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The A B Cs of OFDM
Part 2

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Abstract: The main benefit of OFDM is its ability to elegantly cope with severe multipath channel conditions without needing complex equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a high-data-rate signal into smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

**OFDM's main function is to manipulate orthogonal sinusoids.**

**What are 2 Key Characteristics of Orthogonal Sinusoids?**

1. There must be an integer number of cycles of each subcarrier sinusoid contained in the interval $T_s$.

2. $f = 1/T_s$

where $T_s$ is the data portion of the OFDM symbol.
Other Orthogonality Characteristics

- For reconstructing the correct OFDM subcarriers at the receiver:

  We need to maintain signal orthogonality. This is accomplished by

  1. preserving signal length
  2. preserving constant envelope
  3. preserving an integer number of cycles per gated sinusoid

- Preserving Length

  The use of linear convolution with an $N$-point DFT would create a lengthened output. But, by making the signal (with a CP) appear circularly convolved, the original signal length is preserved.

- Preserving Constant Envelope

  Convoluting a signal with the channel impulse response causes a transient at the start and end of the symbol. Any such transient causes envelope variations. The CP absorbs the starting transient of the current symbol and the stopping transient of the previous symbol. By discarding the CP in the guard interval (the overlapping transient), we thereby preserve a constant envelope for each gated-sinusoid symbol.

- Preserving an Integer Number of Cycles

  Discarding the CP guard interval also preserves the integer number of cycles of each symbol (the way it was originally created).
Orthogonality in the time domain assures orthogonality in the frequency domain, and vice versa. The property is easiest to see in the time domain.

\[ \int_0^T s_1(t) s_2(t) \, dt = 0 \]

S_1 and S_2 cannot possibly interfere with one another.
\[ f = \frac{1}{T_s} \] (BW of each subchannel)

where \( T_s \) is the data portion of the OFDM symbol time.

The data, 16-QAM here, modulates the subcarrier sinc spectra (amplitude and phase). The center frequency of these sincs are the selected OFDM subcarrier values.

They are turned on, and turned off corresponding to the OFDM symbol interval.

Why do we call these gated sinusoids?

Note the real & imaginary axes
An OFDM system, with $N_c = 4$ subcarriers and 16-QAM modulation

Example of the OFDM time/frequency structure, focusing on a grid where $N_c = 4$ candidate subcarriers located at each symbol time.

The subcarrier's amplitude can be zero, as seen in 8 unoccupied grid points here. They have the potential to be assigned.

$\text{potential}$

$\text{partitioned data}$

$\text{Gated sinusoids}$

$\text{superposition of data at successive times}$

$\text{baseband frequency}$

$\text{symbol #1}$

$\text{symbol #2}$

$\text{symbol #3}$

$\text{time}$

$\text{frequency}$

$\text{real}$

$\text{imaginary}$

$\text{CP}$

$f = 1/T_s$
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Periodic Outputs on a Unit Circle
The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

Gated Sinusoid (one sinusoid with an integer number of cycles)

Sinc function spectrum
The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

Gated Sinusoid (one sinusoid with an integer number of cycles)

And if the gated sinusoid is discretely sampled, the transform will be periodic.

Let's plot this periodic spectrum as a power signal on a unit circle.
The DFT of a discretely sampled time sequence yields a continuous periodic spectrum.

Also, if the spectrum is periodic (easily seen on a unit circle), then the transform must have stemmed from discrete samples.

Easily seen, because start equals finish.
Plotting the spectrum on a unit circle helps us visualize (as we go round-and-round the circle) that the spectrum is periodic.

A circular plot only displays the periodic events (or the ssr) of a process (no transients).

After removal of the CP at the receiver, there are no transients.

The CP (and its removal) brings about what this plot portrays, namely (NO boundary transients), which makes it possible to perform circular convolution.
After discarding the discontinuities carried by the cyclic prefix, what remains is a "steady-state" signal as would have arisen from circular convolution.

The steady-state response (ssr) has essentially gotten rid of all the On-Off transients.
The Cyclic Prefix in OFDM Modifies Linear Convolution so that it Appears to be Circular Convolution

- **A property of the Fourier Transform:**
  
  Spectral multiplication of continuous signals \( X(f) H(f) \) corresponds to linear convolution \( x(t) * h(t) \) in time.

- **A property of the Discrete Fourier Transform (DFT):**
  
  Spectral multiplication of sampled signals \( X(k) H(k) \) corresponds to circular convolution \( x(n) \circledast h(n) \) in time (sampling the transform makes the time signal periodic, and sampling the time signal makes the transform periodic).

- **When Using DFTs for implementing OFDM systems:**
  
  A continuous waveform, linearly convolved with the channel impulse response, is modified so that it appears to be circularly convolved with the channel impulse response. This makes the task of equalization simple – spectral scaling during the DFT.

\* The DFT forms a sampled-data spectrum. Samples in the frequency domain correspond to periodicity in the time domain. Any periodic function on a time-line is nicely portrayed as one copy of the function plotted on a unit circle (start and finish are the same point). This makes linear convolution appear to be circular.
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**OFDM Modem Block Diagram**

- An OFDM symbol is made up of a sum of $N$ terms ($N_c$ modulated orthogonal carriers plus null bins). Each $k^{th}$ sample of a symbol can be represented as:

  $$x_k = \sum_{n=0}^{N-1} d_n e^{j \frac{2\pi}{N} nk} \quad n, k = 0, 1, 2, \ldots, N - 1$$

  where some of the $d_n$ values are zero.

$N$ is made larger than $N_c$ by zero-padding $N_c$ in the frequency domain, which raises the output sample rate (time interpolation).

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**IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.**

Parallel group of $N_c$ data symbols (points in 2-space following some $M$-ary modulation) plus null symbols. Each symbol is mapped into a different frequency bin, thus yielding details of each sinusoid to be built.

**Think of the IDFT as a signal generator. Constellation points in, and time waveforms out.**

Don't confuse $N_c$ with $N$. $N_c$ represents the data (constellation points) or subcarriers, and $N$ is the transform size. For building real analog filters, we use zero extensions (null bins) to form the transform such that $N > N_c$. 

---

**CP discarded**

**Using M-ary Modulation**

- S/P
- Constellation Mapping (Modulation)
- N-Point Inverse DFT (IDFT)
- MUX
- N-Point Forward DFT
- Channel
- P/S

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**CP discarded**
This slide shows the OFDM signal processing, making it appear as if the $N$ output wires of the IDFT generate samples of $N$ different tones, which means there would have to be:

- $N$ coherent oscillator/modulators (very costly processing).
- Some of the $N$ tones would have zero amplitudes, leaving $N_c$ enabled tones.

The actual Inverse Fourier Transform processor will output the sum of its $N$ output wires (the superposition of all the enabled $N_c$ tones). Each of the $N$ output wires holds a successive time sample of the same superposition.

With this implementation, it is only possible to see the sum of the $N_c$ tones, but not any one of them. They can only be seen alone after detection at the receiver.
59. In the early "battle" for the best codes (convolutional vs. Reed-Solomon), what are the arguments for each, and why did convolutional win out?

60. In mobile channels, how does the terrain affect fading? How does the mobile-velocity affect it?

61. What is the advantage of circular-convolution versus linear-convolution? How do we trick the channel into performing circular convolution?

62. In OFDM, what is the mitigation technique for precluding ISI? For precluding ICI?

63. Baseband OFDM symbols are typically made up of independent data at positive and negative spectral locations. How is this effected, and how is a real transmission-signal formed?

64. For maintaining orthogonality among the subcarriers in OFDM, the tone spacing was chosen to be $1/T_s$. Why wasn't it chosen to be $1/T_{OFDM}$? (Sklar ADC notes, section 3)

65. How can SC-OFDM still be resistant to multipath when the data symbols are so short? Hint: The time duration of a data pulse is longer than its main lobe.

66. Early skeptics about MIMO, claimed that it violated Shannon's capacity theorem. Why is that not the case?

67. Why won't MIMO work in a multipath-free environment?

68. Often, the signal-processing operations "DFT and IDFT" are called out as "FFT and IFFT," when one means the mathematical transformation. Why is this NOT precise?

69. What are the Key Control Loops needed for system Synchronization? (Fred Harris, "Let's Assume the System is Synchronized.")

70. How do you shape a time waveform to meet system spectral-confinement requirements? Hint: symbol rate, sample rate, window type, filter length, transition BW, out-of-band attenuation.
Complex modulation is required in order to place independent data on the sidebands around zero frequency.

1. The baseband OFDM symbol is complex yielding a spectrum with NO symmetry (independent data) across F(0).

2. Subcarrier Spectra

3. Modulation with such complex signals is not unique to OFDM. The transform of any real baseband signal has Hermitian symmetry properties. But a complex signal does not. Having \( I & Q \) (cosine & sine) channels allows for Single-Sideband separation, such that independent data can be sent across F(0).

The guard interval mitigates ISI.

The CP helps preserve orthogonality, which mitigates ICI.
Data Constellation Points Distributed over Time-Frequency Indices

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Real and Imaginary Signals
and Hermitian Symmetry
A sampled data time series has a continuous periodic spectrum, and the alias-free BW is bounded by the sample rate.

To satisfy Nyquist, the sample rate must at least be equal to the 2-sided BW, which it is in this sketch.

No smearing between spectral copies.

Periodic outputs can easily be plotted on a unit circle.
Spectra of Real and Imaginary Signals: Spectrum of each is Complex. Spectrum of each displays Hermitian Symmetry

Hermitian Transform Properties

- Real signals are typically made up of both cosine and sine components. Hence, the Fourier transform of a real signal is generally complex, and shows cosines on the real axis and sines on the imaginary axis.

- In the frequency domain, the spectrum of a real signal manifests even symmetry on the real axis, and odd symmetry on the imaginary axis (known as Hermitian transform properties). Even and/or odd symmetry in one domain corresponds to the same symmetry properties in the other domain.

- Upper figure shows the spectrum of a real signal having such Hermitian (even/odd) properties.

- Lower figure shows the spectrum of an imaginary signal (\( j \) times a real signal), having anti-Hermitian properties (odd symmetry on the real axis, and even symmetry on the imaginary axis).

Complex Baseband signals have NO spectral symmetry.
Non-Hermitian Spectrum of a Complex Baseband Signal stemming from a Real and an Imaginary Signal

If the spectrum of a signal has no symmetry at all, then that spectrum must have stemmed from a complex time signal.

Adding the complex spectra of real and imaginary time signals yields no spectral symmetry whatever.
Non-Hermitian Spectrum of a Complex Baseband Signal stemming from a Real and an Imaginary Signal

**Example:** \( a + jb \)

From this sketch, one should see that independent signaling across \( F(0) \) is achieved by choosing the proper values of \( a + jb \) or their spectra.

Spectrum has NO symmetry. Since the time-signal is complex, it cannot be sent over a single wire or antenna.

Such a complex baseband time-signal has a spectrum with independent data around zero frequency.
Complex Baseband & Real Band-Centered

OFDM Example:
We start with a complex baseband time signal having independent complex data symbols around zero frequency. Such a signal will require two cables for transmission.

Then, the complex baseband time signal is modulated onto a carrier (real part onto cosine, imaginary part onto sine), thereby producing a real waveform that can be sent on a single cable, or on an antenna.

Hence, there is independent data around zero frequency.

The up- and down-shifted real transmission waveform, that results from complex modulation produces a spectrum with Hermitian symmetry about zero and non-Hermitian symmetry about the carrier frequency.

The RF waveform is formed as the real part of \( A = [x(t) + jy(t)] \exp(j\omega_ct) \), obtained by adding the complex conjugate of \( A \) to itself, and scaling by one-half.

See Sklar text Appendix D, Equations D.4 and D.5
The baseband OFDM symbol is complex yielding a spectrum with NO symmetry across F(0). Modulation with such complex signals is not unique to OFDM. The transform of any real baseband signal has Hermitian symmetry properties. But a complex signal does not. Having I & Q (cosine & sine) channels allows for Single-Sideband separation, such that independent data can be sent across F(0).

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Our "Wish List"
### OFDM Glossary (and Channel Parameters)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_c)</td>
<td>number of subcarriers</td>
</tr>
<tr>
<td>(N &gt; N_c)</td>
<td>transform size (data-symbol samples)</td>
</tr>
<tr>
<td>(N_c = 0.6 , N)</td>
<td>typical subcarrier apportionment</td>
</tr>
<tr>
<td>(N_{cp})</td>
<td>cyclic prefix samples</td>
</tr>
<tr>
<td>(N_L = N + N_{cp})</td>
<td>total samples per OFDM symbol</td>
</tr>
</tbody>
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<tr>
<td>(T_L)</td>
<td>sample time</td>
</tr>
<tr>
<td>(T_s = (N \times T_L))</td>
<td>symbol time (data portion)</td>
</tr>
<tr>
<td>(T_{cp})</td>
<td>cyclic prefix time</td>
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<tr>
<td>(T_{cp} = 0.25 , T_s)</td>
<td>typical CP apportionment</td>
</tr>
<tr>
<td>(T_{OFDM} = (T_s + T_{cp}) = (N_L \times T_L))</td>
<td>OFDM symbol time</td>
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</table>

\[
\Delta f = \frac{1}{T_s}
\]

- frequency difference between adjacent subcarriers

\[
f_s = (N \times \Delta f) = \frac{1}{T_L}
\]

- sample rate

\[
W_{\text{signal}} = (N_c + 1) \, \Delta f
\]

- OFDM modulation BW

#### Channel parameters:

<table>
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<tr>
<td>(T_m)</td>
<td>max multipath delay</td>
</tr>
<tr>
<td>(\sigma_\tau)</td>
<td>rms multipath delay</td>
</tr>
<tr>
<td>(f_0 \approx 1/T_m)</td>
<td>coherence BW</td>
</tr>
<tr>
<td>(f_0(50%) \approx 1/5\sigma_\tau)</td>
<td>coherence BW over which the spectral correlation is at least 0.5</td>
</tr>
<tr>
<td>(f_d)</td>
<td>fading rate (Doppler spectral spreading)</td>
</tr>
<tr>
<td>(T_0 \approx 1/f_d)</td>
<td>coherence time</td>
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Why OFDM?

- **Divide-and-Conquer**
  - Mitigation for frequency-selective fading environments. Parse the single, high-rate channel into $N_c$ low-rate overlapping, and orthogonal, sub-channels.

- Subdivide $W_{\text{signal}}$ by large $N_c$ so that $f = W_{\text{signal}} / N_c \ll f_0$
  - We desire flat faded sub-channels
  - For a fixed $W_{\text{signal}}$
    - Large $N_c$ $\Rightarrow$ Large $T_s = N_c / W_{\text{signal}}$
    - $\Rightarrow$ Reduced relative ISI when $T_s \gg \sigma_t$
  - But, for slow fading, we want $T_s < T_0$, thus $T_0$ defines the upper bound of $N_c$ Otherwise, pulse mutilation

- We want that:
  - $T_m < T_s < T_0$
  - where $T_s =$ time duration of the data portion of OFDM symbol

---

**Alternative Wish-List Statement**

$T_m < T_s < T_0$

---

Coherence BW | Symbol rate | Fading Rate
---|---|---
In General $f_0 > 1/T_s > 1/T_0$ partitioned
For OFDM $f_0 > W_{\text{signal}} / N_c > 1/T_0$

Our "Wish List"
Recall the WSSUS model. We want to avoid frequency-selective fading. Notice our wish-list below. OFDM makes it easy to achieve flat fading.

**Partitioning a high-data-rate**

- Subdivide $W_{\text{signal}}$ by large $N_c$ so that $W_{\text{signal}}/N_c \ll f_0$
- We desire flat faded sub-channels
- Large $N_c \Rightarrow$ Large $T_s = N_c/W_{\text{signal}}$

⇒ Reduced relative ISI when $T_s >> \sigma_t$
But, for slow fading, we want $T_s < T_0$, thus $T_0$ defines the upper bound of $N_c$

- We want that:

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<td>Our &quot;Wish List&quot;</td>
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</table>

Alternative Wish-List Statement

$$T_m < T_s < T_0$$

where $T_s =$ time duration of the data portion of OFDM symbol
\( N_c \) represents the number of potential (candidate) subcarriers, with locations of \( k \cdot f \), where \( k \) is any positive or negative integer.

- We want that:

  **Coherence BW**  
  In General  \( f_0 > 1/T_s \)  
  For OFDM  \( f_0 > W_{\text{signal}} / N_c \)  

  **Fading Rate**  
  \( > 1/T_0 \)  

  **Symbol Rate**  
  \( > 1/T_0 \)

Our "Wish List" to preclude frequency-selective & fast fading

where  \( W_{\text{signal}} / N_c = f \)  
subchannel BW

\( \frac{\text{Total OFDM bandwidth}}{\text{number of candidate subcarriers}} \)
Why OFDM?

- **Divide-and-Conquer**
  - Mitigation for frequency-selective fading environments. Parse the single, high-rate channel into $N_c$ low-rate overlapping, and orthogonal, sub-channels.

---

**How large should $N_c$ be?**

⇒ Reduced relative ISI when $T_s >> \sigma_t$

But, for slow fading, we want $T_s < T_0$, thus $T_0$ defines the upper bound of $N_c$.

- We want that:
  - Coherence BW in general $f_0 > 1/T_s > 1/T_0$
  - Fading Rate
  - OFDM $f_0 > W_{\text{signal}} / N_c > 1/T_0$

**Our "Wish List"**

---

Without OFDM: signaling rate $= 1/T_s = W_{\text{signal}}$

With OFDM, for each subcarrier: signaling rate $= W_{\text{signal}} / N_c$

$T_s = \text{time duration of the data portion of OFDM symbol}$
How Large should $N_c$ be? What Range should it fall in?

\[ f = \frac{1}{T_s} = \text{OFDM symbol rate} \]

\[
f_0 > \frac{W_{\text{signal}}}{N_c} > \frac{1}{T_0}
\]

Take the reciprocal of each term, invert the inequalities, and then multiply by the signal Bandwidth.

\[
\frac{W_{\text{signal}}}{f_0} < N_c < T_0 W_{\text{signal}}
\]

The left-side inequality allows computation of minimum $N_c$. The right-side inequality allows computation of maximum $N_c$. 


OFDM System Design Example

Assume a wireless channel with an rms delay spread $\sigma_\tau$ of 5 $\mu$sec, and a coherence time $T_0$ of 50 $\mu$sec. If the transmission BW is 20 MHz, find the min and max number of OFDM subcarriers needed in order to insure flat-fading and slow-fading effects.

\[
\begin{align*}
\text{coherence BW} & \quad \text{signaling rate} & \quad \text{fading rate} \\
F_0 & > \frac{1}{T_s} & > \frac{1}{T_0} \\
W_{\text{signal}} & > \frac{W_{\text{signal}}}{N_c} & > \frac{1}{T_0} \\
\frac{W_{\text{signal}}}{F_0} & < N_c & < T_0 \frac{W_{\text{signal}}}{T_0}
\end{align*}
\]

Note that a coherence time of 50 microseconds is very short. A typical mobile system's coherence time is in the order of milliseconds.

General "Wish List"

\[W_{\text{signal}}/N_c = f = 1/T_s\]

OFDM "Wish List"
Using the $f_0 (50\%)$ approximation

$$f_0 \approx \frac{1}{5 \sigma_r} = \frac{1}{5 \times 5 \times 10^{-6}} = 40 \text{ kHz}$$

$$(N_c)_{\text{min}} \geq \frac{W_{\text{signal}}}{f_0} = \frac{20 \text{ MHz}}{40 \text{ kHz}} = 500$$

$$(N_c)_{\text{max}} \leq T_0 W_{\text{signal}} = 50 \times 10^{-6} \times 20 \times 10^6 = 1000$$

Number of subcarriers should fall in the range of 500 to 1000.

In this example, one of the given parameters is NOT realistic. Which one?
Spaced-Frequency Correlation Function shows the spectral correlation of received narrow-band signals spaced $\Delta f$ apart.

Separated by $\Delta f$

Increasing $\Delta f$.

$f_0(90\%)$ and $f_0(50\%)$

The positioning is a random process, dependent on the nature of the propagation path (the terrain).

$f_0(90\%)$ is defined as the spectral interval over which the spaced-freq correlation function has a correlation of at least 0.9.

$f_0(50\%)$ is defined as the spectral interval over which the spaced-freq correlation function has a correlation of at least 0.5.
The parameter $T_0 = 50 \times 10^{-6}$ seconds is not realistic because:

Assume a carrier frequency of $f_c = 300$ MHz

$$\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{3 \times 10^8} = 1 \text{ m}$$

$$T_0 \approx \frac{0.5 \times \lambda}{\nu} = \frac{0.5}{\nu} = 50 \times 10^{-6} \text{ very short coherence time}$$

$$\nu = \frac{0.5}{T_0} = \frac{0.5}{50 \times 10^{-6}} = 10 \times 10^3 \text{ m/s} = 22,369.4 \text{ miles/hour}$$

Coherence time (average channel-state time duration) is a function of velocity. For the given $T_0$ of only $50 \times 10^{-6}$ sec, the velocity needs to be unreasonably large.

This makes sense. Coherence time is inversely proportional to velocity. When there is no motion, coherence time is infinite.
Abstract: The main benefit of OFDM is its ability to cope with severe multipath channel conditions without needing complex equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a high-data-rate signal into smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

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- Testing for Orthogonality.
- **Tricking the Channel.**
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).
Transforming Linear Convolution to Circular Convolution

This is our tool for tricking the channel.
by rearranging the past and the future

• The cyclic-prefix-end matches the signal-front. There will no longer be a transient at the original time signal's starting edge. The transient now resides at the new starting edge of the cyclic prefix which will be tossed.

Integer number of cycles per symbol interval
Hence back-end of CP = front-end of symbol
Continuous edge between added CP and old starting edge
Transient at new starting edge

- End of cycle at back end
- Start of cycle at front end
- Continuous edge
- Transient at new starting edge

MOVING THE DISCONTINUITIES
An Example of Tricking the Channel

(Converting Linear Convolution to Circular Convolution)
Example: Use of the CP makes linear convolution appear circular

Sampled data \( x(k) = [1, 2, 3, 2, 1] \)

\[
\begin{align*}
x(k) & \text{ transmitted over channel } h(k) = [3, 2, 1] \\
\end{align*}
\]

Time-reverse one of the functions. Here, \( h(k) \) was reversed. Then perform multiply, add, and shift (product integration).

\[
\begin{align*}
1 & 2 3 2 1 \\
1 & 2 3 & = 3 \\
1 & 2 3 2 1 \\
1 & 2 3 & = 8 \text{ etc.}
\end{align*}
\]
Note: The original five data samples have grown to seven.

Circularized time index $k$

On the circle, Outer Counter-Clockwise values are the positive bins. Inner Clockwise values are the Negative Bins.

We have unfurled the circularized time index $k$ in order to see $h(-k)$ as a function of positive time. In effect, we have rendered $h(-k)$ causal by repositioning each of its sample values to be placed in positive bins.

Repositioning neg. samples

0 is still 0.
-1 becomes +4.
-2 becomes +3.

$h(-k)$ bins at the start of convolution with circular indices

On the circle, time -2 corresponds to bin 3, and time -1 corresponds to bin 4.

Repositioning neg. samples

0 is still 0.
-1 becomes +4.
-2 becomes +3.

Time-reversed channel impulse response

$\text{Linear convolution output } x(k)*h(k) = [3, 8, 14, 14, 10, 4, 1]$

What would a circularly convolved $x(k) \boxtimes h(k)$ look like?
Example: Use of the CP makes linear convolution appear circular

Sampled data \( x(k) = [1, 2, 3, 2, 1] \)

\[
x(k) \text{ transmitted over channel } h(k) = [3, 2, 1]
\]

Circular Convolution

\[
\begin{array}{cccccc}
1 & 2 & 3 & 2 & 1 \\
3 & 0 & 0 & 1 & 2
\end{array}
\]

sampled data sequence \( x(k) \)

circularized reversed impulse response \( h(k) \)
Circular Convolution yields a very different looking output

7  9  14  14  10  This is what we want.

Unlike linear convolution, length is preserved.

Circular Convolution Output

Even though linear convolution and circular convolution look very different, watch how we can trick the channel into performing circular convolution.
How can we make the channel's linear convolution look like circular convolution? We trick it with a CP (longer than the channel delay spread) making the signal look periodic.

Add a CP to the original sampled sequence so that \( x'(k) = [2, 1, 1, 2, 3, 2, 1] \), and perform linear convolution with the channel impulse response. The resulting sequence [6, 7, 7, 9, 14, 14, 10, 4, 1] is seen below. **We need a trick**, because a signal \( x(k) \) transmitted over a channel with impulse response \( h(k) \) is received as the linear convolution \( x(k) \ast h(k) \). There is NO mechanism in nature that yields circular convolution.

This signal [6, 7, 7, 9, 14, 14, 10, 4, 1] results from a **linear** convolution with channel \( h(k) \). From the figure below, we see the similarity between the **circularly convolved** signal \( x(k) \otimes h(k) \) and the **linearly convolved** signal \( x'(k) \ast h(k) \).
How can we make the channel's linear convolution look like circular convolution? We trick it with a CP (longer than the channel delay spread) making the signal look periodic.

Add a CP to the original sampled sequence so that \( x'(k) = [2, 1, 1, 2, 3, 2, 1] \), and perform linear convolution with the channel impulse response. The resulting sequence \([6, 7, 7, 9, 14, 14, 10, 4, 1]\) is seen below. **We need a trick**, because a signal \( x(k) \) transmitted over a channel with impulse response \( h(k) \) is received as the linear convolution \( x(k) \ast h(k) \). There is NO mechanism in nature that yields circular convolution.

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\[
\begin{align*}
\text{from Rudyard Kipling:} \\
\text{East is east, and west is west, and never the twain shall meet. We need a trick (the CP) to bring the} \\
\text{beginning and the end together.}
\end{align*}
\]

Once a signal is launched, there is nothing in nature allowing us to fetch it back.

This signal \([6, 7, 7, 9, 14, 14, 10, 4, 1]\) results from a linear convolution with channel \(h(k)\). From the figure below, we see the similarity between the circularly convolved signal \(x(k) \mathcal{O} h(k)\) and the linearly convolved signal \(x'(k) \ast h(k)\).
Remember why we're doing this.

To help preserve orthogonality.

Isn't this clever?

The receiver removes the signal samples at both ends of \( x'(k) * h(k) \), and feeds the result to the DFT unit. After DFT processing, we obtain the desired frequency-domain result \( X(n) H(n) \). The CP has forced the linear convolution of \( x'(k) * h(k) \) in the channel to yield the desired appearance of a circularly-convolved signal.
OFDM Modem Block Diagram

- An OFDM symbol is made up of a sum of \( N \) terms (\( N_C \) modulated orthogonal carriers plus null bins). Each \( k^{th} \) sample of a symbol can be represented as:

\[
x_k = \sum_{n=0}^{N-1} d_n e^{\frac{j 2\pi nk}{N}} \quad n, k = 0, 1, 2, \ldots, N - 1
\]

where some of the \( d_n \) values are zero

\( N \) is made larger than \( N_C \) by zero-padding \( N_C \) in the frequency domain, which raises the output sample rate (time interpolation).

IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.

Input Data Symbols

Using M-ary Modulation eg., BPSK, MPSK, MQAM

Parallel group of \( N_C \) data symbols (points in 2-space following some \( M \)-ary modulation) plus null symbols. Each symbol is mapped into a different frequency bin, thus yielding details of each sinusoid to be built.

Single-carrier Spectrum of \( \{d\} \)

Multi-carrier Spectrum of \( \{x\} \)

Don't confuse \( N_C \) with \( N \). \( N_C \) represents the data (constellation points) or subcarriers, and \( N \) is the transform size. For building real analog filters, we use zero extensions (null bins) to form the transform such that \( N > N_C \).
The receiver removes the signal samples at both ends of $x'(k) \ast h(k)$, and feeds the result to the DFT unit. After DFT processing, we obtain the desired frequency-domain result $X(n)H(n)$. The CP has forced the linear convolution of $x'(k) \ast h(k)$ in the channel to yield the desired appearance of a circularly-convolved signal. This was accomplished by removing discontinuities at the boundaries, thereby maintaining orthogonality, and making equalization easy (multiplication by a complex scalar).
That's why we moved the discontinuities to the CP in the first place.

Removing the CP at the receiver, means that we've removed the discontinuities.

- The cyclic-prefix-end matches the signal-front. There will no longer be a transient at the original time signal's starting edge. The transient now resides at the new starting edge of the cyclic prefix which will be tossed.

- Integer number of cycles per symbol interval
- Hence back-end of CP = front-end of symbol
- Continuous edge between added CP and old starting edge
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OFDM Applications

Standard 802.11a

Wireless Local Area Networks (WLAN)
### OFDM Glossary (and Channel Parameters)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_c$</td>
<td>number of subcarriers</td>
</tr>
<tr>
<td>$N &gt; N_c$</td>
<td>transform size (data-symbol samples)</td>
</tr>
<tr>
<td>$N_c = 0.6 N$</td>
<td>typical subcarrier apportionment</td>
</tr>
<tr>
<td>$N_{cp}$</td>
<td>cyclic prefix samples</td>
</tr>
<tr>
<td>$N_L = N + N_{cp}$</td>
<td>total samples per OFDM symbol</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_L$</td>
<td>sample time</td>
</tr>
<tr>
<td>$T_S = (N \times T_L)$</td>
<td>symbol time (data portion)</td>
</tr>
<tr>
<td>$T_{cp}$</td>
<td>cyclic prefix time</td>
</tr>
<tr>
<td>$T_{cp} = 0.25 T_S$</td>
<td>typical CP apportionment</td>
</tr>
<tr>
<td>$T_{OFDM} = (T_S + T_{cp}) = (N_L \times T_L)$</td>
<td>OFDM symbol time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f = 1/T_S$</td>
<td>frequency difference between adjacent subcarriers</td>
</tr>
<tr>
<td>$f_s = (N \times \Delta f) = 1/T_L$</td>
<td>sample rate</td>
</tr>
<tr>
<td>$W_{signal} = (N_c + 1) \Delta f$</td>
<td>OFDM modulation BW</td>
</tr>
</tbody>
</table>

### Channel parameters:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_m$</td>
<td>max multipath delay</td>
</tr>
<tr>
<td>$\sigma_\tau$</td>
<td>rms multipath delay</td>
</tr>
<tr>
<td>$f_0 \approx 1/T_m$</td>
<td>coherence BW</td>
</tr>
<tr>
<td>$f_0(50%) \approx 1/5\sigma_\tau$</td>
<td>coherence BW over which the spectral correlation is at least 0.5</td>
</tr>
<tr>
<td>$f_d$</td>
<td>fading rate (Doppler spectral spreading)</td>
</tr>
<tr>
<td>$T_0 \approx 1/f_d$</td>
<td>coherence time</td>
</tr>
</tbody>
</table>
OFDM 802.11a

Uplink and Downlink are Symmetrical

- $N_c = 52$ Subcarriers (48 data + 4 pilot) in each 20 MHz Channel
- BPSK with rate 1/2 code: $1/2 \times 48 \times 250$ ksymb/s = 6 Mbps
- 64-QAM with rate 3/4 code: $3/4 \times 48 \times 6 \times 250$ ksymb/s = 54 Mbps

Channel Spacing: 20 MHz (func of reqd ACI)

Efficient spectral spacing to reduce ACI. Thus, the spectral tails (guard bands) are about 4 MHz.
OFDM Parameters for 802.11 (Local Area Network)

Typical Example (various ways to express parameters)

\[ N_c = \text{no. subcarriers} = 52 \quad \text{Modulation BW} \quad W_{\text{signal}} \approx N_c \times \Delta f \approx 16 \text{ MHz} \]

\[ N = \text{transform size} \quad (N > N_c) = 64 \text{ (or 128) data samples} \]

\[ N_{cp} = \text{cyclic prefix samples} = 16 \text{ (based on } N = 64) \quad T_{cp} \gg \sigma_c \]

\[ N_L = N + N_{cp} \text{ total samples per OFDM symbol} = 80 \]

\[ T_L = \text{sample time} = 1/f_s = 1/(20 \text{ MHz}) = 0.05 \mu s \]

\[ f_s = \frac{1}{T_L} = \frac{N_L}{T_{OFDM}} = \frac{N_{cp}}{T_{cp}} = \frac{N}{T_s} = N \times \Delta f = 20 \text{ MHz samp rate} \quad 64 \times 312.5 \text{ kHz} \]

\[ T_s = 1/\Delta f \approx N_c/W_{\text{signal}} = N \times T_L = 64 \times 0.05 \mu s = 3.2 \mu s \]

\[ T_{OFDM} = T_s + T_{cp} = N_L \times T_L = 80 \times 0.05 \mu s = 4 \mu s \]

where \( \Delta f \) the spacing between subcarriers is \((1/3.2) \text{ MHz} = 312.5 \text{ kHz}\)
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where \( \Delta f \) the spacing between subcarriers is \((1/3.2) \text{ MHz} = 312.5 \text{ kHz} \)
OFDM Transmission Bandwidth (802.11 Example)

• We define a time interval $T_s$ over which the signaling is to be orthogonal. For 802.11, $T_s = 3.2 \ \mu s$

• Choose a quantity of subcarriers $N_c$ (dependent on multipath channel). For 802.11, $N_c = 52$ (48 data plus 4 pilot)

• The reciprocal of $T_s$ ($1/T_s = \Delta f$) gives the spacing between FFT bins (frequency-domain samples) = 312.5 kHz

• $W_{\text{signal}} = \text{OFDM Data BW} = (N_c + 1) \times \Delta f = 53 \times 312.5 \text{ kHz} = 16.56 \text{ MHz}$

• How does the transform size $N$ enter the picture? In 802.11, typically $N = 64, 128$

• Increasing the transform size (FFT bins) extends the transform in the frequency domain. There will be more unoccupied bins, making the filtering less costly.
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• How does the transform size $N$ enter the picture? In 802.11, typically $N = 64, 128$

• Increasing the transform size (FFT bins) extends the transform in the frequency domain. There will be more unoccupied bins, making the filtering less costly. Time-resolution improves. Sampling rate increases (requiring interpolation). Spacing between spectral copies increases (eases analog filtering).
**OFDM Parameters (802.11 Typical Example)**

\[ N_c = 52 \]

\[ W_{\text{signal}} = (N_c + 1) \Delta f \]

\[ \Delta f = \frac{1}{T_s} = 1/3.2 \mu s \]

\[ = 312.5 \text{ kHz} \]

\[ W_{\text{signal}} = 53 \times 312.5 \text{ kHz} \]

\[ = 16.56 \text{ MHz} \]

**Transform size**

\[ N = 64 \]

**Samples:**

\[ \begin{align*}
N_{cp} & = 16 \\
N & = 64 \\
N_L & = N_{cp} + N = 80 \\
T_{cp} & = 0.8 \mu s \\
T_s & = 3.2 \mu s \\
T_{\text{OFDM}} & = T_{cp} + T_s = 4 \mu s \\
R_{\text{OFDM}} & = \frac{1}{T_{\text{OFDM}}} = 250 \text{ ksymb/s} 
\end{align*} \]

**Samp rate and time**

\[ f_s = \frac{N}{T_s} = \frac{64}{3.2 \mu s} = 20 \text{ MHz} = \frac{1}{T_L} \]

\[ T_{\text{OFDM}} = N_L \times T_L = 80 \times 0.05 \mu s = 4 \mu s \]
Consider an 802.11 OFDM system, having the following parameter values: 64-point transform, with 48 message-occupied bins, OFDM symbol time = 4 \( \mu \)sec, CP time = 0.8 \( \mu \)sec, data modulation is 16-QAM, Error-correcting code rate = \( \frac{3}{4} \). Find the following:

(a) Sampling rate  
(b) Sample time  
(c) Code-bits per subcarrier  
(d) Code-bits per OFDM symbol  
(e) Data-bits per OFDM symbol  
(f) Data rate  

(g) If the channel max delay spread = 20 samples, determine if the given CP time of 0.8 \( \mu \)sec will be long enough to mitigate the channel ISI.
Solution to 802.11 OFDM Exercise

(a) Tone spacing: \( \Delta f = \frac{1}{T_s} = \frac{1}{T_{\text{OFDM}} - T_{\text{CP}}} = \frac{1}{3.2 \mu \text{sec}} = 312.5 \text{ kHz} \)

Sample rate: \( f_s = N \times \Delta f = 64 \times \Delta f = 64 \times 312.5 \text{ kHz} = 20 \text{ MHz} \)
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Sample rate: \( f_s = N \times \Delta f = 64 \times 312.5 \ \text{kHz} = 20 \ \text{MHz} \)

(b) \( T_{\text{samp}} = \frac{1}{f_s} = 50 \ \text{nano-sec} \)
Solution to 802.11 OFDM Exercise

(a) Tone spacing:  \( \Delta f = \frac{1}{T_s} = \frac{1}{T_{OFDM} - T_{CP}} = \frac{1}{3.2 \, \mu \text{sec}} = 312.5 \, \text{kHz} \)

Sample rate:  \( f_s = N \times \Delta f = 64 \times \Delta f = 64 \times 312.5 \, \text{kHz} = 20 \, \text{MHz} \)

(b)  \( T_{samp} = \frac{1}{f_s} = 50 \, \text{nano-sec} \)

(c) 16-QAM yields 4 code-bits per subcarrier.
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(d) 48 message-occupied subcarriers per OFDM symbol
   \( 48 \times 4 = 192 \, \text{code-bits per OFDM symbol} \)
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(a) Tone spacing: \( \Delta f = \frac{1}{T_s} = \frac{1}{T_{OFDM} - T_{CP}} = \frac{1}{3.2 \mu \text{sec}} = 312.5 \text{ kHz} \)

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(e) Rate \( \frac{3}{4} \) code: \( \frac{3}{4} \times 192 = 144 \text{ bits per OFDM symbol} \)
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(a) Tone spacing: \( \Delta f = \frac{1}{T_s} = \frac{1}{T_{OFDM} - T_{CP}} = \frac{1}{3.2 \mu \text{sec}} = 312.5 \text{ kHz} \)

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(e) Rate \( \frac{3}{4} \) code: \( \frac{3}{4} \times 192 = 144 \text{ bits per OFDM symbol} \)

(f) Data rate: \( \frac{144 \text{ bits} / \text{symbol}}{4 \mu \text{sec} / \text{symbol}} = 36 \text{ Mbps} \)
Solution to 802.11 OFDM Exercise

(a) Tone spacing: $\Delta f = \frac{1}{T_s} = \frac{1}{T_{OFDM} - T_{CP}} = \frac{1}{3.2 \, \mu\text{sec}} = 312.5 \, \text{kHz}$

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$48 \times 4 = 192 \, \text{code-bits per OFDM symbol}$

(e) Rate $\frac{3}{4}$ code: $\frac{3}{4} \times 192 = 144 \, \text{bits per OFDM symbol}$

(f) Data rate: $\frac{144 \, \text{bits/symbol}}{4 \, \mu\text{sec/symbol}} = 36 \, \text{Mbps}$

(g) Max delay spread $20 \, \text{samples} \times T_{samp} = 20 \times 50 \, \text{nano-sec} = 1 \, \mu\text{sec}$

$T_{CP} = 0.8 \, \mu\text{sec} < 1 \, \mu\text{sec}$. Therefore, we need a longer cyclic prefix.
Sending an 84-bit data sequence as 21
16-QAM symbols sent as 3 OFDM symbols

84-bit data sequence:

```
1001010000111100011000001001011011111101001010000100001
0000001101111101000011000111
```

<table>
<thead>
<tr>
<th>1001</th>
<th>0100</th>
<th>0001</th>
<th>1110</th>
<th>0011</th>
<th>0000</th>
<th>0100</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>4</td>
<td>1</td>
<td>14</td>
<td>3</td>
<td>0</td>
<td>4</td>
</tr>
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</table>

OFDM data symbol 1

<table>
<thead>
<tr>
<th>1011</th>
<th>0111</th>
<th>1110</th>
<th>1001</th>
<th>0100</th>
<th>0010</th>
<th>0001</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>7</td>
<td>14</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

OFDM data symbol 2

<table>
<thead>
<tr>
<th>0000</th>
<th>0011</th>
<th>0111</th>
<th>1101</th>
<th>0000</th>
<th>1100</th>
<th>0111</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>7</td>
<td>13</td>
<td>0</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

OFDM data symbol 3
## Gray-Coded 16-QAM

<table>
<thead>
<tr>
<th></th>
<th>0000</th>
<th>0100</th>
<th>1100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>7</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Shows the locations of the 4-bit data sequences in the 2-D constellation.
Gray-Coded 16-QAM

message symbols: 9 4 1 14 3 0 4
Sending an 84-bit data sequence as 21 16-QAM symbols sent as 3 OFDM symbols

Message symbols partitioned onto subcarriers
OFDM is Particularly Useful for High Data Rates, Such as Pictures or Video

• “A picture is worth 1000 words.” Is this age-old expression correct? Yes, in the behavioral sense. But what about the task (i.e., bandwidth requirements) of sending a picture versus 1000 words of text?

• A high-quality 8x10 photo is made up of about 3 Megapixels. A good quality color photo requires 8 bits per primary color per pixel, or 24 bits per pixel. Thus such an 8x10 photo can be represented as a sequence of 72 Megabits.

• English text on average has about 4.5 letters per word. Using ASCII with parity, each letter is made up of 8 bits. Thus on average, one English word, on average, can be represented as a sequence of 36 bits.

• How many such 36-bit words comprise one good quality 8x10 color picture?

$$\frac{72 \times 10^6 \text{ bits per picture}}{36 \text{ bits per word}} = 2 \text{ million words per } 8 \times 10 \text{ picture}$$

• Therefore, sending one such picture via your cellphone is the equivalent of sending about six 500 page textbooks.

• 1000 words = 36,000 bits. What size high-quality picture is worth a thousand words? Smaller than \(\frac{1}{4}\) inch x \(\frac{1}{4}\) inch.

• That’s why source coding of images & BW efficiency is important.
59. In the early "battle" for the best codes (convolutional vs. Reed-Solomon), what are the arguments for each, and why did convolutional win out? (Sklar DIG notes, section 8)

60. In mobile channels, how does the terrain affect fading? How does the mobile-velocity affect it? (Sklar ADC notes, section 2)

61. What is the advantage of circular-convolution versus linear-convolution? How do we trick the channel into performing circular convolution? (Sklar ADC notes, section 3)

62. In OFDM, what is the mitigation technique for precluding ISI? For precluding ICI? (Sklar ADC notes, section 3)

63. Baseband OFDM symbols are typically made up of independent data at positive and negative spectral locations. How is this effected, and how is a real transmission-signal formed? (Sklar ADC notes, section 3)

64. For maintaining orthogonality among the subcarriers in OFDM, the tone spacing was chosen to be $1/T_s$. Why wasn't it chosen to be $1/T_{OFDM}$?

65. How can SC-OFDM still be resistant to multipath when the data symbols are so short? (Sklar ADC notes, sec. 3) Hint: The time duration of a data pulse is longer than its main lobe.

66. Early skeptics about MIMO, claimed that it violated Shannon's capacity theorem. Why is that not the case? (Sklar ADC notes, section 4)

67. Why won't MIMO work in a multipath-free environment? (Sklar ADC notes, section 4)

68. Often, the signal-processing operations "DFT and IDFT" are called out as "FFT and IFFT," when one means the mathematical transformation. Why is this NOT precise?

69. What are the Key Control Loops needed for system Synchronization? (Fred Harris, "Let's Assume the System is Synchronized," Sklar ADC notes, section 8)

70. How do you shape a time waveform to meet system spectral-confinement requirements? (Sklar DIG notes, sec 9 & ADC notes, sec 1) Hint: symbol rate, sample rate, filter length, transition BW, out-of-band attenuation.
Data Length Defines Sinc Width: Spectral Spacing Matches Width

Fourier Transform of an infinitely long sinusoid is an impulse function. Fourier Transform of a gated-sinusoid is a sinc function.

Fourier Transform of an infinitely long sinusoid is an impulse function.
Fourier Transform of a gated-sinusoid is a sinc function.

With the proper pulse spacing of $1/T$, the sequence is orthogonal, characterized by the peak of each pulse experiencing zero-value interference from neighboring pulses. Hence there is NO ISI.

Ref: fred harris, lecture on OFDM, San Diego State University
Spectral spacing is preserved because we haven't changed the cycles per interval (frequency). We've just extended the length (as in the case of a cyclic prefix).

One might ask, with the extended length (reduced sinc width) why not reduce the sub-carrier spacing, thereby forming a family of extended orthogonal waveforms? Answer: Orthogonality in space is not what we're striving for. We want orthogonal waveforms at the receiver, after the extension is discarded.
Extended Data Length Reduces Sinc Width: Spectral Spacing Preserved

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OFDM Applications

Long Term Evolution (LTE)
An OFDM Long Term Evolution (LTE) Application

LTE (Wide Area Network) Resources: Grid, Block, Element

Note: LTE standards involve Multiple Access

- **Resource Grid**
  - Several resource blocks (RB’s).
  - Number of RB’s adjusted to cover available BW.

- **Resource Block (RB)**
  - 7 by 12 elements.
  - Transmissions are allocated in discrete RB’s.

- **Resource element**
  - One symbol width in time
  - One 15 kHz sub-carrier in frequency.

\[ f = 15 \text{ kHz} \]
Frames, slots, OFDM symbols, & data symbols

**OFDM in LTE**

Note that whichever slot type is chosen (short or long CP) the data portion of the OFDM symbol is always the same size.

A frame is made up of 20 slots. A slot is made up of 7 (or 6) symbols.

An OFDM symbol is an IFFT wave-shape output + CP

One slightly longer CP to force the slot size to be 500 μsec

Note that whichever slot type is chosen (short or long CP) the data portion of the OFDM symbol is always the same size.

**Frames, slots, OFDM symbols, & data symbols**

$D_f = \frac{1}{T_s}$
3 OFDM Symbols
(3/7 of a 7-symbol LTE slot)
LTE allows for modulation types: QPSK, 16-QAM, and 64-QAM

In LTE 20 MHz channels, there are 1201 candidate subcarriers per OFDM symbol (not 7 as shown here).
OFDM in LTE (six channel bandwidth options - Release 8)

Transition BW reqmt of filters dictate that the candidate subcarriers $N_c$ be approx 60% of the FFT size $N$.

<table>
<thead>
<tr>
<th>Transmission BW</th>
<th>1.4</th>
<th>3.0</th>
<th>5.00</th>
<th>10.00</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation BW (MHz)</td>
<td>1.08</td>
<td>2.7</td>
<td>4.5</td>
<td>9.0</td>
<td>13.5</td>
<td>18.0</td>
</tr>
<tr>
<td>Slot Duration</td>
<td>500 µsec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub Carrier Spacing $f_s$</td>
<td>FIXED</td>
<td>15 kH</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample Rate $f_s$ = 15 kHz x Transform Size $N$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulation BW approx = 15 kHz x No. of occupied subcarriers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>1.92 MHz</td>
<td>3.84 MHz</td>
<td>7.68 MHz</td>
<td>15.36 MHz</td>
<td>23.04 MHz</td>
<td>30.72 MHz</td>
</tr>
<tr>
<td>FFT Size $N$</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>1536</td>
<td>2048</td>
</tr>
<tr>
<td>Number of Candidate Sub-Carriers (approx 60% of FFT size) $N_c$</td>
<td>73</td>
<td>181</td>
<td>301</td>
<td>601</td>
<td>901</td>
<td>1201</td>
</tr>
<tr>
<td>Number of OFDM Symbols Per Slot</td>
<td>7-Short Cyclic Prefix</td>
<td>6-Long Cyclic Prefix</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short CP Length (in clock samples)</td>
<td>1-5.21 µsec 10 Samples</td>
<td>1-5.21 µsec 20 Samples</td>
<td>1-5.21 µsec 40 Samples</td>
<td>1-5.21 µsec 80 Samples</td>
<td>1-5.21 µsec 120 Samples</td>
<td>1-5.21 µsec 160 Samples</td>
</tr>
<tr>
<td></td>
<td>6-4.69 µsec 9 Samples</td>
<td>6-4.69 µsec 18 Samples</td>
<td>6-4.69 µsec 36 Samples</td>
<td>6-4.69 µsec 72 Samples</td>
<td>6-4.69 µsec 108 Samples</td>
<td>6-4.69 µsec 144 Samples</td>
</tr>
<tr>
<td>Long CP Length</td>
<td>16.67 µsec 32 Samples</td>
<td>16.67 µsec 64 Samples</td>
<td>16.67 µsec 128 Samples</td>
<td>16.67 µsec 256 Samples</td>
<td>16.67 µsec 384 Samples</td>
<td>16.67 µsec 512 Samples</td>
</tr>
</tbody>
</table>

In 802.11a, channel spacing is 20 MHz (func of reqd ACI). Also $N = 64$ or 128. Note the greater flexibility in LTE with 6 channel options.
### LTE Channel Bandwidth Configurations

Slot time = 0.5 ms. Sub-frame time = 1 ms. Frame time = 10 ms

Transition BW Filter requirements dictate that $N_c \approx 0.6 N$. BW of RB = 180 kHz

Guard-band BW equals 10% of Channel BW except for 1.4 MHz case (22.85%)

Subcarrier spacing $\Delta f = 15$ kHz.

Data portion of Symbol $T_s = 1/\Delta f = 66.667 \, \mu\text{sec}$

<table>
<thead>
<tr>
<th>Transmission BW (W_{xmt} MHz) 2-sided</th>
<th>1.4</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation Bandwidth (W_m MHz)</td>
<td>1.08</td>
<td>2.70</td>
<td>4.50</td>
<td>9.00</td>
<td>13.50</td>
<td>18.00</td>
</tr>
<tr>
<td>Guard Band for each side (kHz)</td>
<td>160</td>
<td>150</td>
<td>250</td>
<td>500</td>
<td>750</td>
<td>1000</td>
</tr>
<tr>
<td>Number of RBs #RBs = W_m / 180 kHz</td>
<td>6</td>
<td>15</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Transform size</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>1536</td>
<td>2048</td>
</tr>
<tr>
<td>Occupied subcarriers $N_c$</td>
<td>72</td>
<td>180</td>
<td>300</td>
<td>600</td>
<td>900</td>
<td>1200</td>
</tr>
<tr>
<td>Sample rate (MHz) DFT BW $f_s = N \Delta f$</td>
<td>1.92</td>
<td>3.84</td>
<td>7.68</td>
<td>15.36</td>
<td>23.04</td>
<td>30.72</td>
</tr>
<tr>
<td>Sample time (µ sec) $T_L = 1/f_s$</td>
<td>0.5208</td>
<td>0.2604</td>
<td>0.1302</td>
<td>0.0651</td>
<td>0.0434</td>
<td>0.0326</td>
</tr>
<tr>
<td>Samples per slot</td>
<td>960</td>
<td>1920</td>
<td>3840</td>
<td>7680</td>
<td>11520</td>
<td>15360</td>
</tr>
</tbody>
</table>
Sampling Requirements

• The 2-sided baseband spectrum of an OFDM symbol contains independent data on each of its two sides.

• Hence, OFDM baseband modulation doesn’t have Hermitian symmetry properties, and is complex. When up-shifted onto a transmission carrier wave, it then becomes a real time-signal with even Hermitian symmetry.

• To meet the Nyquist sampling criterion, the required sampling rate has often been described as: exceeding twice the signal's single-sided baseband bandwidth.

• In the modern era, it is more appropriate to describe a signal's bandwidth in terms of its 2-sided spectrum. Thus we recast the Nyquist sampling criterion as: The sampling rate must exceed the 2-sided baseband bandwidth.

• The sampling rate $f_s$ defines the distance between spectral copies. This distance must exceed the 2-sided bandwidth in order to avoid spectral-overlap of copies.

• Hence Nyquist sampling in OFDM (which will be complex) is best described as requiring a sampling rate that must exceed the 2-sided baseband bandwidth.

• We verify that it does indeed meet that rate for both 802.11 and LTE. In 802.11a, the 20 MHz channel has a 2-sided signal BW of approx 16 MHz and a sampling rate of 20 MHz. In LTE, the 20 MHz channel has a 2-sided signal BW of approx. 18 MHz and a sampling rate of 30.72 MHz.

• The chosen sampling rate is forced by the transform size to allow for the proper spacing between FFT bins to sustain orthogonality of gated complex sine waves.

$$f_s = N \times \Delta f$$
samples/slot = slot time x $f_s$

**OFDM in LTE: Example of FFT size 2048**

500 µsec slot contains 15,360 samples \((\text{sample rate} = 30.72 \text{ MHz})\)

1-Slot, 7-OFDM Symbols, Short CP

An OFDM symbol, made up of multiple tones, comprises a channel.

15,360 samples in slot comprised of seven symbols

<table>
<thead>
<tr>
<th>Type of Cyclic Prefix</th>
<th>Cyclic Prefix Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>160 for Slot 0</td>
</tr>
<tr>
<td></td>
<td>144 for Slots 1, 2, ..., 6</td>
</tr>
<tr>
<td>Long</td>
<td>512 for Slots 0, 1, ..., 6</td>
</tr>
</tbody>
</table>

In this example, each symbol is the sum of $N_C = 1201$ gated sinusoids, where each sinusoid has $N = 2048$ samples. Hence, the composite time waveform has $N = 2048$ samples/symbol.

- Duration for the IFFT part of the symbol (without CP) is a fixed 66.667 µsec. The reciprocal of this duration is 15 kHz, which is the spacing between the $\sin(x)/x$ channels or tones.
- The sample rate (BW spanned by the FFT) is this spacing times the number of channels.

In this example sample rate equals $15 \text{ kHz} \times 2048 = 30.72 \text{ MHz}$. Thus, within one slot, the number of samples amount to sample rate times the slot time = 15,360 samples.

- Since channel spacing is fixed, an increase in BW requires more tones, which means a larger size FFT, which in turn necessitates an increase in sample rate.
- Users need to be aligned in time, frequency, and signal strength by the base station.

Users are typically assigned several specific subcarriers (a segment of a channel) for a specific time interval.
Key OFDM Relationships

\[ f_s = \frac{N}{T_s} = N \times \Delta f \]
\[ \Delta f = \frac{1}{T_s} \]
\[ T_{\text{OFDM}} = T_s + T_{\text{CP}} \]

\[ f_s \text{ and } T_s \text{ are user independent parameters.} \]

Once chosen, you've locked in \( N \) and \( \Delta f \):
\[ N > N_c \]
\[ W_{\text{signal}} = N_c + 1) \Delta f \]

Describe the effects of increasing the number \( N \) of the \( N \)-point IDFT transform. Will it increase the sample rate? Will it increase the OFDM BW?

Describe the effects of decreasing \( \Delta f \) the frequency spacing between tones. And show the ways in which it can happen.
**Key OFDM Relationships**

\[ f_s = \frac{N}{T_s} = N \times \Delta f \quad \Delta f = \frac{1}{T_s} \quad T_{\text{OFDM}} = T_s + T_{\text{CP}} \]

\( f_s \) and \( T_s \) are user independent parameters. Once chosen, you've locked in \( N \) and \( \Delta f \): \( N > N_c \quad W_{\text{signal}} = (N_c + 1) \Delta f \)

Describe the effects of increasing the number \( N \) of the \( N \)-point IDFT transform. Will it increase the sample rate? Will it increase the OFDM BW?

An increase in \( N \) by itself, will increase the sample rate, but will NOT increase the BW because we are not sending samples; we are sending waveforms. The increased sample rate will result in greater time-domain resolution, and will spread the spectral periodic copies further apart. This makes the analog filtering easier and less costly.

Describe the effects of decreasing \( \Delta f \) the frequency spacing between tones. And show the ways in which it can happen.

The effect of decreased \( \Delta f \) will increase the length of the OFDM symbol \( T_s \) making it less vulnerable for a given channel ISI. Increased symbol length can come about by decreasing \( \Delta f \), or increasing \( N \) (for fixed \( f_s \)), or by increasing \( N_c \) (for fixed \( W_{\text{signal}} \)).
Using the Table of LTE Channel BW Configurations
Show that when using 64-QAM, the Max OFDM Uncoded Data Rate that can be supported for LTE ≈ 100 Mbps

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Largest Channel BW</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Largest Transform Size</td>
<td>2048</td>
</tr>
<tr>
<td>Candidate Subcarriers</td>
<td>1200</td>
</tr>
<tr>
<td>64-QAM</td>
<td>6 bits/subcarrier</td>
</tr>
<tr>
<td>Bits per OFDM Symbol</td>
<td>6 x 1200 = 7200 bits</td>
</tr>
<tr>
<td>Slot Time</td>
<td>0.5 ms</td>
</tr>
<tr>
<td>Symbol Time</td>
<td>0.5 ms / 7 = 71.429 µsec</td>
</tr>
<tr>
<td>Max Data Rate</td>
<td>7200 bits / 71.429 µsec ≈ 100 Mbps</td>
</tr>
<tr>
<td>Overhead reduces this to</td>
<td>≈ 86 Mbps</td>
</tr>
</tbody>
</table>
Another Way to Compute the Max OFDM Uncoded Data Rate using Resource Blocks

Max Channel BW 20 MHz
Max Occupied BW, \( W_m \) 18 MHz

\[ \#\text{RBs} = \frac{W_m}{180 \text{ kHz/RB}} = 100 \text{ RBs} \]

Each RB has 12 x 7 = 84 REs

100 RBs have 8400 REs

64-QAM yields 6 bits per RE

Bits per RB = 6 x 84 = 504 bits

Bits per 100 RBs = 50400 bits

Bit Rate = 50400 bits / 0.5 ms ≈ 100 Mbps

Overhead reduces this to ≈ 86 Mbps
An OFDM Long Term Evolution (LTE) Application

LTE (Wide Area Network) Resources: Grid, Block, Element

Note: LTE standards involve Multiple Access

- **Resource Grid**
  - Several resource blocks (RB’s).
  - Number of RB’s adjusted to cover available BW.

- **Resource Block (RB)**
  - 7 by 12 elements.
  - Transmissions are allocated in discrete RB’s.

- **Resource element**
  - one symbol width in time
  - one 15 kHz sub-carrier in frequency.

\[ f = 15 \text{ kHz} \]
Power vs Bandwidth OFDM Trade-Off for a 50 Mbps D/L Channel for an LTE System (neglecting overhead)

<table>
<thead>
<tr>
<th>Smaller BW - needs Larger SNR</th>
<th>Larger BW - needs Smaller SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel BW</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Transform Size</td>
<td>1024</td>
</tr>
<tr>
<td>Candidate Subcarriers</td>
<td>600</td>
</tr>
<tr>
<td><strong>64-QAM bits/subcarrier:</strong></td>
<td><strong>6 bits</strong></td>
</tr>
<tr>
<td><strong>64-QAM bits/subcarrier:</strong></td>
<td><strong>6 bits</strong></td>
</tr>
<tr>
<td>Bits/OFDM symb: 6 x 600 = 3600 bits</td>
<td>4 x 900 = 3600 bits</td>
</tr>
<tr>
<td>OFDM Symbol Time = 71.429 µsec</td>
<td>71.429 µsec</td>
</tr>
</tbody>
</table>
| Data Rate = \[
\frac{3600 \text{ bits}}{71.429 \mu\text{sec}} \approx 50 \text{ Mbps}
\] | \[
\frac{3600 \text{ bits}}{71.429 \mu\text{sec}} \approx 50 \text{ Mbps}
\] |

A service provider typically responds to a wireless user's request for service (say a 50 Mbps channel) based upon the user's SNR. If the user's SNR is large, the provider will respond with a small BW channel (10 MHz here). If the SNR is small, the provider will offer a larger BW (15 MHz here) to support the requested bit rate.

Shown above is a power versus bandwidth OFDM trade-off for a 50 Mbit/s downlink LTE channel (neglecting overhead). Either a 10 MHz channel and 64-QAM modulation or a 15 MHz channel and 16-QAM modulation can support the user's 50 Mbps request.
OFDM Parameters (LTE Typical Example)

- Transform size: $N = 2048$

- $N_c = 1201$
  - Includes DC Subcarrier

- Transmission BW

- $W_{signal} = (N_c + 1) \Delta f$
  - $\Delta f = 1/T_s = 1/66.67 \mu s$
  - $W_{signal} = 1202 \times 15 \text{ kHz}$

- Time duration:
  - $N_{cp}$
  - $N$
  - $N_L = N_{cp} + N = 2192$
  - $T_{cp}$
  - $T_s$
  - $4.69 \mu s$
  - $66.67 \mu s$

- Samples:
  - $N_{cp} = 144$
  - $N = 2048$

- Samp time
  - $T_L = T_{slot} / \text{samp/slot} = 500 \mu s / 15,360 = 0.03255 \mu s$

- $T_{OFDM} = N_L \times T_L = 71.4 \mu s$

- Samp rate
  - $f_s = \frac{N}{T_s} = \frac{1}{T_L} = 30.72 \text{ MHz}$

- Slot time
  - $T_{slot} = 7 \times T_{OFDM} = 500 \mu s$

- $R_{OFDM} = 1/T_{OFDM} = 14 \text{ ksymb/s}$
LTE Resources Scheduling Example

- Consider the slot and subcarrier architecture in the LTE resource blocks.
- Note that the slot time is 0.5 ms, that the subcarrier bandwidth (and separation between subcarriers) is 15 kHz. There are $12 \times 7 = 84$ REs/RB.

Find the theoretical peak downlink and uplink bit rates for a 20 MHz channel. The downlink uses 4 x 4 MIMO. The uplink is a single beam from UE to BS.

Assume that the modulation is 64-QAM, and that there is no error-correction coding. Consider that reference and control channels use about 25% of the resources.
LTE Throughput Solution

**Downlink**
20 MHz channel corresponds to 100 RBs, which contains $84 \times 100 = 8,400$ REs.

Theoretical Peak Data Rate (bits/ms) = number of REs per subframe x number of bits per modulation symbol (64-QAM for this example)

$$\text{bits/sec} = \frac{8400 \times 2}{1 \text{ ms/subframe}} \times 6 \text{ bits/symbol} = 100.8 \text{ Mbps}$$

In a MIMO 4 x 4 system, peak data rate = $4 \times 100.8 \text{ Mbps} = 403.2 \text{ Mbps}$

About 25% of resources are needed for reference and control signals.
That leaves: $\approx 300 \text{ Mbps}$ as the peak data rate.

**Uplink**
For a single beam from UE to BS, the peak data rate follows above computation, minus the MIMO, that is: $100.8 \text{ Mbps}$. After considering a 25% reduction for reference and control, the peak data rate is: $\approx 75 \text{ Mbps}$.
Abstract: The main benefit of OFDM is its ability to cope with Severe multipath channel conditions without needing Complex Equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a High-data-rate signal into Smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).

- Single-Carrier OFDM (SC-OFDM).
Single-Carrier OFDM (SC-OFDM)
33. Why do binary and 4-ary orthogonal FSK manifest the same bandwidth-efficiency relationship? (Section 9.5.1)

34. Describe the subtle energy and rate transformations of received signals: from data-bits to channel-bits to symbols to chips. (Section 9.7.7)

35. Define the following terms: Baud, State, Communications Resource, Chip, Robust Signal. (Sections 1.1.3 and 7.2.2, Chapter 11, and Sections 12.3.2 and 12.4.2)

36. What are the two fading mechanisms that characterize small-scale fading? (Sec 15.2)

37. For a mobile fading channel, why is signal dispersion independent of fading rapidity? (Section 15.4.1.1)

38. What is the key difference between Rician fading and Rayleigh fading? (Sec 15.2.2)

39. In the context of a fading channel, define the terms: delay spread, coherence bandwidth, coherence time, Doppler spread. How are they related? (Sec 15.3 and 15.4)

40. What if any, are the differences in the terms: Doppler spread, spectrum spreading, fading bandwidth, fading rapidity, fading rate? (Section 15.4.2)

41. Why does signal distortion due to fading yield more serious degradation than a loss in SNR? (Section 15.5)

42. Why is OFDM useful for high-data-rate fading channels? (Sec. 15.5.1 & 15.20 Prob 15.20). What benefit of SC-OFDM makes it a natural for mobile systems?

43. To provide diversity between two fixed platforms, how large an interleaver span is needed? (Section 15.5.6)
33. Why do binary and 4-ary orthogonal FSK manifest the same bandwidth-efficiency relationship? (Section 9.5.1)
34. Describe the subtle energy and rate transformations of received signals: from data-bits to channel-bits to symbols to chips. (Section 9.7.7)
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**SC-OFDM offers improved PAPR, which facilitates the efficient operation of power amplifiers.**
SC-FDMA (DFT-Spread FDMA) versus OFDMA

- OFDMA systems achieve robustness in the presence of multipath by transmitting on $M$ orthogonal frequency carriers, each operating at $R/M$ bits/s. $M = Nc$ (earlier slides)

- OFDMA exhibits very pronounced envelope fluctuations (the output sum of gated sinusoids can yield a variety of amplitudes) resulting in high PAPR.

- This requires highly linear power amplifiers to avoid excessive IM distortion. Amplifiers have to operate with a large backoff from their peak power, resulting in lower power efficiency.

- SC-FDMA is a modified version of OFDMA for U/L transmission in the LTE of cellular systems. In the SC-FDMA output, individual $(\sin x)/x$ basis functions are transmitted sequentially (each peaks at a different time), thus reducing the PAPR.

- The Cyclic Prefix (CP) acts as a guard time. If the length of CP $> T_m$ then there is no intersymbol interference (ISI). Since the CP is a copy of the last part of the symbol, it converts linear convolution (with the channel) into circular convolution, which in the frequency domain is a pointwise multiplication of DFT frequency samples.

- Equalization consists of dividing the DFT of the received signal by the DFT of the channel impulse response (a simple scaling – same for OFDM and SC-OFDM).

- For an ideal channel (no distortion), note that the receiver for SC-OFDM would only need to sample the received waveform at the appropriate times.
PAPR in OFDMA can be Large

**OFDMA signal generation**

QPSK example using M=4 subcarriers

Each of M subcarriers is encoded with one QPSK symbol

M subcarriers can transmit M QPSK symbols in parallel

The amplitude of the combined four subcarrier signal varies widely depending on the symbol data being transmitted

Data Symbols with LTE Spectral Spacing

Ref: Moray Rumney, Agilent, March 20, 2008, "SC-FDMA The New LTE Uplink Explained"
On Trading Excess Bandwidth for Reduced Peak to Average Power Ratio in Single Carrier Shaped Dirichlet Kernel OFDM

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Motivation for reducing Peak-to-Average Power Ratio (PAPR)

ABSTRACT
The waveform of choice for OFDM signaling is the sinusoid with an integer number of cycles per interval and with an appended cyclic prefix to obtain circular convolution with the channel. This combination makes the channel inversion particularly simple; performed as a ratio between the DFT of the received signal and the DFT of the channel. In fact, this relationship is valid for any periodic function formed as a sum of the basis sinusoids of the DFT. One particularly simple example of this class of signals is the Dirichlet kernel (the periodically extended sinc function). This kernel is used in single carrier OFDM [1]. An advantage of this kernel relative to the complex sinusoid kernel is a 3.4 dB reduction in peak to average power ratio (PAPR). We show here that a windowed version of this kernel exhibits a significantly lower, in fact up to a 10.0 dB reduction in PAPR. The cost to obtain the reduced PAPR is excess bandwidth but that may be a fair trade to obtain higher average transmitted power for a given peak power limited amplifier. With average amplitude 1/4-th of peak amplitude, average power is 1/16-th of peak power. Power amplifiers are DC to AC converters and power pulled from the DC power supply, which is approximately constant, not delivered to the load is dissipated in the power amplifier. Amplifiers are very inefficient in their transduction process of turning DC power to signal power when they operate at small fractions of their peak power level. Typical efficiencies for an amplifier operating with an IEEE 802.11a signal are on the order of 18 % [1]. Thus an amplifier required to supply 1 Watt would have a peak power capability of 16 watts and would be pulling 5.5 Watts from the power supply while squandering 4.5 Watts, raising the temperature of its heat sinks, while delivering 1 Watt to its external load.
What are Basis Functions?

A common mathematical representation of a signal is a linear combination of elementary functions called basis functions.

In communication systems, examples of the most popular basis functions are gated sinusoids, square-root Nyquist pulses, and Nyquist pulses (also called (sin x)/x, sinc, raised cosine, and Dirichlet functions). Dirichlet is a periodically extended sinc function.

In OFDM, the basis functions in the time domain are gated sinusoids. In the frequency domain, they are sinc pulses.

In SC-OFDM, the basis functions in the time domain are sinc pulses. In the frequency domain they are gated sinusoids.
Why SC-OFDM Offers Improved PAPR Performance Over Standard OFDM

Standard OFDM:
The output sum of gated sinusoids can yield a variety of amplitudes. Hence there is a large PAPR.

SC-OFDM:
Individual Dirichlet basis functions are transmitted sequentially, each peaking at a different time. Hence there is a reduced PAPR.
Generating OFDM: Sinc functions representing narrowband data phasors are inputted into the IFFT. Output time signal: Superposition of gated sinusoids.

Generating SC-OFDM: Sinc functions representing time waveforms are transformed (via the DFT) to wideband gated sinusoids. Such wideband spectra are inputted into the IFFT. Output time signal: Staggered sinc functions.

**Figure 5** Simplified model of SC-FDMA generation and reception


SC-FDMA: Hybrid scheme, combining the low PAPR of single-carrier transmission systems with the long symbol time and flexible frequency allocation of OFDM. Note that: DFT is a transform, and FFT is an algorithm. These names are often used inconsistently.
For SC-OFDM, the output waveform is made up of Dirichlet basis functions.

For an IDEAL CHANNEL, a simple sampling of the SC-OFDM output waveform would yield the original 2-space points. Not so with OFDM.

SC-OFDM survives frequency selective fading (data symbol is represented with spectral diversity)

OFDM precludes gated sinusoids

OFDM (parallel data) yields large PAPR.

SC-OFDM (sequential data) reduces PAPR.

Each of 4 points in 2-space is assigned a frequency

**Figure 2** Comparison of OFDMA and SC-FDMA transmitting a series of QPSK data symbols

The signal length is a multiple of its sinusoidal basis functions (there are an integer number of cycles per gated sinusoid).

Waveform Formation

OFDM vs SC-OFDM

IDFT Input freq samples

SC-OFDM

Zero padding of the IDFT input freq samples, raises the sample rate $f_s$.

DFT Input time samples

Output waveform is a sequence of sinc functions

DFT Output freq samples

IDFT Output time samples

Visualisation:
sinc functions

NOT actually built

Visualisation:
sinc functions

NOT actually built

Visualisation:
sinc functions

NOT actually built
SC-OFDM: Same Time-Frequency Plot for DFT and IDFT

Input time samples to DFT
Input frequency samples to IDFT
Output frequency samples from DFT
Output time samples from IDFT

(1) Start Here
(2) Input time samples to DFT
(3) Input frequency samples to IDFT
(4) Finish Here

Start Here
Output time samples from IDFT

staggered sinc functions in time domain

gated sinusoid functions in frequency domain

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Each SC-OFDM time symbol is formed as a sum of constant envelope sinusoids cooperating over the same long OFDM symbol time. Each main lobe and tails extends over the same long-duration OFDM time interval.

In SC-OFDM, the narrow sinc-function main lobes are misleading, giving the impression that short symbols are sent. Just like OFDM sends the superposition of (long) gated sinusoids, here too SC-OFDM sends the superposition of (long) gated sinc functions.
Fourier Refresher

(a) DC waveform

(b) Pulse

(c) Sinusoid

(d) Higher freq Sinusoid

(e) Gated sinusoid

Time

Frequency

\[
\begin{align*}
\text{DC waveform} & : \ldots \quad t \\
\text{Pulse} & : \ldots \\
\text{Sinusoid} & : \cos 2\pi f_c t \\
\text{Higher freq Sinusoid} & : \cos 2\pi f_c t \\
\text{Gated sinusoid} & : \cos 2\pi f_c t
\end{align*}
\]

\[
\begin{align*}
-\infty < t < \infty \\
-\infty < t < \infty \\
-\infty < t < \infty \\
-\frac{T}{2} < t \leq \frac{T}{2}
\end{align*}
\]
Fourier Refresher (cont’d.)

Infinite BW
White (uncorrelated)
Example: AWGN

Wideband Spectrum of Gated Sinusoids
Phase slope = 0

Wideband Spectrum of Gated Sinusoids
Phase slope = \(-T_1\)

Wideband Spectrum of Gated Sinusoids
Phase slope = \(-T_2\)
SC-OFDM

- A data sequence is transformed to a sequence of 2-space points \( \{p_k\} \) following some \( M \)-ary modulation scheme. Next,
- The DFT transforms a time block of such points to a summation of \textit{wide BW} spectral samples (complex). One data point (with zeros at all other times in the block) yields an \( M_i(f) \angle \theta_i \)
- The sample rate is raised (zero extension)
- The IFFT transforms each wide BW spectrum to an offset \((\sin x)/x\) pulse (a phase offset in the frequency domain yields a time offset in the time domain)
- Superposition of the offset \((\sin x)/x\) pulses yields the SC-OFDM output time waveform
- Such waveforms carry constellation amplitudes, which are retrieved by the MF at the receiver
- For an IDEAL CHANNEL, sampling the output would \textit{yield the original} 2-space points
OFDM and SC-OFDM

Data is characterized by a set of points in 2-space that are mapped into coefficients of a sinusoidal basis set that describe the details of each sinusoid to be built.

**Staggered Sinc Functions**

Each data symbol is mapped to a separate subcarrier (i.e., carried by a sinc function in frequency).

**SC-OFDM**

Data is characterized by a set of points that can be visualized as \((\sin x)/x\) functions, offset in time.

Each data symbol is mapped to multiple subcarriers (i.e., carried by a sinc function in time).

**Visualization:** \((\sin x)/x\) (not actually built)

It is the transform of a gated sinusoid.

**Actually built:** Time offset \((\sin x)/x\) pulses

Zero extensions provide frequency padding.
The IFFT performs a Dual-like function in SC-OFDM compared to ordinary OFDM.

Data is characterized by a set of points in 2-space that are mapped into coefficients of a sinusoidal basis set that describe the details of each sinusoid to be built.

Visualization: \((\sin x)/x\) (not actually built)

It is the transform of a gated sinusoid.

Actually built: Time offset \((\sin x)/x\) pulses

Each data symbol is mapped to a separate subcarrier (i.e., carried by a sinc function)
Thank You

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