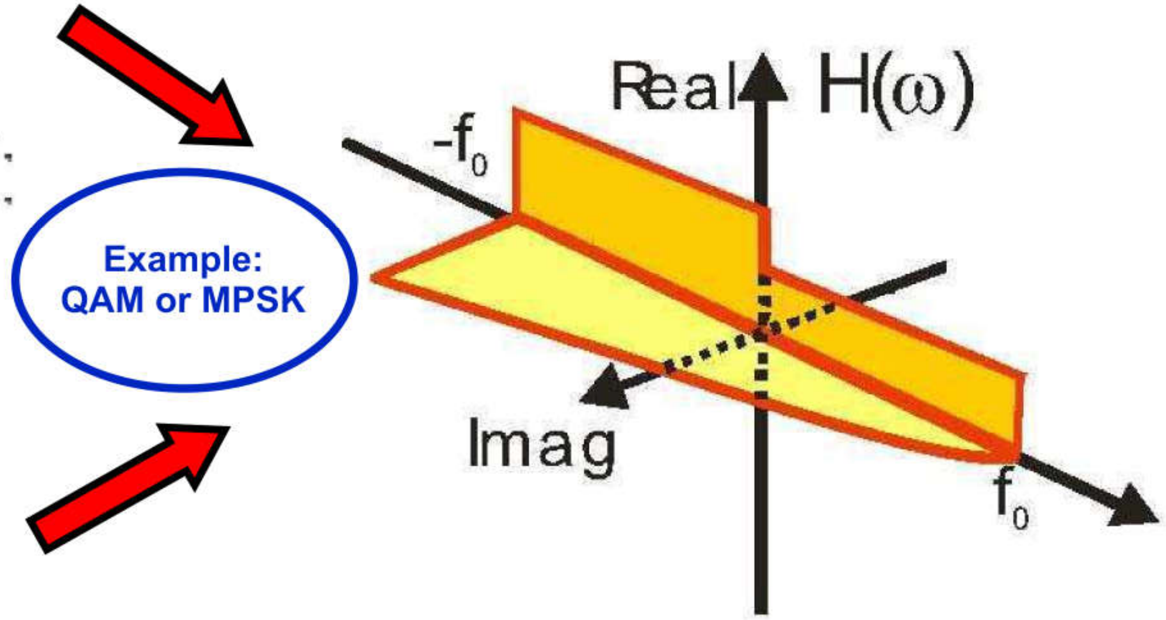
If the spectrum of a signal has no symmetry at all, then that spectrum must have stemmed from a complex time signal.

Non-Hermitian Spectrum of a Complex Baseband Signal stemming from a Real and an Imaginary Signal

Example: $a + jb$







Example:
QAM or MPSK

Spectrum has NO symmetry.
Since the time-signal is complex,
it cannot be sent over a single wire or antenna.

Such a complex baseband time-signal has a spectrum with independent data around zero frequency.

Complex Baseband & Real Band-Centered

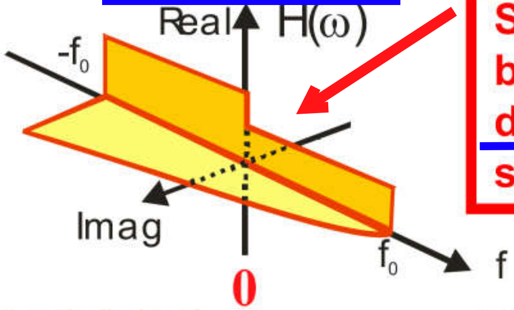
Hence, there is independent data around zero frequency

OFDM Example:

*

We start with a complex baseband time signal having independent complex data symbols around zero frequency. Such a signal will require two cables for transmission.

Baseband

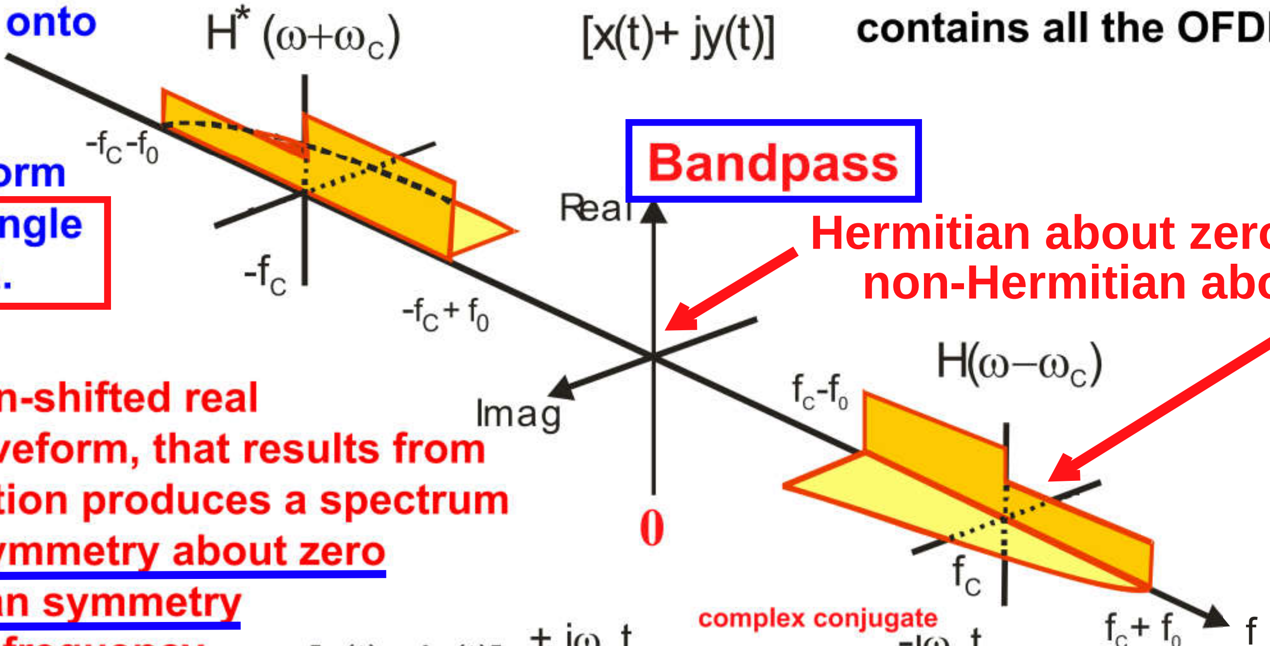


Spectrum of a complex baseband time signal does not have Hermitian symmetry about zero.

Then, the complex baseband time signal is modulated onto a carrier (real part onto cosine, imaginary part onto sine), thereby producing a real waveform that can be sent on a single cable, or on an antenna.

OFDM transmission BW contains all the OFDM sucarriers

Bandpass



Hermitian about zero and non-Hermitian about carrier

The up- and down-shifted real transmission waveform, that results from complex modulation produces a spectrum with Hermitian symmetry about zero and non-Hermitian symmetry about the carrier frequency.

complex conjugate

$$[x(t)+jy(t)]\,e^{+j\omega_c t}+[x(t)-jy(t)]\,e^{-j\omega_c t}$$

RF waveform $x(t)\cos(\omega_c t)-y(t)\sin(\omega_c t)$

See Sklar text Appendix D, Equations D.4 and D.5

The RF waveform is formed as the real part of A = $[x(t) + jy(t)]\exp(j\omega_c t)$, obtained by adding the complex conjugate of A to itself, and scaling by one-half.

Thank You

See you next week: March 25, 2021

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