IEEE Presentation
March 25, 2021

The ABCs of OFDM

Part 2

by Dr. Bernard Sklar
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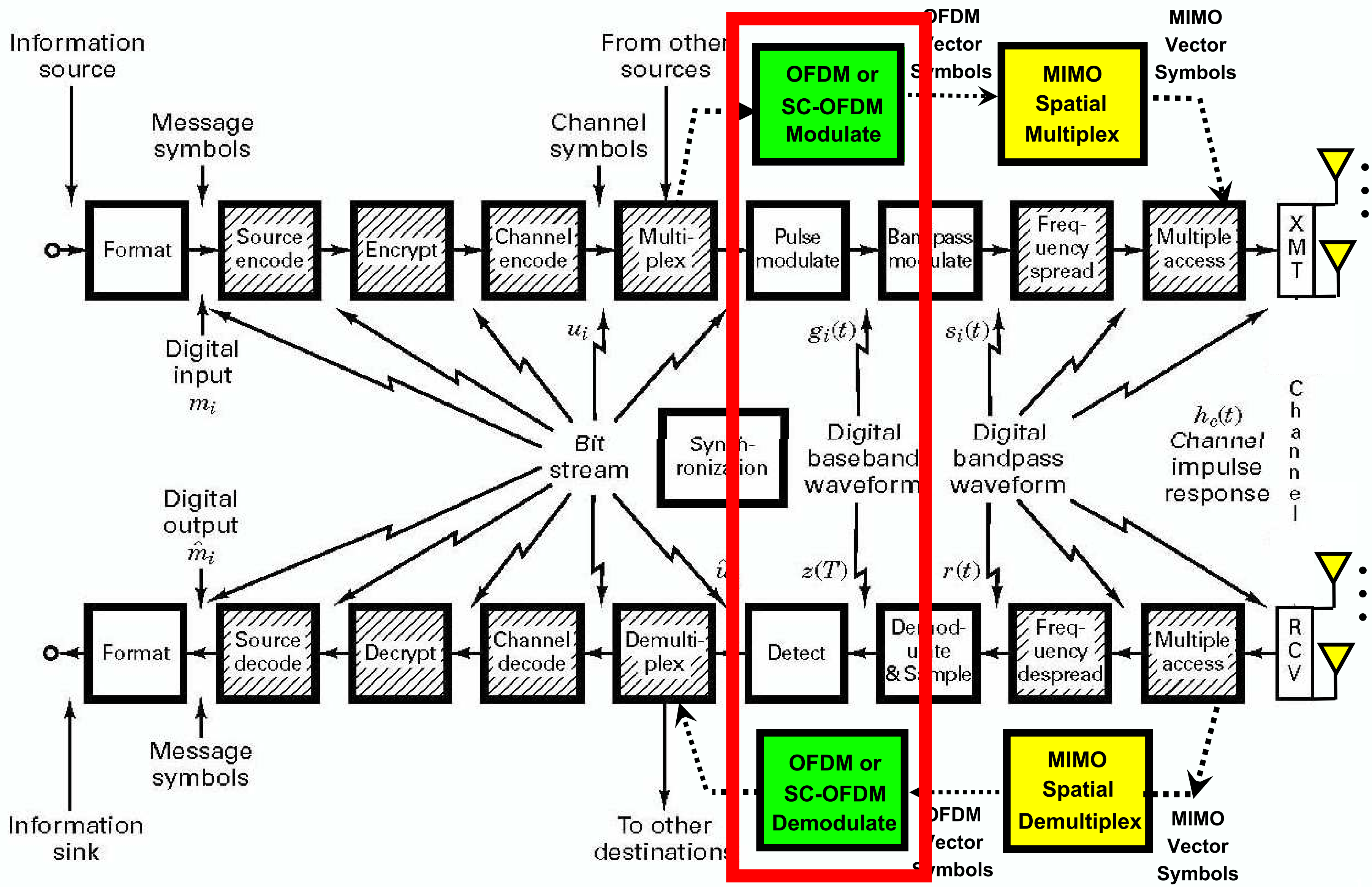
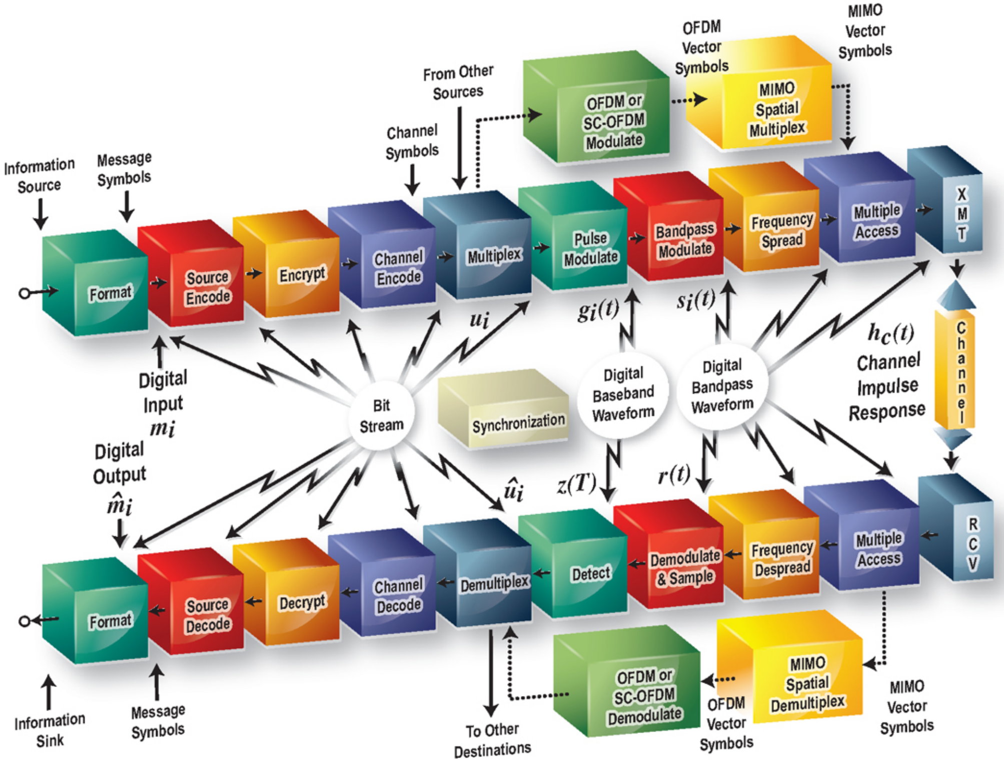

Information source
Message symbols
Channel symbols
From other sources
OFDM or SC-OFDM Modulate
OFDM Vector Symbols
MIMO Spatial Multiplex
MIMO Vector Symbols
Format
Source encode
Encrypt
Channel encode
Multi-plex
Pulse modulate
Bandpass modulate
Freq-uency spread
Multiple access
XMT
Digital input
m_i
u_i
$g_i(t)$
$s_i(t)$
Bit stream
Synch-ronization
Digital baseband waveform
Digital bandpass waveform
$h_c(t)$
Channel impulse response
Channel
Digital output
$\hat{m}_i$
$\hat{u}_i$
$z(T)$
$r(t)$
Format
Source decode
Decrypt
Channel decode
Demulti-plex
Detect
Demod-ulate & Sample
Freq-uency despread
Multiple access
RCV
Message symbols
Information sink
To other destinations
OFDM or SC-OFDM Demodulate
OFDM Vector Symbols
MIMO Spatial Demultiplex
MIMO Vector Symbols

THIRD EDITION

Digital Communications

Fundamentals and Applications



Bernard Sklar *and* fred harris

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- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Example
- Importance of the Cyclic Prefix (
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
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OFDM's main function is to manipulate orthogonal sinusoids.

What are 2 Key Characteristics of Orthogonal Sinusoids?

1. There must be an integer number of cycles of each sub-carrier sinusoid contained in the interval T_S

2. $\Delta f = 1/T_S$

where T_S is the data portion of the OFDM symbol

Other Orthogonality Characteristics

- **For reconstructing the correct OFDM subcarriers at the receiver:**

  We need to maintain signal orthogonality. This is accomplished by

  1. **preserving signal length**
  2. **preserving constant envelope**
  3. **preserving an integer number of cycles per gated sinusoid**

- **Preserving Length**

  The use of linear convolution with an *N*-point DFT would create a lengthened output. But, by making the signal (with a CP) appear circularly convolved, the original signal length is preserved.

- **Preserving Constant Envelope**

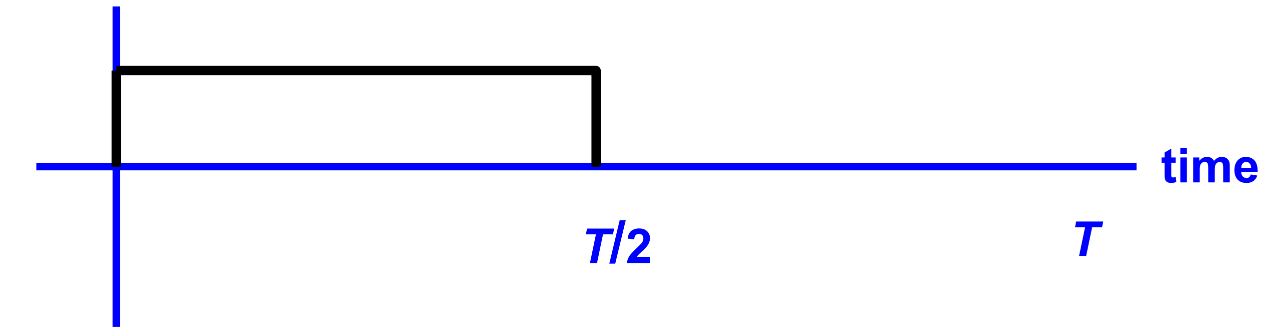
  Convolving a signal with the channel impulse response causes a transient at the start and end of the symbol. Any such transient causes envelope variations. The CP absorbs the starting transient of the current symbol and the stopping transient of the previous symbol. By discarding the CP in the guard interval (the overlapping transient), we thereby preserve a constant envelope for each gated-sinusoid symbol.

- **Preserving an Integer Number of Cycles**

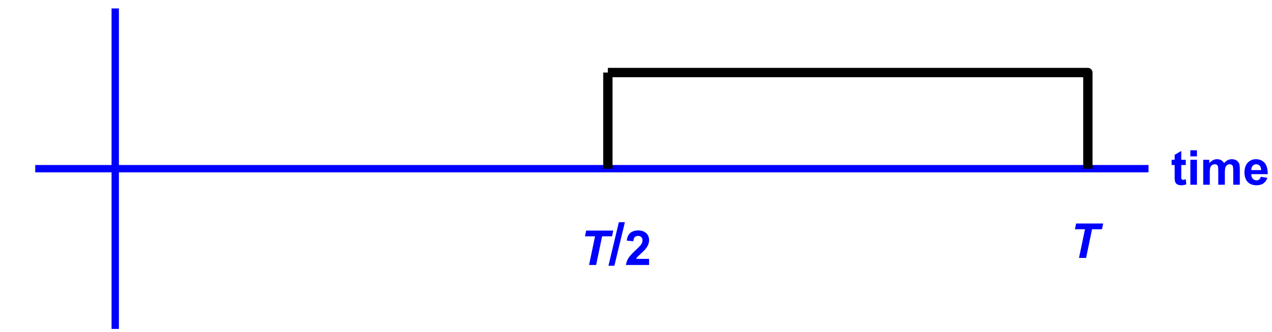
  Discarding the CP guard interval also preserves the integer number of cycles of each symbol (the way it was originally created).

Reminder: Testing for Orthogonality

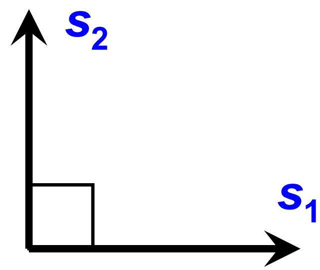
Orthogonality in the time domain assures orthogonality in the frequency domain, and vice versa. The property is easiest to see in the time domain.



S_1 and S_2 cannot possibly interfere with one another.

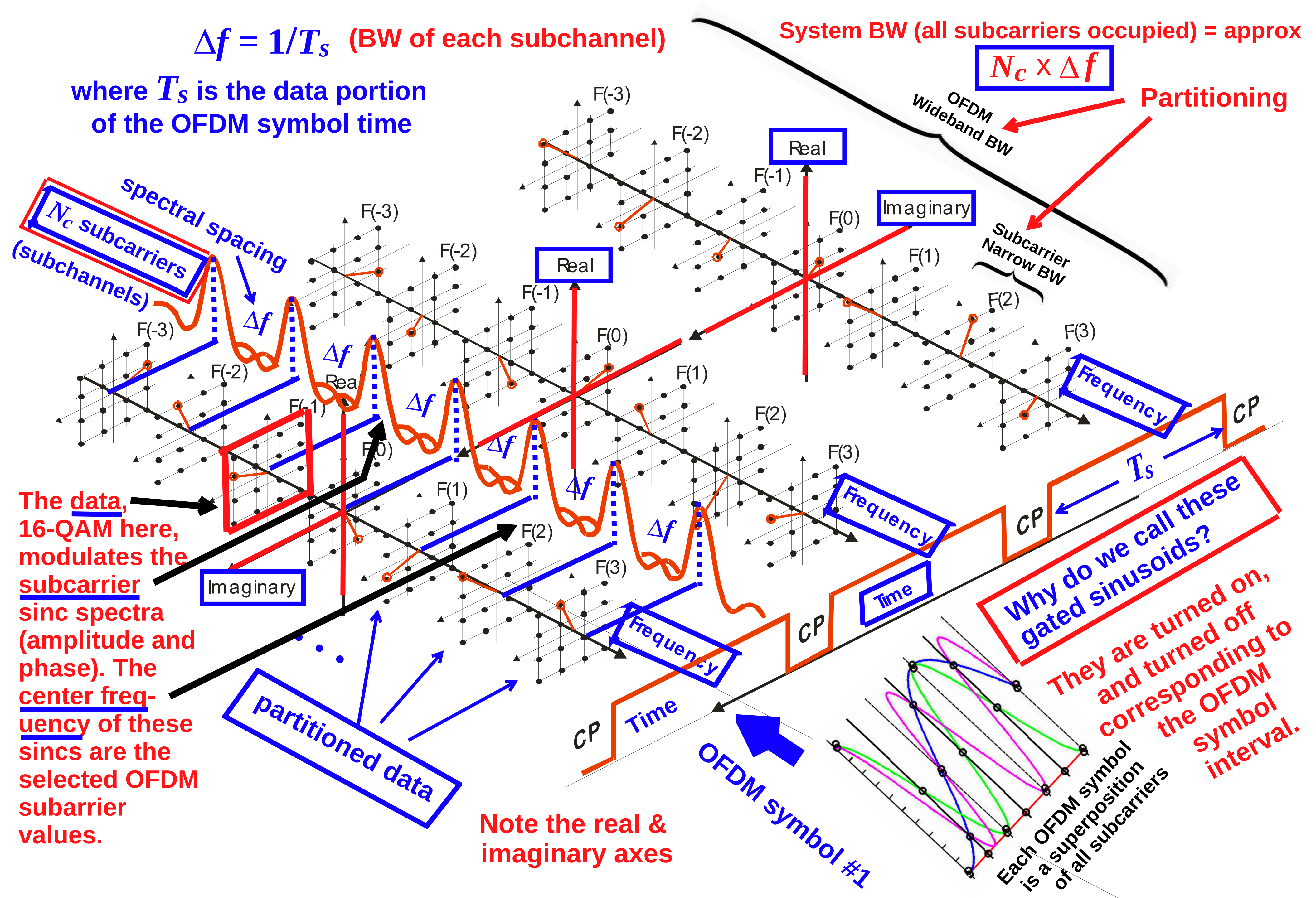


Vector Representation



$$\boxed{\int_0^T s_1(t)\, s_2(t)\, dt = 0}$$

Cross-Correlation Inner Product equals zero.

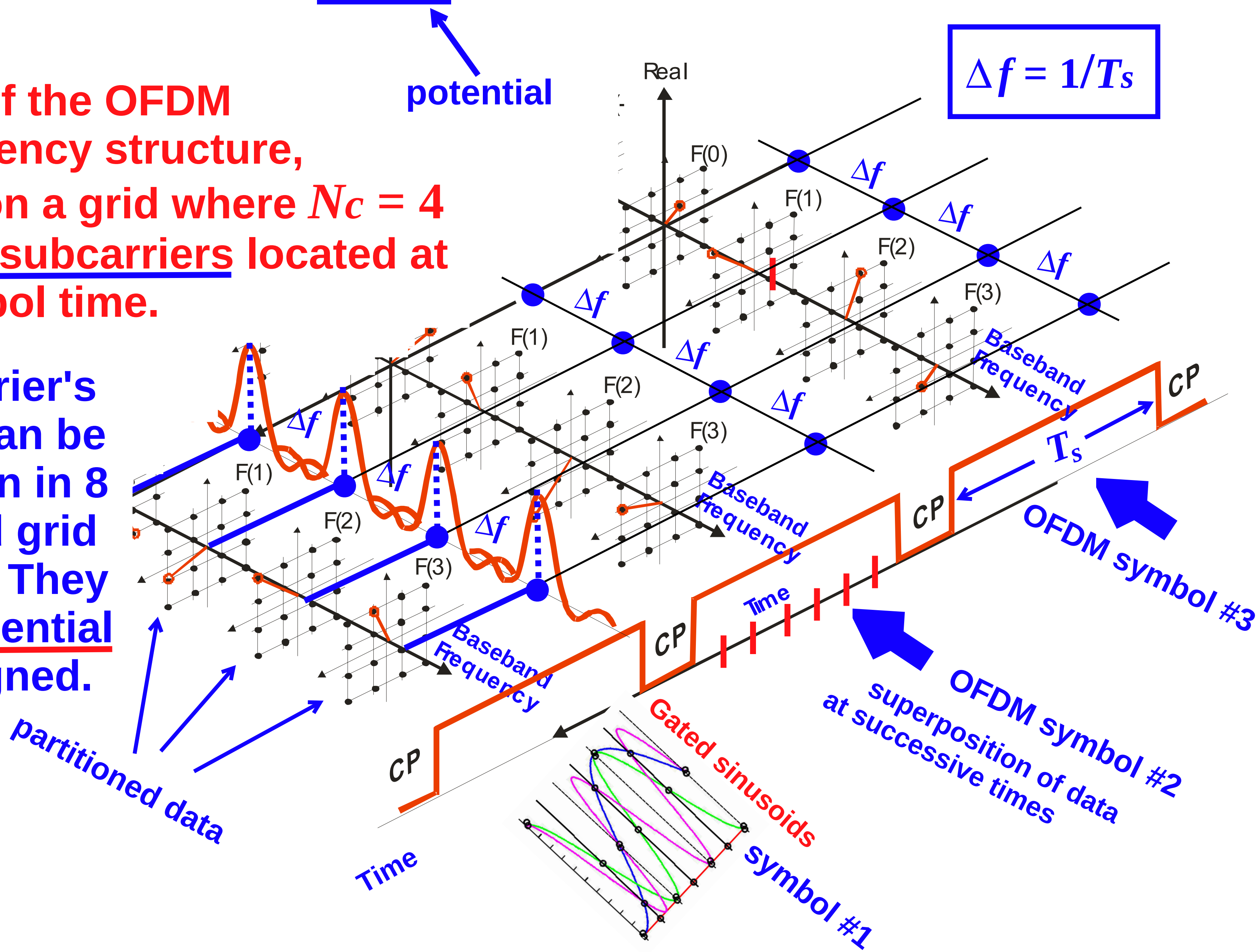


Δf = 1/Ts (BW of each subchannel)
where Ts is the data portion of the OFDM symbol time
System BW (all subcarriers occupied) = approx
Nc x Δf
OFDM Wideband BW
Partitioning
Subcarrier Narrow BW
Nc subcarriers (subchannels)
spectral spacing
Δf
F(-3)
F(-2)
F(-1)
F(0)
F(1)
F(2)
F(3)
Real
Imaginary
Frequency
Time
CP
Ts
The data, 16-QAM here, modulates the subcarrier sinc spectra (amplitude and phase). The center frequency of these sincs are the selected OFDM subarrier values.
partitioned data
Note the real & imaginary axes
OFDM symbol #1
Why do we call these gated sinusoids?
They are turned on, and turned off corresponding to the OFDM symbol interval.
Each OFDM symbol is a superposition of all subcarriers

An OFDM system, with $\boxed{N_c = 4}$ subcarriers and 16-QAM modulation

Example of the OFDM time/frequency structure, focusing on a grid where $N_c = 4$ candidate subcarriers located at each symbol time.

The subcarrier's amplitude can be zero, as seen in 8 unoccupied grid points here. They have the potential to be assigned.



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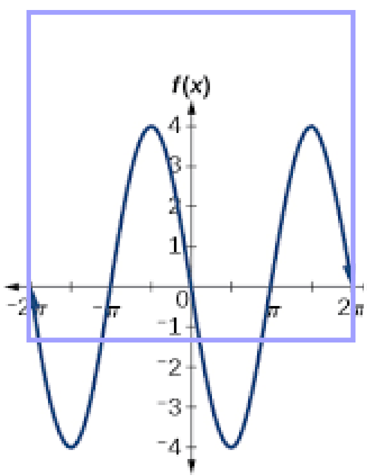
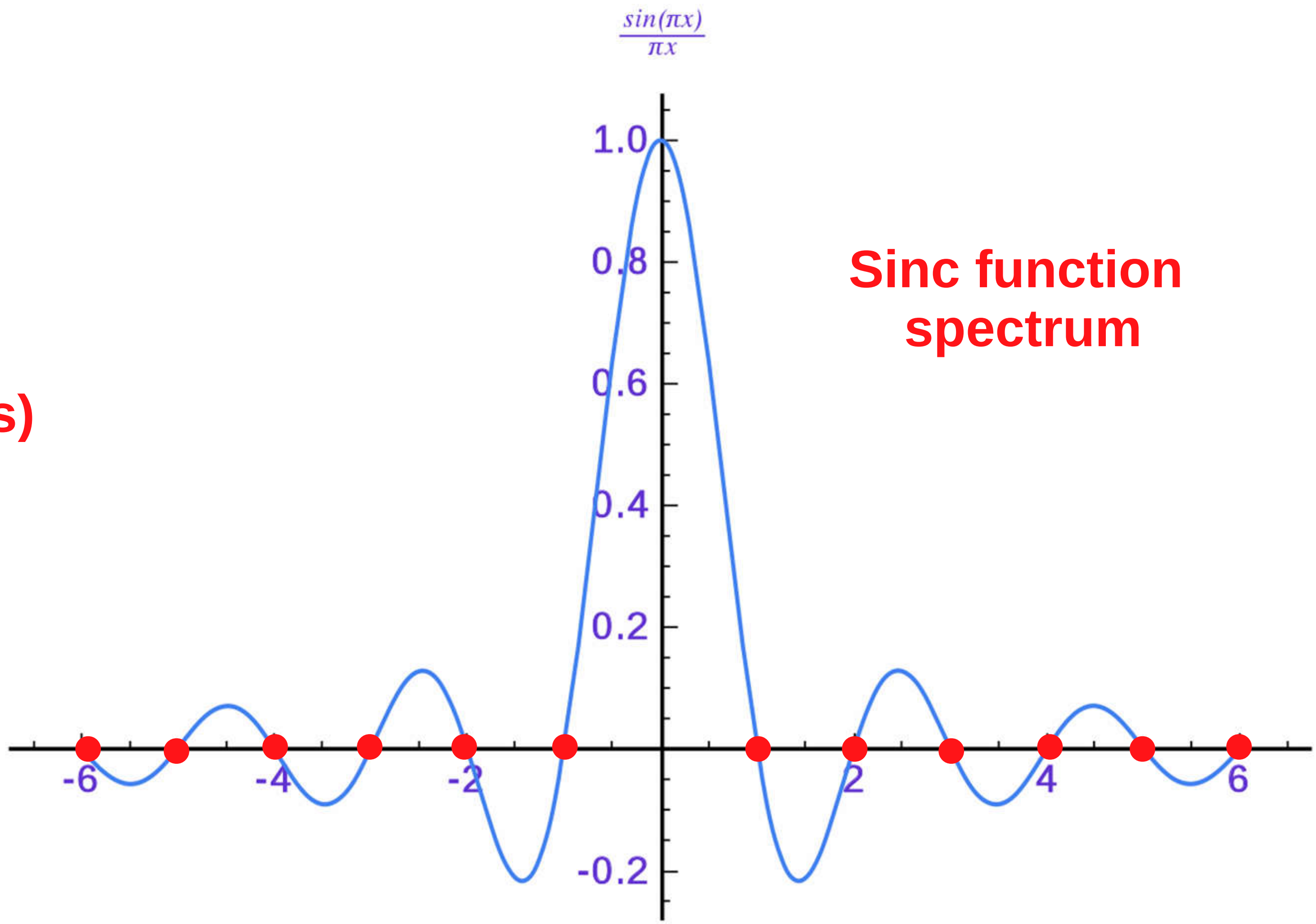
Part 2 starts here.

Periodic Outputs on a Unit Circle

The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

Gated Sinusoid (one sinusoid with an integer number of cycles)



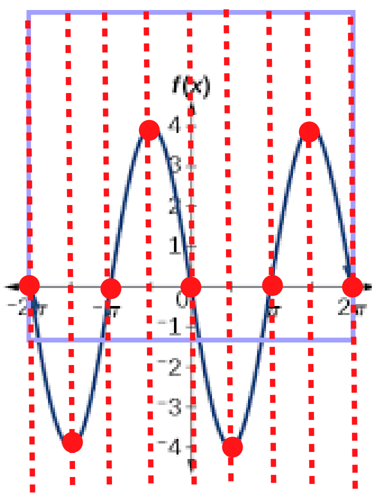


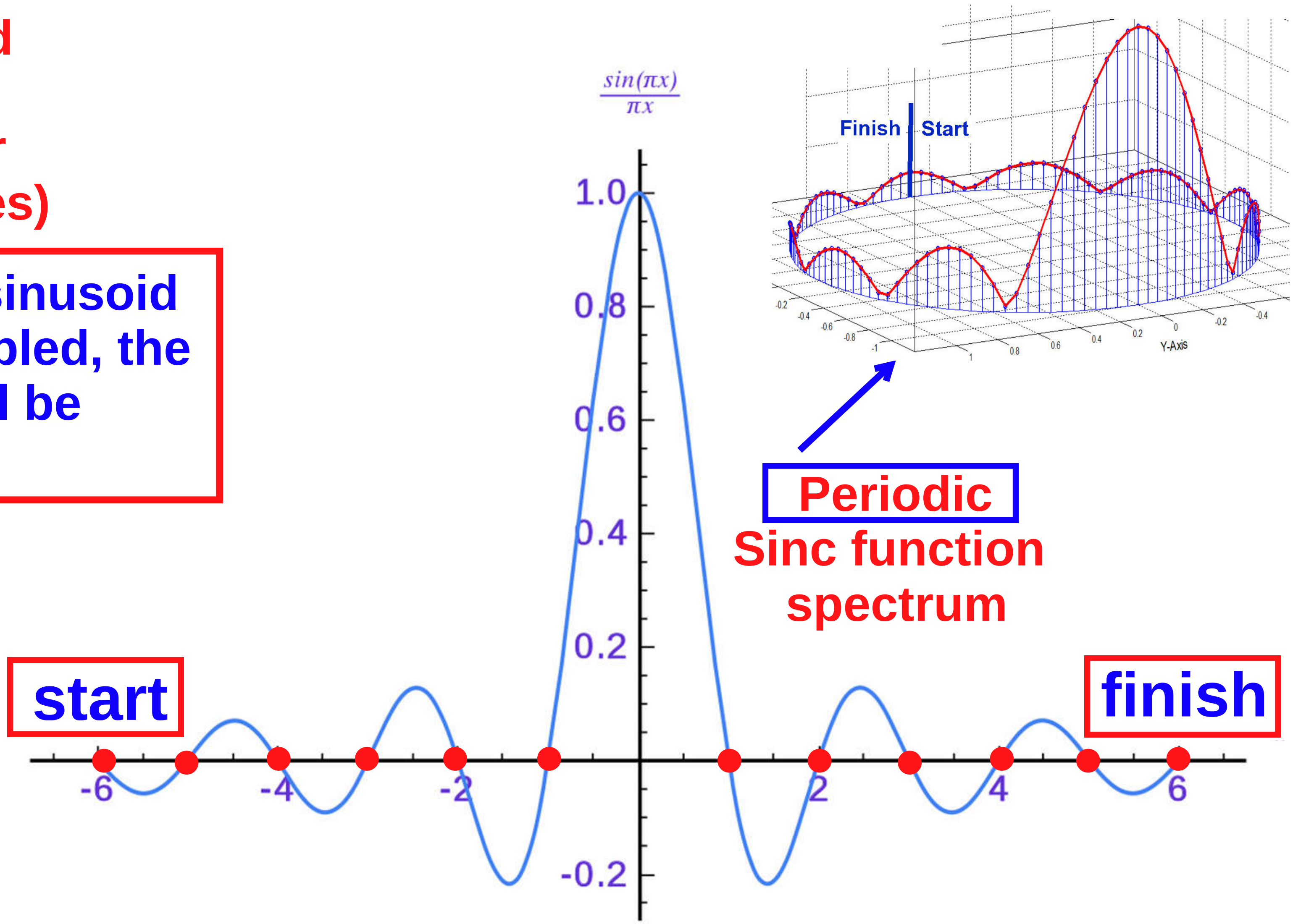
Sinc function spectrum

The Fourier Transform of a rectangular-windowed (gated) sinusoid is a sinc function, having equally spaced zeroes.

Gated Sinusoid (one sinusoid with an integer number of cycles)

And if the gated sinusoid is discretely sampled, the transform will be periodic.



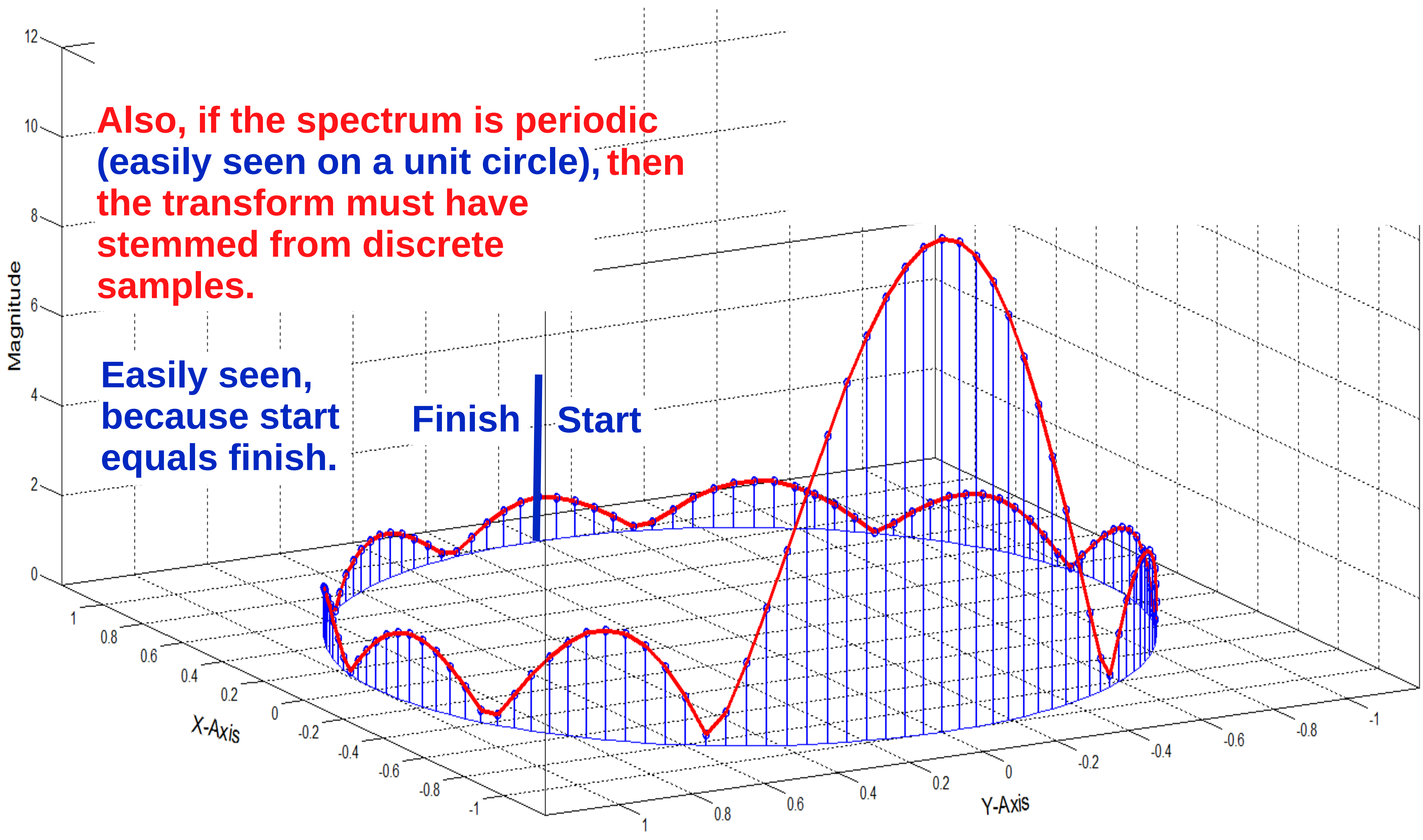


Periodic Sinc function spectrum

Let's plot this periodic spectrum as a power signal on a unit circle.

SIN(X)/X ON UNIT CIRCLE

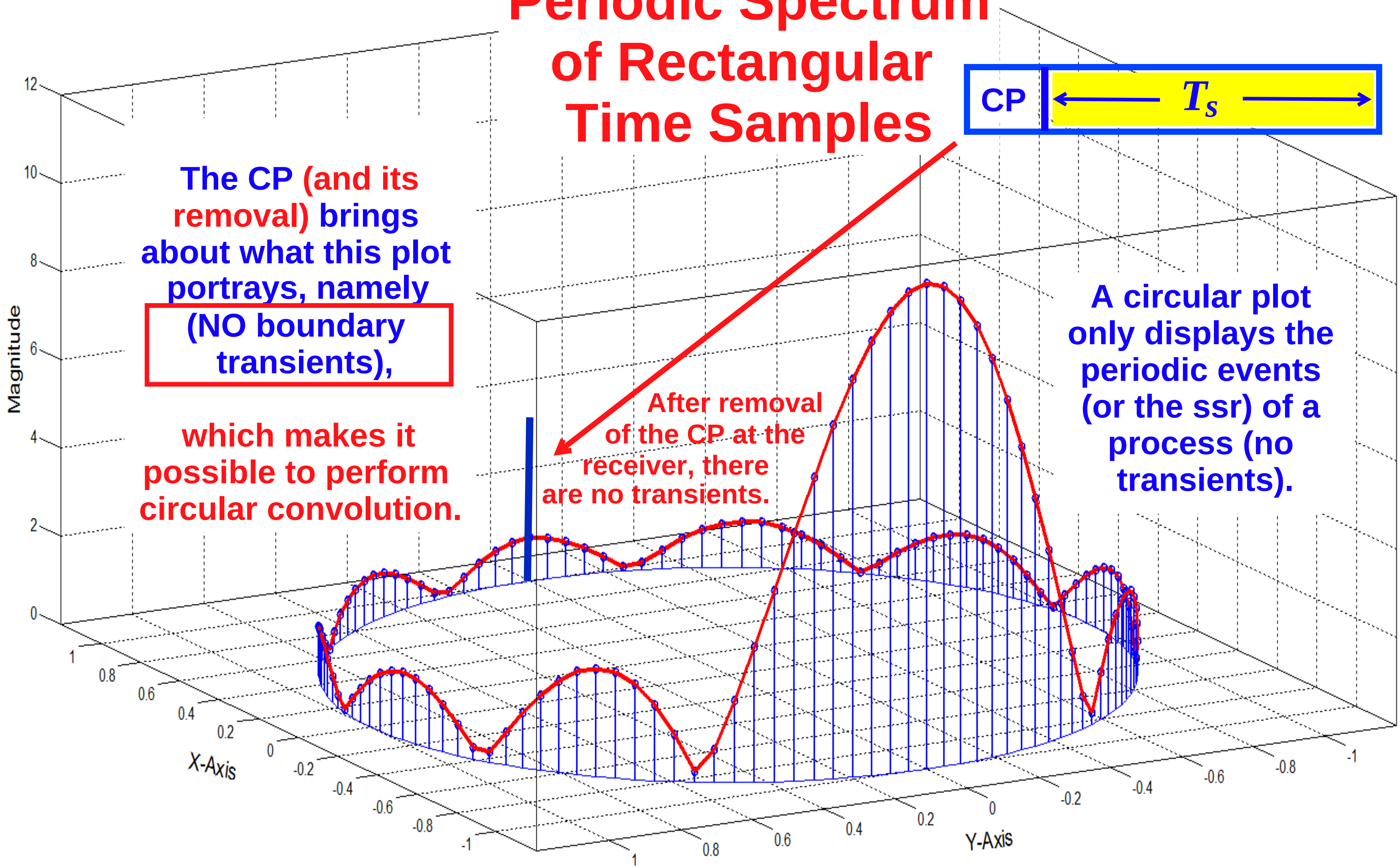
The DFT of a discretely sampled time sequence yields a continuous periodic spectrum.



SIN(X)/X ON UNIT CIRCLE

Periodic Spectrum of Rectangular Time Samples

CP | T_s



The CP (and its removal) brings about what this plot portrays, namely (NO boundary transients),

which makes it possible to perform circular convolution.

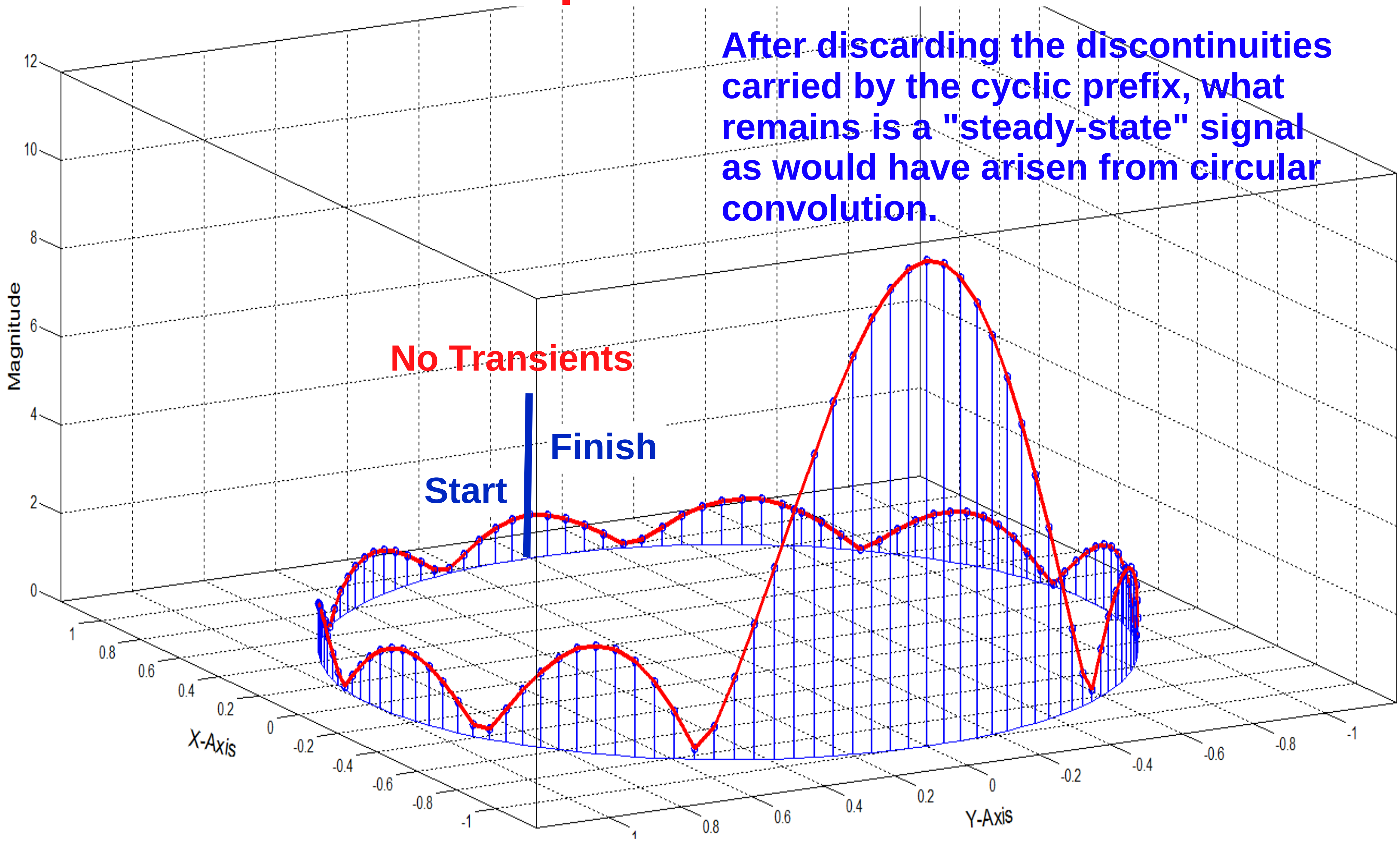
After removal of the CP at the receiver, there are no transients.

A circular plot only displays the periodic events (or the ssr) of a process (no transients).

Plotting the spectrum on a unit circle helps us visualize (as we go round-and-round the circle) that the spectrum is periodic.

SIN(X)/X ON UNIT CIRCLE

Periodic Spectrum at the Receiver



The steady-state response (ssr) has essentially gotten rid of all the On-Off transients.

The Cyclic Prefix in OFDM Modifies Linear Convolution so that it Appears to be Circular Convolution

- **A property of the Fourier Transform:**

  Spectral multiplication of continuous signals $X(f)H(f)$ corresponds to linear convolution $x(t) * h(t)$ in time.

- **A property of the Discrete Fourier Transform (DFT):**

  Spectral multiplication of sampled signals $X(k)H(k)$ corresponds to circular convolution $x(n) \circledast h(n)$ in time (sampling the transform makes the time signal periodic, and sampling the time signal makes the transform periodic).

- **When Using DFTs for implementing OFDM systems:**

  A continuous waveform, linearly convolved with the channel impulse response, is modified so that it appears to be circularly convolved with the channel impulse response. This makes the task of equalization simple – spectral scaling during the DFT.

* **The DFT forms a sampled-data spectrum. Samples in the frequency domain correspond to periodicity in the time domain. Any periodic function on a time-line is nicely portrayed as one copy of the function plotted on a unit circle (start and finish are the same point). This makes linear convolution appear to be circular.**

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OFDM Modem Block Diagram

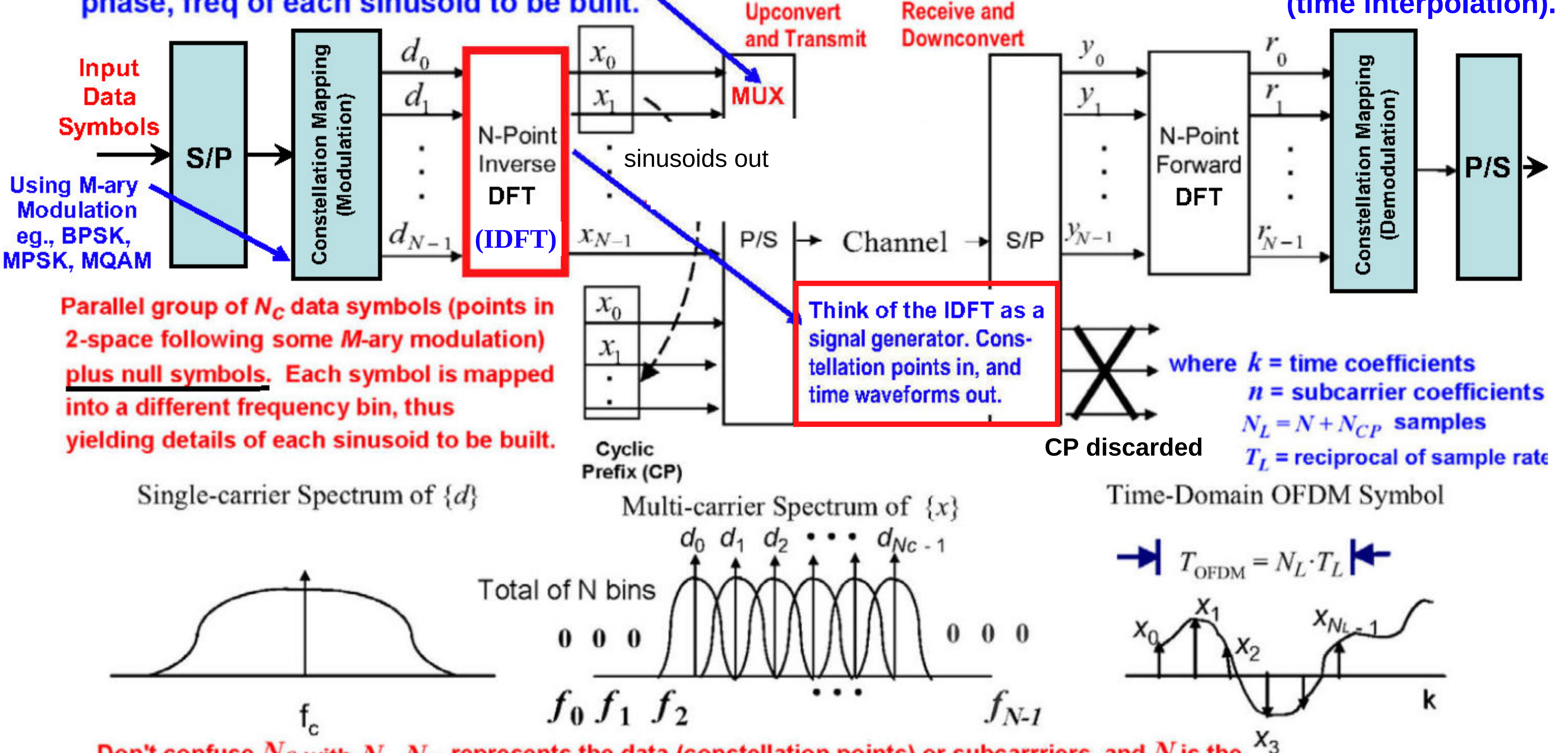
- An OFDM symbol is made up of a sum of N terms (N_C modulated orthogonal carriers plus null bins). Each k^{th} sample of a symbol can be represented as:

N is made larger than Nc by zero-padding Nc in the frequency domain, which raises the output sample rate (time interpolation).

IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.

$$x_k = \sum_{n=0}^{N-1} d_n e^{j\frac{2\pi}{N}nk} \qquad n, k = 0,1,2,\dots, N-1$$

where some of the d_n values are zero



Don't confuse N_C with N. N_C represents the data (constellation points) or subcarriers, and N is the transform size. For building real analog filters, we use zero extensions (null bins) to form the transform such that $N > N_C$.

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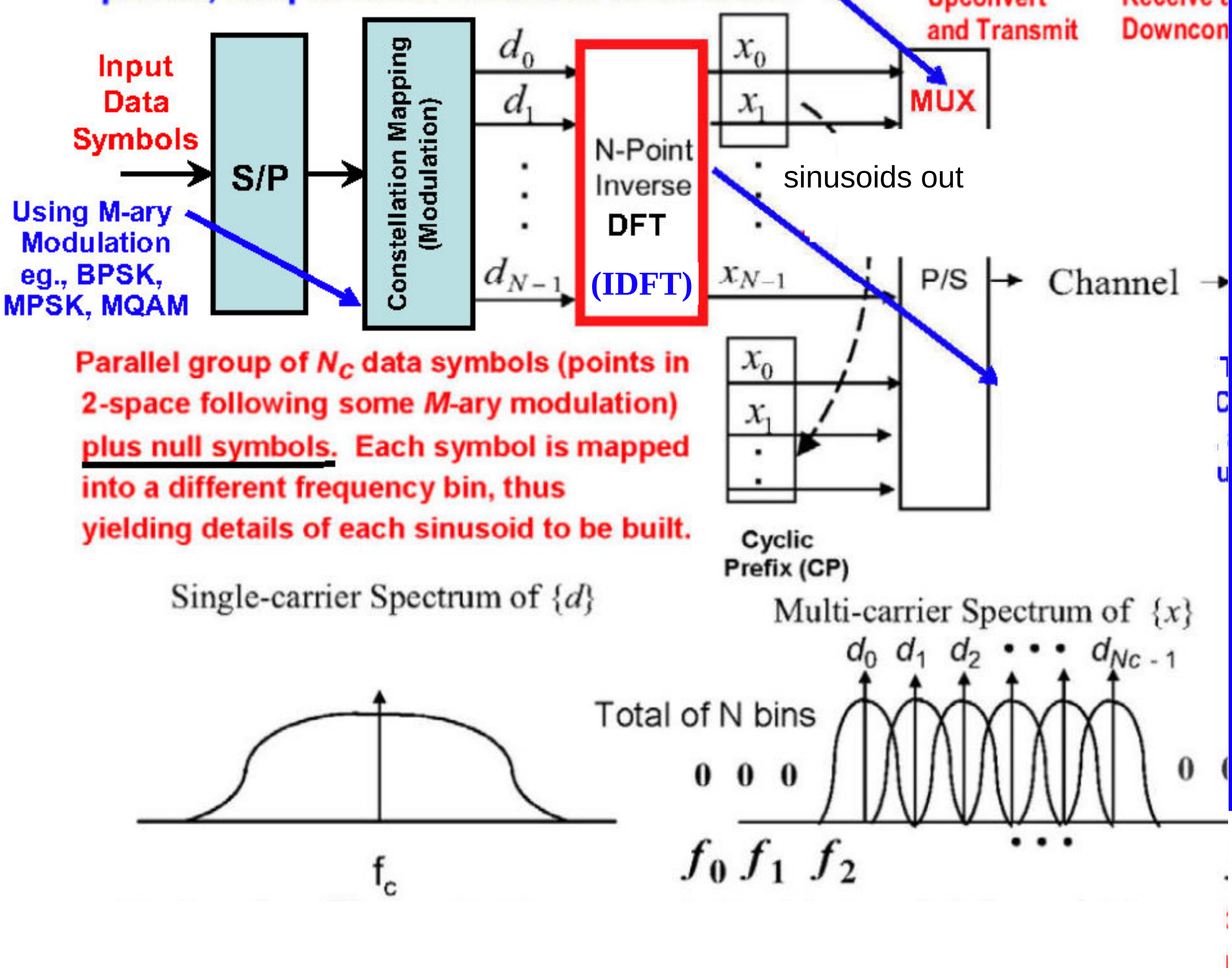


This slide shows the OFDM signal processing, making it appear as if the N output wires of the IDFT generate samples of N different tones, which means there would have to be:

N coherent oscillator/modulators (very costly processing). Some of the N tones would have zero amplitudes, leaving N_C enabled tones.

The actual Inverse Fourier Transform processor will output the sum of its N output wires (the superposition of all the enabled N_C tones). Each of the N output wires holds a successive time sample of the same superposition.

With this implementation, it is only possible to see the sum of the N_C tones, but not any one of them. They can only be seen alone after detection at the receiver.

59. In the early "battle" for the best codes (convolutioal vs. Reed-Solomon), what are the arguments for each, and why did convolutional win out?

60. In mobile channels, how does the terrain affect fading? How does the mobile-velocity affect it?

61. What is the advantage of circular-convolution versus linear-convolution? How do we trick the channel into performing circular convolution?

62. In OFDM, what is the mitigation technique for precluding ISI? For precluding ICI?

63. Baseband OFDM symbols are typically made up of independent data at positive and negative spectral locations. How is this effected, and how is a real transmission-signal formed?

64. For maintaining orthogonality among the subcarriers in OFDM, the tone spacing was chosen to be $1/T_S$. Why wasn't it chosen to be $1/T_{\mathrm{OFDM}}$? (Sklar ADC notes, section 3)

65. How can SC-OFDM still be resistant to multipath when the data symbols are so short? Hint: The time duration of a data pulse is longer than its main lobe.

66. Early skeptics about MIMO, claimed that it violated Shannon's capacity theorem. Why is that not the case?

67. Why won't MIMO work in a multipath-free environment?

68. Often, the signal-processing operations "DFT and IDFT" are called out as "FFT and IFFT," when one means the mathematical transformation. Why is this NOT precise?

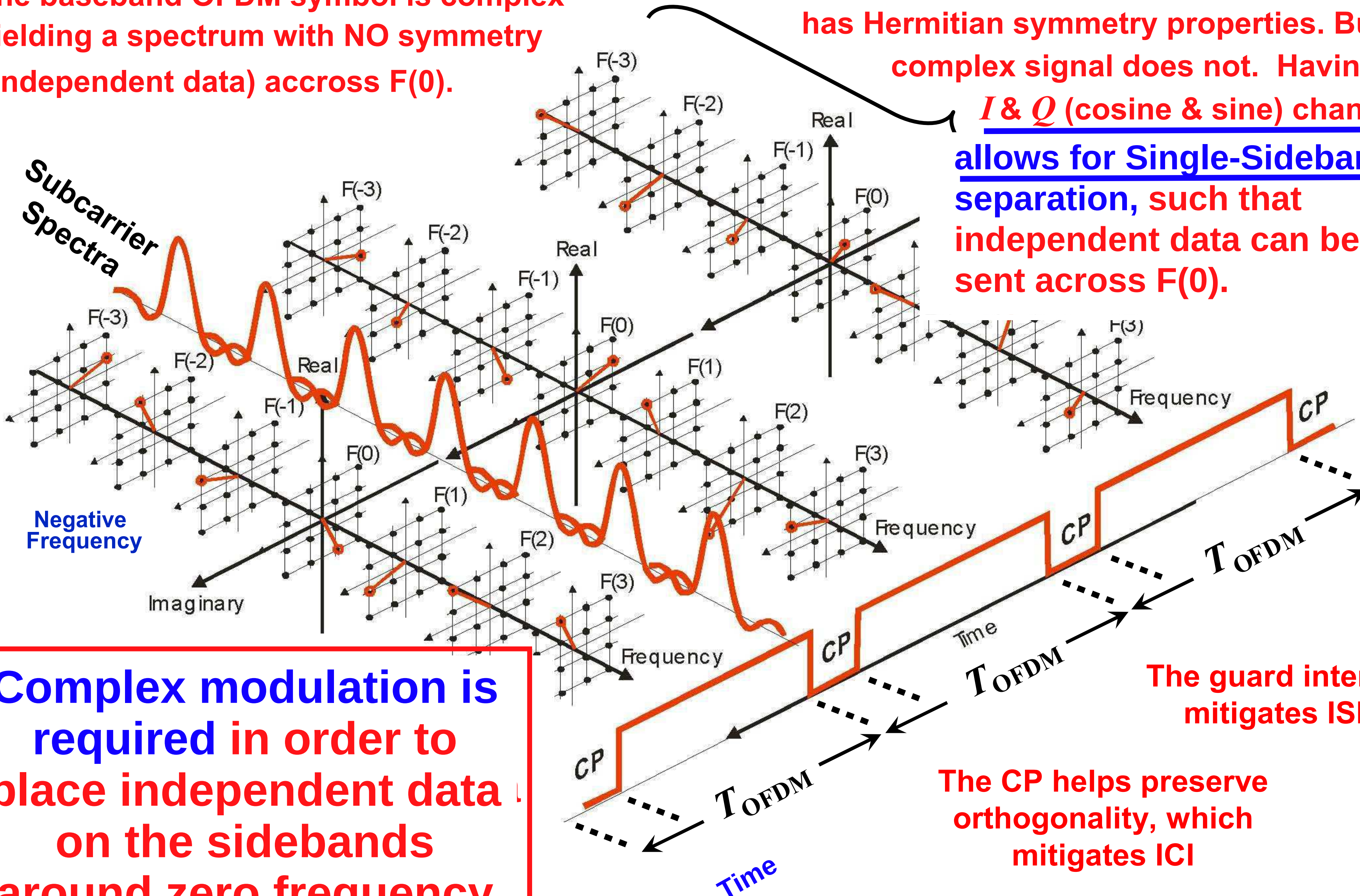
69. What are the Key Control Loops needed for system Synchronization? (fred harris, "Let's Assume the System is Synchronized.")

70. How do you shape a time waveform to meet system spectral-confinement requirements? Hint: symbol rate, sample rate, window type, filter length, transition BW, out-of-band attenuation.

Data Constellation Points Distributed over Time-Frequency Indices

1. The baseband OFDM symbol is complex yielding a spectrum with NO symmetry (independent data) accross F(0).

3. Modulation with such complex signals is not unique to OFDM. The transform of any real baseband signal has Hermitian symmetry properties. But a complex signal does not. Having *I* & *Q* (cosine & sine) channels allows for Single-Sideband separation, such that independent data can be sent across F(0).



2.

Complex modulation is required in order to place independent data on the sidebands around zero frequency.

The guard interval mitigates ISI.

The CP helps preserve orthogonality, which mitigates ICI

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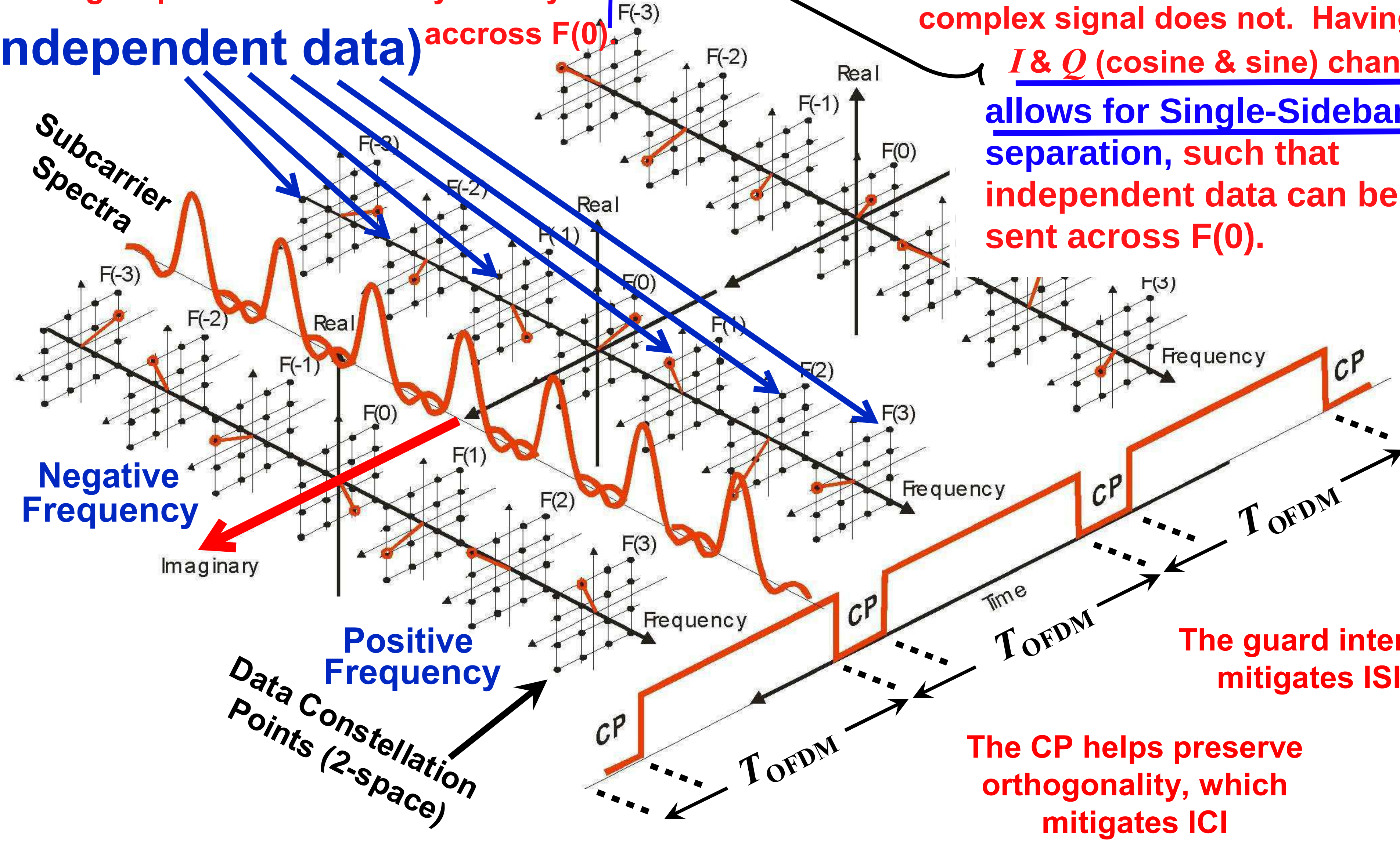
(independent data)



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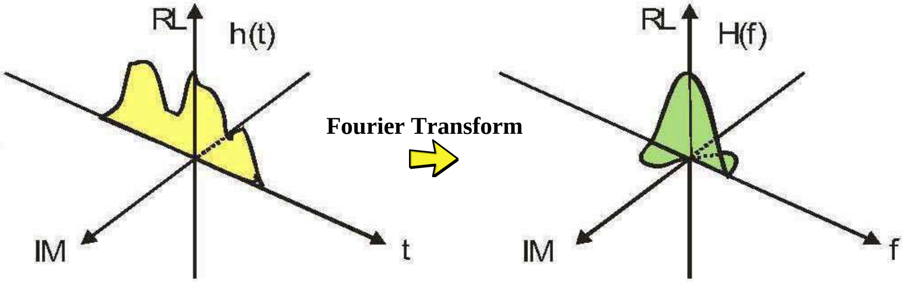
Real and Imaginary Signals

and Hermitian Symmetry

Real Time Signal

Complex Spectrum
with even & odd symmetry



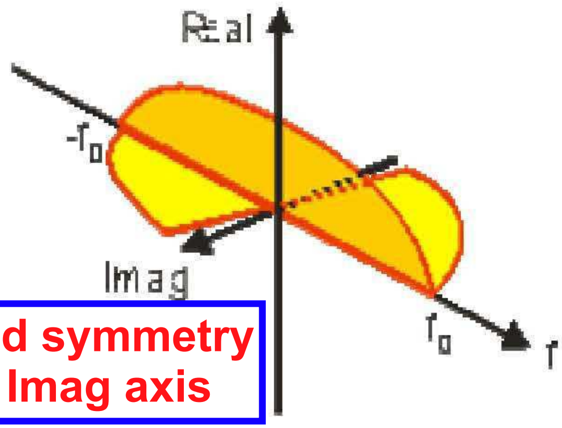
Similarly, an Imaginary Time signal has a Complex Spectrum
with even & odd symmetry.

Spectra of Real and Imaginary Signals: Spectrum of each is Complex.

Spectrum of each displays Hermitian Symmetry

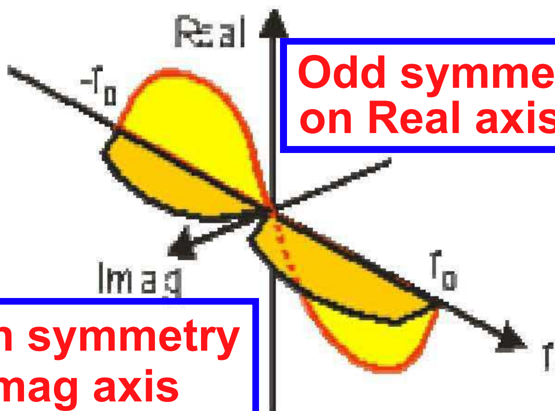
Real signal means there is NO j term

Even symmetry on Real axis



Odd symmetry on Imag axis

spectrum of Real Signal:



Odd symmetry on Real axis

Even symmetry on Imag axis

spectrum of Imaginary Signal:

Hermitian Transform Properties

- Real signals are typically made up of both cosine and sine components. Hence, the Fourier transform of a real signal is generally complex, and shows cosines on the real axis and sines on the imaginary axis.

- In the frequency domain, the spectrum of a real signal manifests even symmetry on the real axis, and odd symmetry on the imaginary axis (known as Hermitian transform properties). Even and/or odd symmetry in one domain corresponds to the same symmetry properties in the other domain.

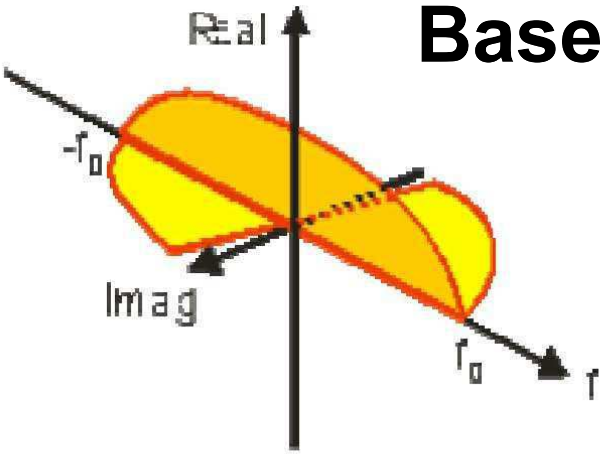
- Upper figure shows the spectrum of a real signal having such Hermitian (even/odd) properties.

- Lower figure shows the spectrum of an imaginary signal (j times a real signal), having anti-Hermitian properties (odd symmetry on the real axis, and even symmetry on the imaginary axis).

Complex Baseband signals have NO spectral symmetry.

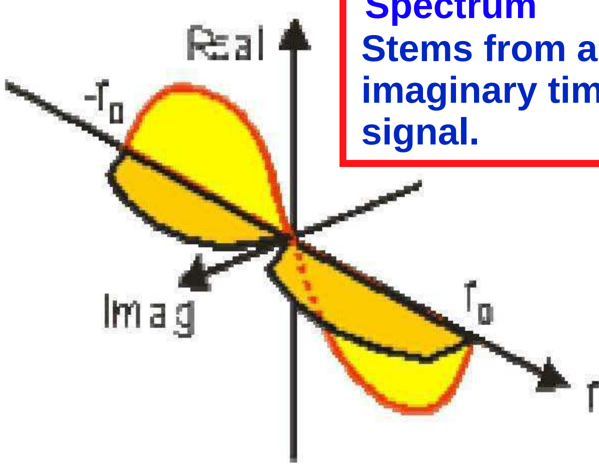
Spectrum Stems from a real time signal.

Non-Hermitian Spectrum of a Complex Baseband Signal stemming from a Real and an Imaginary Signal



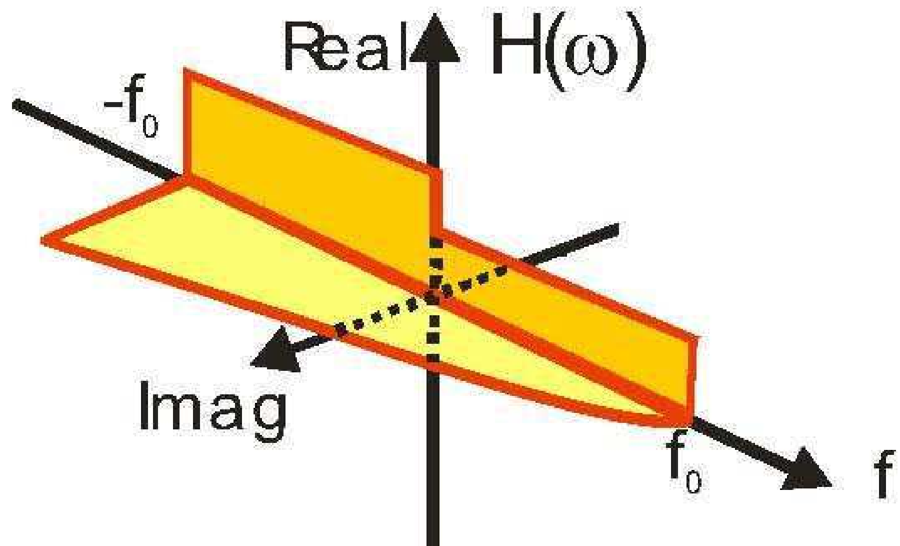
Real Signal:

Spectrum Stems from an imaginary time signal.



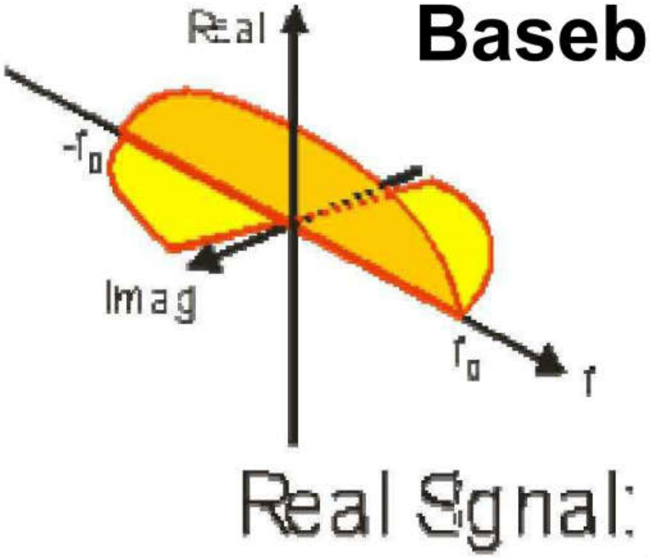
Imaginary Signal:

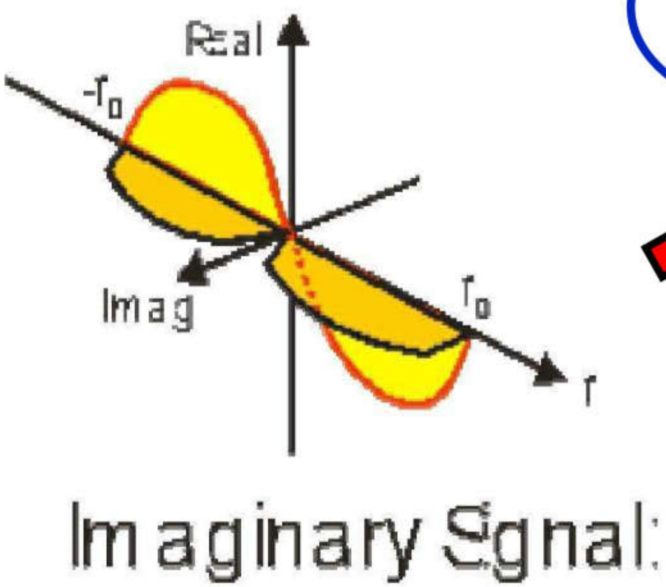
Adding the complex spectra of real and imaginary time signals yields no spectral symmetry whatever.



If the spectrum of a signal has no symmetry at all, then that spectrum must have stemmed from a complex time signal.

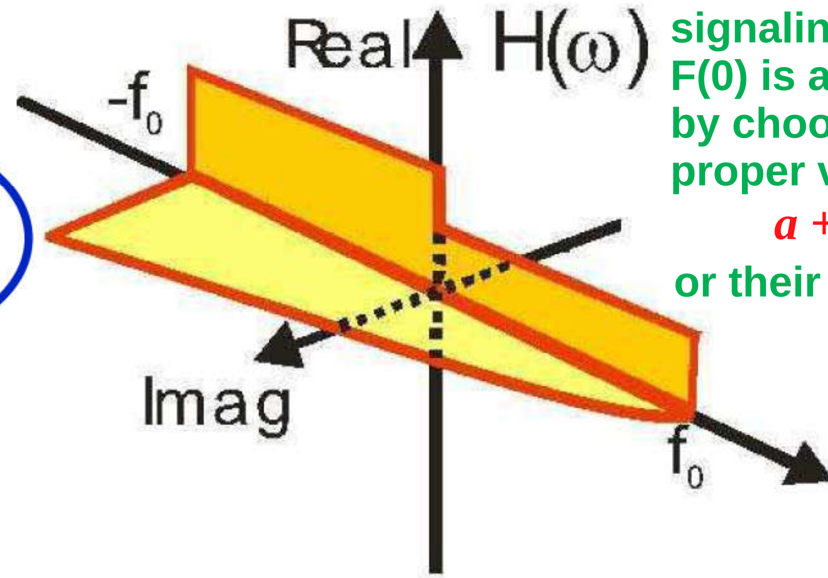
Non-Hermitian Spectrum of a Complex Baseband Signal stemming from a Real and an Imaginary Signal





Example: *a + jb*

Example: QAM or MPSK



From this sketch, one should see that independent signaling across F(0) is achieved by chooosing the proper values of *a + jb* or their spectra.

Spectrum has NO symmetry.
Since the time-signal is complex, it cannot be sent over a single wire or antenna.

Such a complex baseband time-signal has a spectrum with independent data around zero frequency.

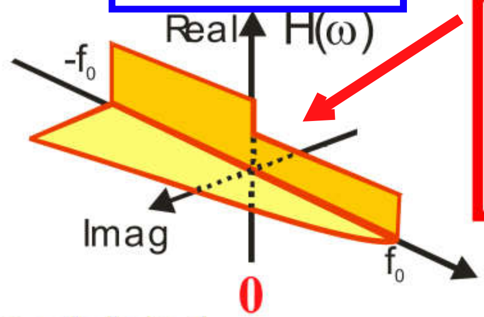
Complex Baseband & Real Band-Centered

Hence, there is independent data around zero frequency

OFDM Example:
We start with a complex baseband time signal having independent complex data symbols around zero frequency. Such a signal will require two cables for transmission.

*

Baseband

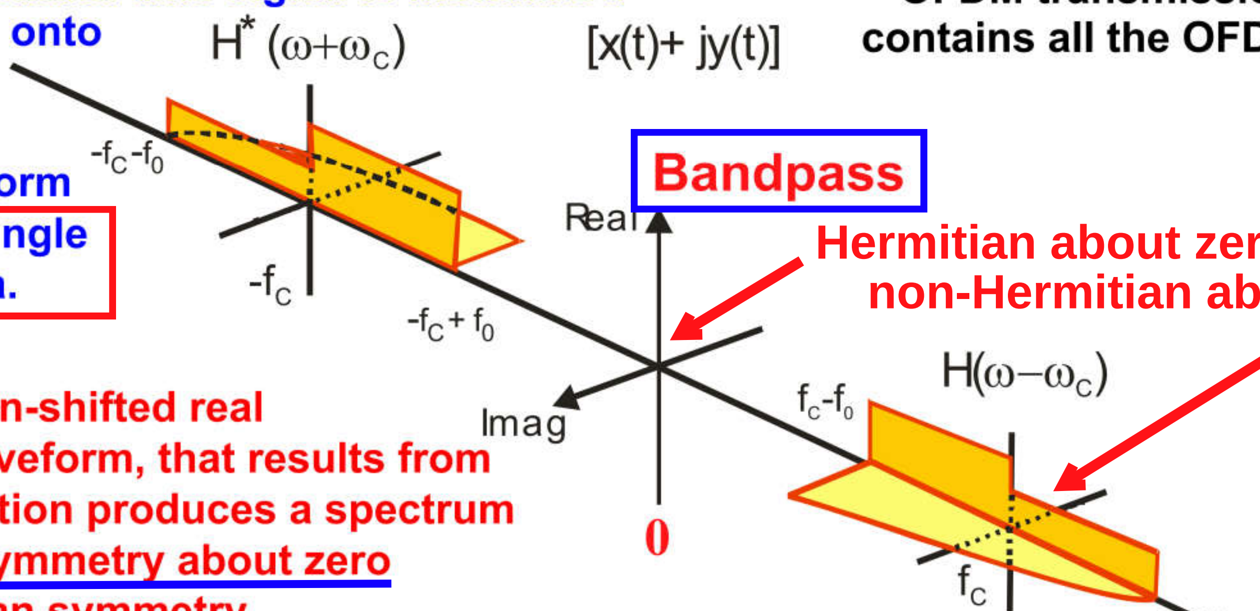


Spectrum of a complex baseband time signal does not have Hermitian symmetry about zero.

OFDM transmission BW contains all the OFDM sucarriers

Then, the complex baseband time signal is modulated onto a carrier (real part onto cosine, imaginary part onto sine), thereby producing a real waveform that can be sent on a single cable, or on an antenna.

Bandpass



Hermitian about zero and non-Hermitian about carrier

The up- and down-shifted real transmission waveform, that results from complex modulation produces a spectrum with Hermitian symmetry about zero and non-Hermitian symmetry about the carrier frequency.

complex conjugate

$$[x(t)+ jy(t)]\, e^{+j\omega_C t} + [x(t)-jy(t)]\, e^{-j\omega_C t}$$

RF waveform $x(t)\cos(\omega_C t) - y(t)\sin(\omega_C t)$

See Sklar text Appendix D, Equations D.4 and D.5

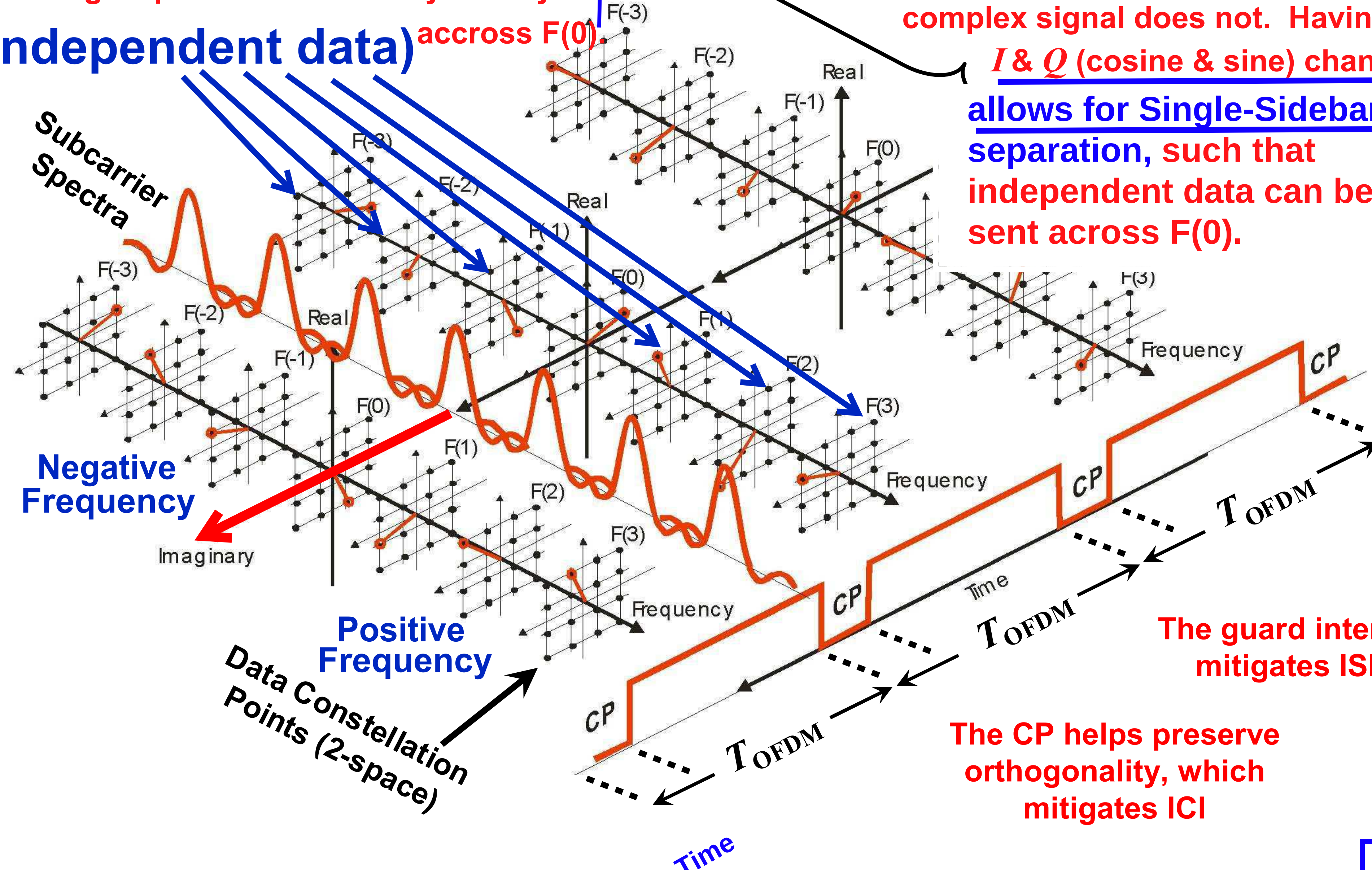
The RF waveform is formed as the real part of A = [x(t) + jy(t)] exp($j\omega_c t$), obtained by adding the complex conjugate of A to itself, and scaling by one-half.

Data Constellation Points Distributed over Time-Frequency Indices

The baseband OFDM symbol is complex yielding a spectrum with NO symmetry

(independent data) accross F(0).

Modulation with such complex signals is not unique to OFDM. The transform of any real baseband signal has Hermitian symmetry properties. But a complex signal does not. Having *I* & *Q* (cosine & sine) channels allows for Single-Sideband separation, such that independent data can be sent across F(0).



The guard interval mitigates ISI.

The CP helps preserve orthogonality, which mitigates ICI

IEEE Virtual Presentation
The ABCs of OFDM
By Dr. Bernard Sklar

Part 1 March 18, 2021
Part 2 March 25, 2021

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- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

Our "Wish List"

OFDM Glossary (and Channel Parameters)

N_c number of subcarriers

$N > N_c$ transform size (data-symbol samples)

$N_c = 0.6\,N$ typical subcarrier apportionment

N_{cp} cyclic prefix samples

$N_L = N + N_{cp}$ total samples per OFDM symbol

T_L sample time

$T_s = (N \times T_L)$ symbol time (data portion)

T_{cp} cyclic prefix time

$T_{cp} = 0.25\,T_s$ typical CP apportionment

$T_{\mathrm{OFDM}} = (T_s + T_{cp}) = (N_L \times T_L)$ OFDM symbol time

---

$\Delta f = 1/T_s$ frequency difference between adjacent subcarriers

$f_s = (N \times \Delta f) = 1/T_L$ sample rate

$W_{\mathrm{signal}} = (N_c + 1)\,\Delta f$ OFDM modulation BW

Channel parameters:

T_m max multipath delay

σ_τ rms multipath delay

$f_0 \approx 1/T_m$ coherence BW

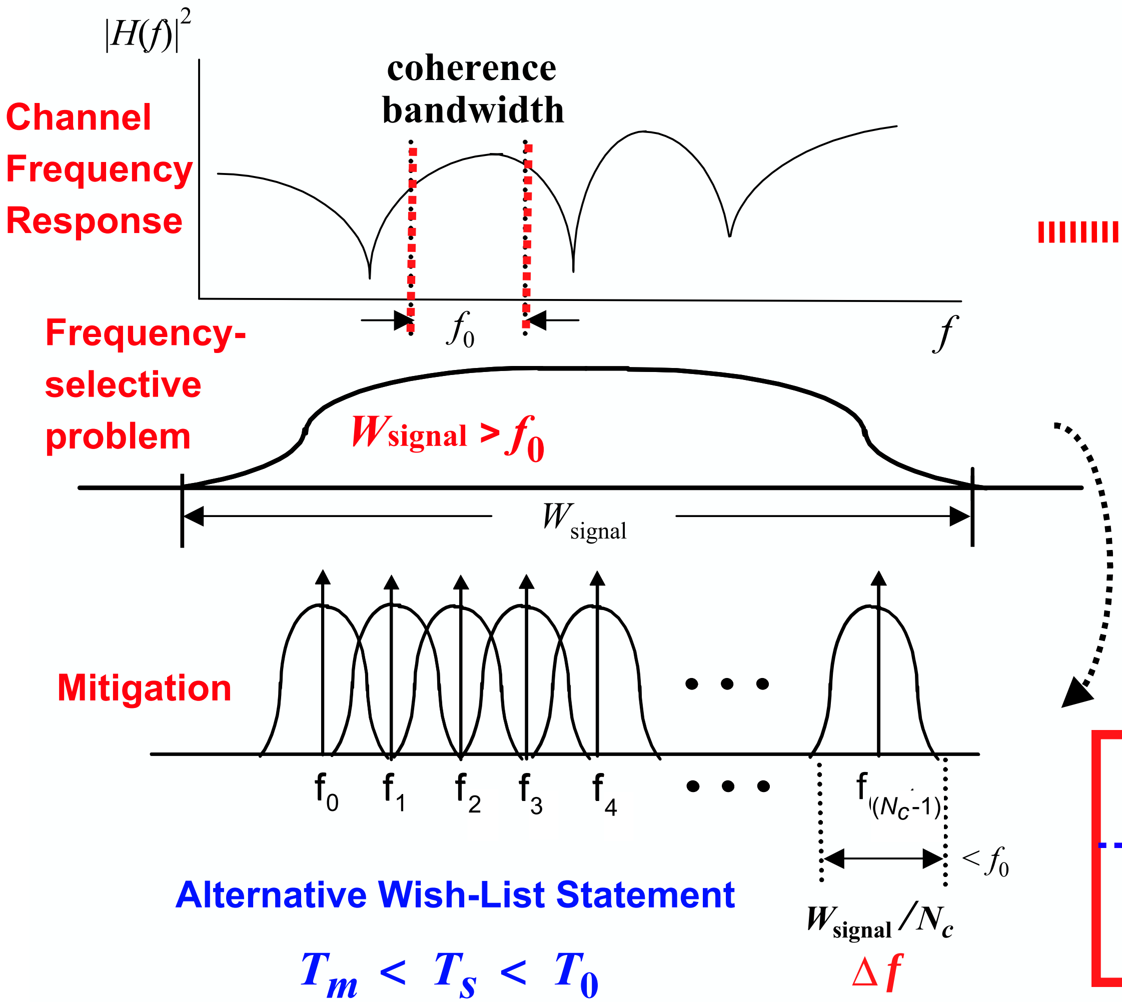
$f_0(50\%) \approx 1/5\sigma_\tau$ coherence BW over which the spectral correlation is at least 0.5

f_d fading rate (Doppler spectral spreading)

$T_0 \approx 1/f_d$ coherence time

Why OFDM?

- Divide-and-Conquer
  - Mitigation for frequency-selective fading environments. Parse the single, high-rate channel into N_c low-rate overlapping, and orthogonal, sub-channels.



Alternative Wish-List Statement

$$T_m < T_s < T_0$$

- Subdivide W_{signal} by large N_c so that
  $\Delta f = W_{\text{signal}}/N_c << f_0$

  ⇒ We desire flat faded sub-channels

  For a fixed W_{signal}
- Large $N_c \Rightarrow$ Large $T_s = N_c/W_{\text{signal}}$

$\Rightarrow$ Reduced *relative* ISI when $T_s >> \sigma_\tau$
But, for slow fading, we want
$T_s < T_0$, thus T_0 defines the upper bound of N_c **Otherwise, pulse mutilation**

- We want that:

	Coherence BW		Symbol rate		Fading Rate
In General	f_0	$>$	$1/T_s$	$>$	$1/T_0$
For OFDM	f_0	$>$	W_{signal}/N_c (partitioned)	$>$	$1/T_0$

Our "Wish List"

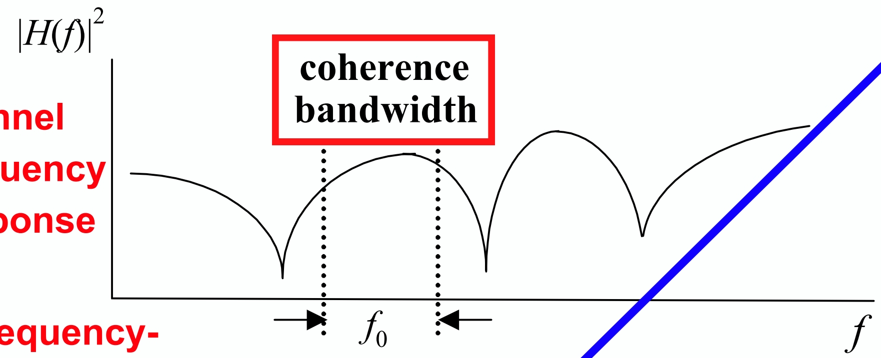
where T_s = time duration of the data portion of OFDM symbol

Why OFDM?

Recall the WSSUS model. We want to avoid frequency-selective fading. Notice our wish-list below. OFDM makes it easy to achieve flat fading.

- Divide-and-Conquer
  - Mitigation for frequency-selective fading environments. Parse the single, high-rate channel into N_c low-rate overlapping, and orthogonal, sub-channels.

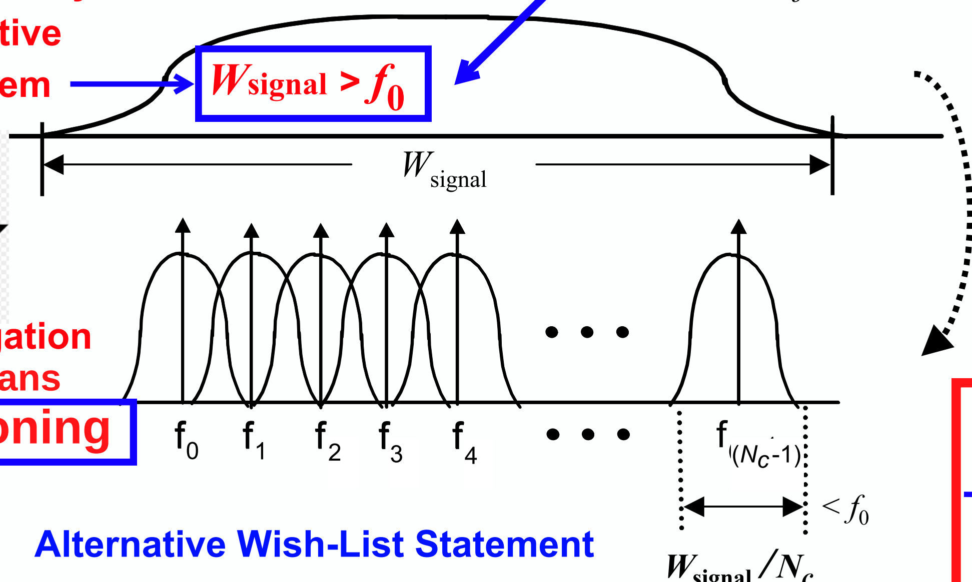
Channel Frequency Response



Frequency-selective problem

Mitigation means

partitioning



Alternative Wish-List Statement

$$T_m < T_s < T_0$$

where T_s = time duration of the data portion of OFDM symbol

partitioning a high-data-rate

- Subdivide W_{signal} by large N_c so that $W_{signal}/N_c << f_0$

  We desire flat faded sub-channels

- Large $N_c \Rightarrow$ Large $T_s = N_c / W_{signal}$

$\Rightarrow$ Reduced *relative* ISI when $T_s >> \sigma_\tau$
But, for slow fading, we want $T_s < T_0$, thus T_0 defines the upper bound of N_c Otherwise, pulse mutilation

- We want that:

	Coherence BW		Symbol rate		Fading Rate
In General	f_0	>	$1/T_s$	>	$1/T_0$
For OFDM	f_0	>	partitioned W_{signal}/N_c	>	$1/T_0$

Our "Wish List"

$\boxed{N_c}$ represents the number of potential (candidate) subcarriers, with locations of $k\Delta f$, where k is any positive or negative integer.

- We want that:

Coherence BW | | Fading Rate

In General $f_0 > 1/T_s > 1/T_0$

Symbol Rate

For OFDM $f_0 > W_{\text{signal}}/N_c > 1/T_0$

Our "Wish List"

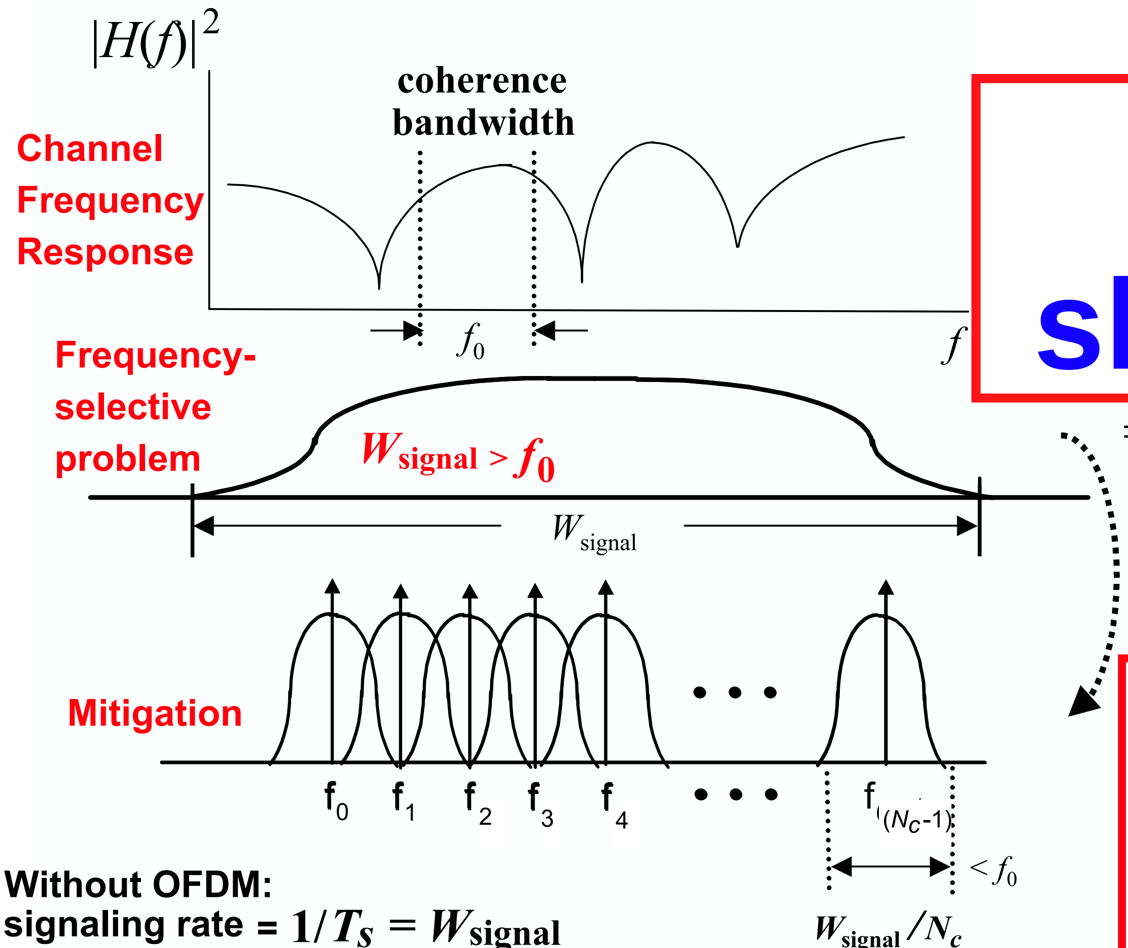
to preclude frequency-selective & fast fading

where $\boxed{W_{\text{signal}}/N_c = \Delta f} = \dfrac{\text{Total OFDM bandwidth}}{\text{number of candidate subcarriers}}$

subchannel BW

Why OFDM?

- Divide-and-Conquer
  - Mitigation for frequency-selective fading environments. Parse the single, high-rate channel into N_c low-rate overlapping, and orthogonal, sub-channels.



How large should N_c be?

$\Rightarrow$ Reduced *relative* ISI when $T_s >> \sigma_\tau$
But, for slow fading, we want $T_s < T_0$, thus T_0 defines the upper bound of N_c

- We want that:

	Coherence BW		Fading Rate
in general	$f_0 >$	$1/T_s$	$> 1/T_0$
OFDM	$f_0 >$	W_{signal}/N_c	$> 1/T_0$

Our "Wish List"

Without OFDM:
signaling rate = $1/T_s = W_{signal}$

With OFDM, for each subcarrier: signaling rate = W_{signal}/N_c

T_s = time duration of the data portion of OFDM symbol

How Large should N_c be?
What Range should it fall in?

$$f_0 > \underbrace{\frac{W_{\text{signal}}}{N_c}}_{\Delta f \,=\, 1/T_S \,=\, \text{OFDM symbol rate}} > \frac{1}{T_0}$$

Take the reciprocal of each term, invert the inequalities, and then multiply by the signal Bandwidth.

$$\frac{W_{\text{signal}}}{f_0} < N_c < T_0 W_{\text{signal}}$$

The left-side inequality allows computation of minimum N_c.

The right-side inequality allows computation of maximum N_c.

OFDM System Design $\boxed{\text{Example}}$

Assume a wireless channel with an rms delay spread σ_τ of 5 µsec, and a coherence time T_0 of 50 μsec. If the transmission BW is 20 MHz, find the min and max number of OFDM subcarriers needed in order to insure flat-fading and slow-fading effects.

coherence BW   signaling rate   fading rate

$$f_0 > \frac{1}{T_s} > \frac{1}{T_0}$$

General "Wish List"

$$W_{\text{signal}} / N_c = \Delta f = 1/T_s$$

Note that a coherence time of 50 microseconds is very short. A typical mobile system's coherence time is in the order of milliseconds.

$$f_0 > \frac{W_{\text{signal}}}{N_c} > \frac{1}{T_0}$$

OFDM "Wish List"

$$\boxed{\frac{W_{\text{signal}}}{f_0} < N_c < T_0 W_{\text{signal}}}$$

Solution Using the $f_0(50\%)$ approximation

$$f_0 \approx \frac{1}{5\sigma_\tau} = \frac{1}{5\times 5\times 10^{-6}} = 40\ \text{kHz}$$

$$(N_c)_{\min} \geq \frac{W_{\text{signal}}}{f_0} = \frac{20\ \text{MHz}}{40\ \text{kHz}} = 500$$

$$(N_c)_{\max} \leq T_0 W_{\text{signal}} = 50\times 10^{-6}\times 20\times 10^{6} = 1000$$

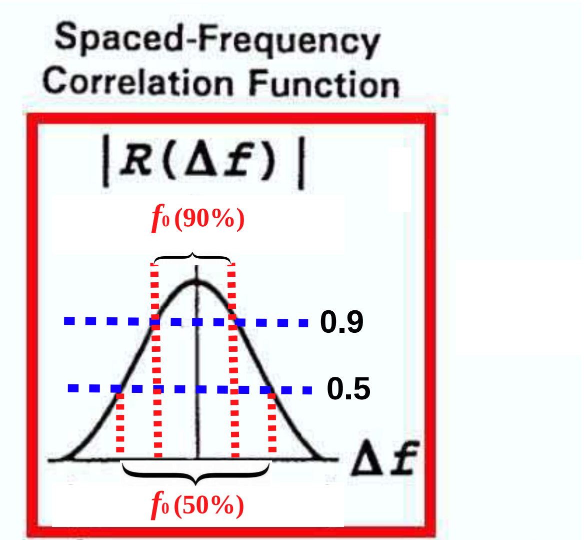
Number of subcarriers should fall in the **range of 500 to 1000.**

In this example, one of the given parameters is NOT reallistic. Which one?

$f_0(90\%)$

and

$f_0(50\%)$



$f_0 \cong 1/T_m$ Coherence Bandwidth

The larger the correlation, the narrower the correlation BW

The parameter T_0 = 50 x 10^{-6} seconds is not realistic because:

Assume a carrier frequency of $fc = 300 \text{ MHz}$

$$\lambda = \frac{c}{fc} = \frac{3 \times 10^8}{3 \times 10^8} = 1\, m \qquad T_0 \approx \frac{0.5 \times \lambda}{v} = \frac{0.5}{v} = 50 \times 10^{-6}$$

very short coherence time

$$v = \frac{0.5}{T_0} = \frac{0.5}{50 \times 10^{-6}} = 10 \times 10^3\, m/s \;=\; 22{,}369.4 \text{ miles/hour}$$

Coherence time (average channel-state time duration) is a function of velocity.
For the given T_0 of only 50 x 10^{-6} sec, the velocity needs to be unreasonably large.

This makes sense. Coherence time is inversely proportional to velocity. When there is no motion, coherence time is infinite.

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Transforming Linear Convolution to Circular Convolution

This is our tool for tricking the channel.

by rearranging the past
and the future

MOVING THE DISCONTINUITIES

End of cycle at back end

Start of cycle at front end

continuous edge

transient at new starting edge

- The cyclic-prefix-end matches the signal-front. There will no longer be a transient at the original time signal's starting edge. The transient now resides at the new starting edge of the cyclic prefix which will be tossed.

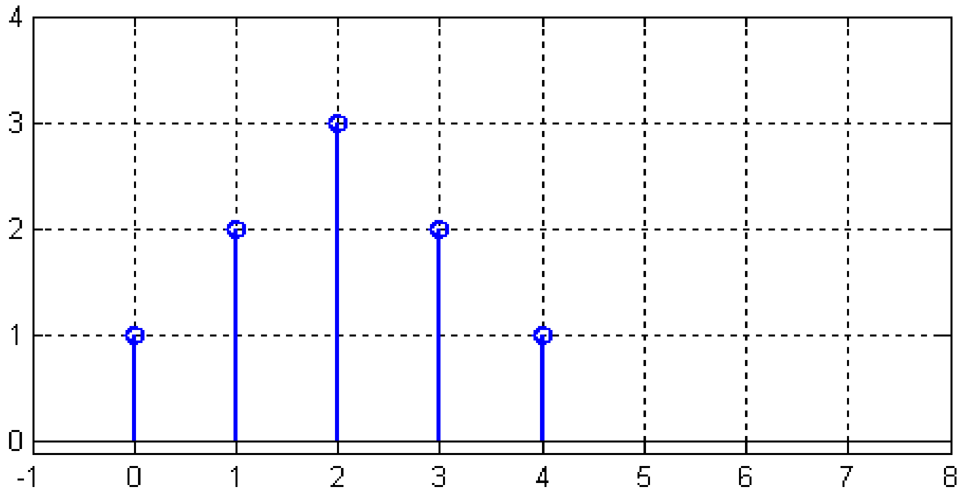
- Integer number of cycles per symbol interval
- Hence back-end of CP = front-end of symbol
- Continuous edge between added CP and old starting edge
- Transient at new starting edge

An Example of Tricking the Channel

(Converting Linear Convolution to Circular Convolution)

Example: Use of the CP makes linear convolution appear circular

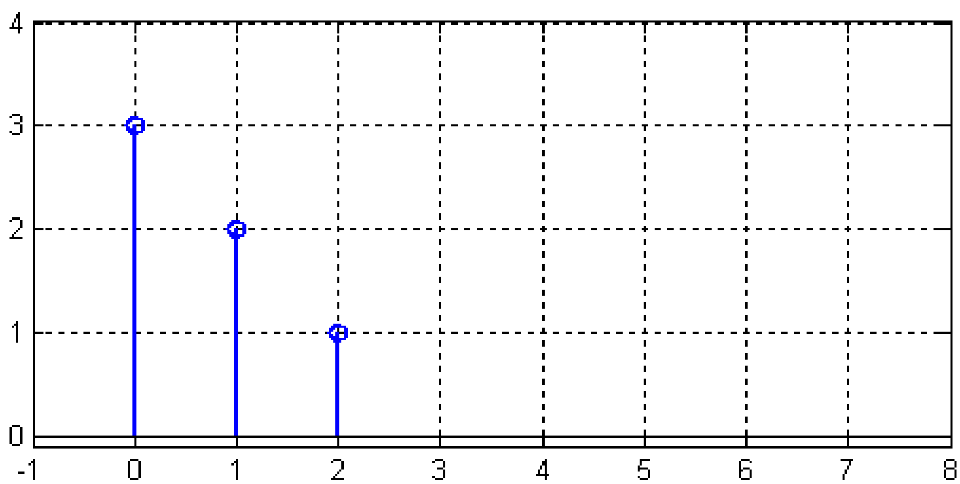
Sampled data $x(k)$ = [1, 2, 3, 2, 1]



$x(k)$ transmitted over channel $h(k)$ = [3, 2, 1]

$h(k)$

Channel impulse response



Time-reverse one of the functions. Here, $h(k)$ was reversed. Then perform multiply, add, and shift (product integration)

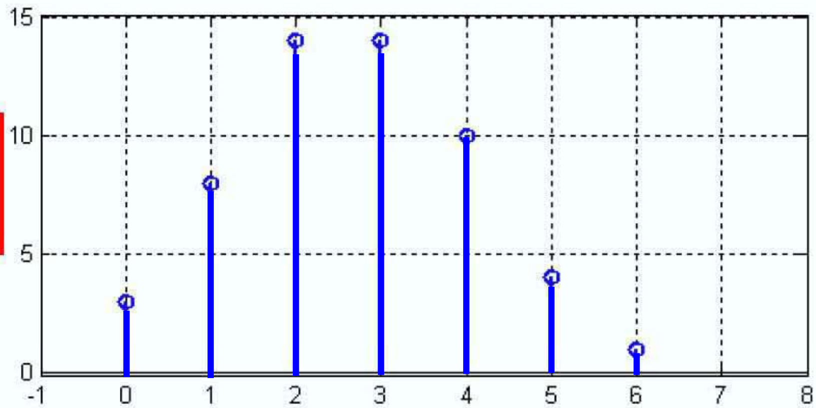
$$\begin{array}{cccccccl} & & 1 & 2 & 3 & 2 & 1 & \\ 1 & 2 & 3 & & & & & = 3 \end{array}$$

$$\begin{array}{cccccccl} & 1 & 2 & 3 & 2 & 1 & & \\ 1 & 2 & 3 & & & & & = 8 \text{ etc.} \end{array}$$

Linearly convolved output $x(k)*h(k)$

$= [3, 8, 14, 14, 10, 4, 1]$

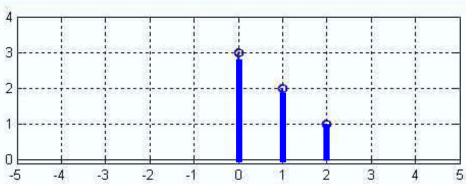
Linear Convolution

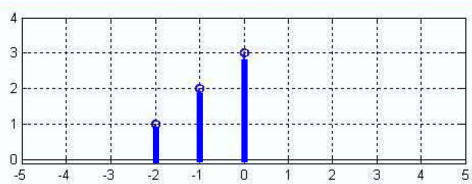


Note: The original five data samples have grown to seven.

What would a circularly convolved $x(k) \circledast h(k)$ look like?

$h(k)$

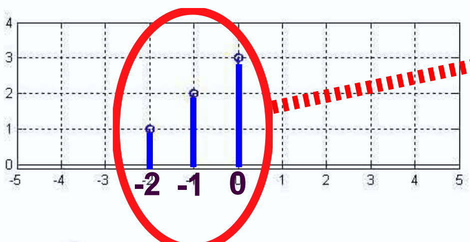


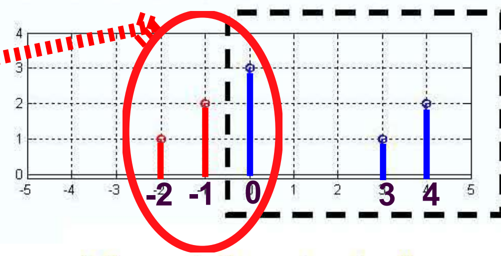


$h(-k)$

Time-reversed channel impulse response

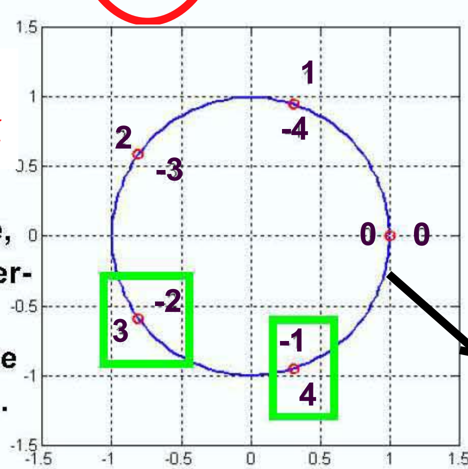
$h(-k)$





Repositioning neg. samples
0 is still 0.
-1 becomes +4.
-2 becomes +3.

Circularized time index k



On the Circle, Outer Counter-Clockwise values are the positive bins. Inner Clockwise values are the Negative Bins.

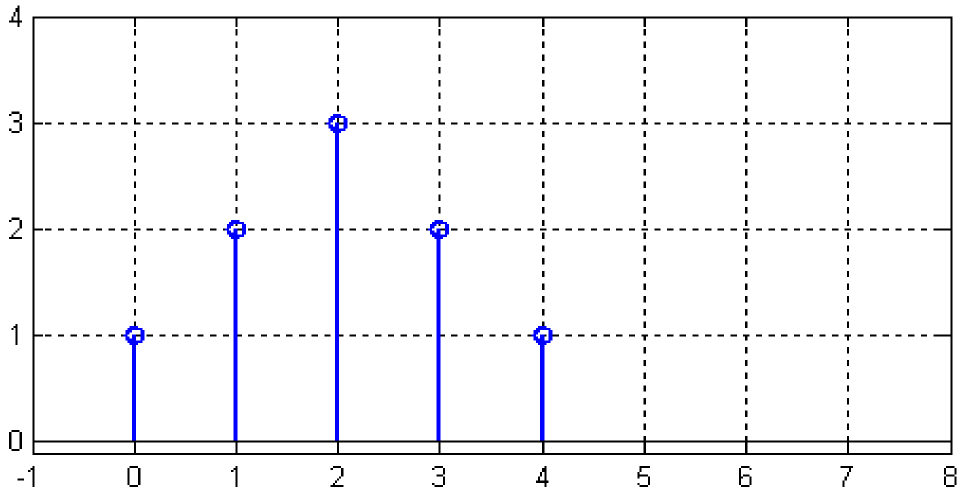
$h(-k)$ bins at the start of convolution with circular indices

On the circle, time -2 corresponds to bin 3, and time -1 corresponds to bin 4.

We have unfurled the circularized time index k in order to see $h(-k)$ as a function of positive time. In effect, we have rendered $h(-k)$ causal by repositioning each of its sample values to be placed in positive bins.

Example: Use of the CP makes linear convolution appear circular

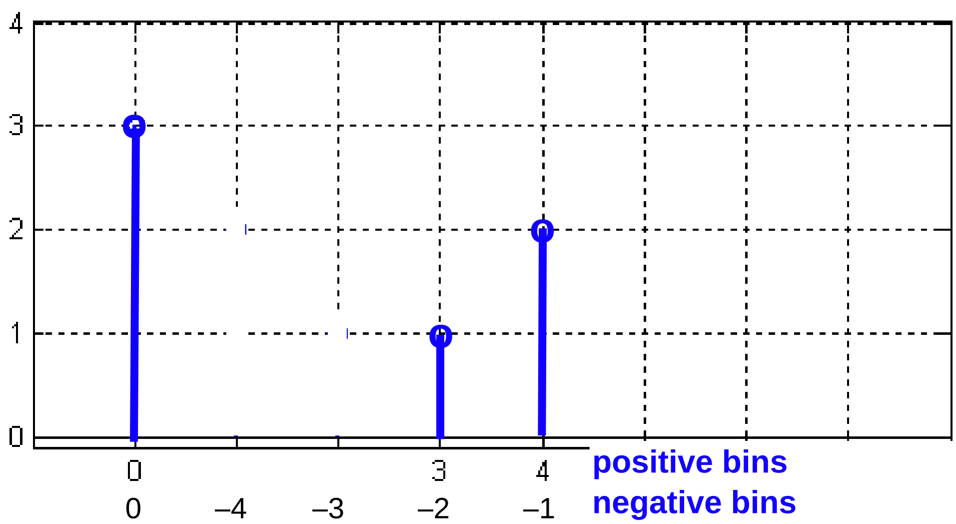
Sampled data $x(k)$ = [1, 2, 3, 2, 1]



$x(k)$ transmitted over channel $h(k)$ = [3, 2, 1]

$h(-k)$

Reversed Channel impulse response with circular indices



Circular Convolution

1	2	3	2	1	sampled data sequence $x(k)$
3	0	0	1	2	circularized reversed impulse response $h(k)$

Sampled data — 1 2 3 2 1

Channel impulse response with circular indices — 3 0 0 1 2 = 7

1 2 3 2 1
2 3 0 0 1 = 9

1 2 3 2 1
1 2 3 0 0 = 14

1 2 3 2 1
0 1 2 3 0 = 14

1 2 3 2 1
0 0 1 2 3 = 10

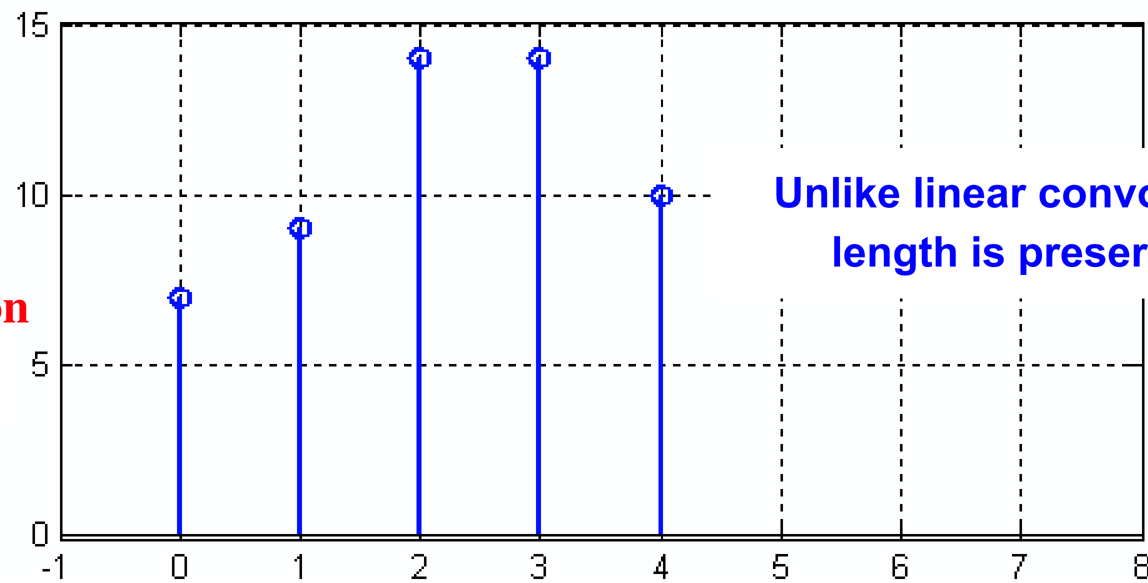
Circular Convolution Output

The next shift returns the process to step 1.

Circular Convolution yields a very different looking output

7 9 14 14 10 **This is what we want.**

Circular Convolution Output



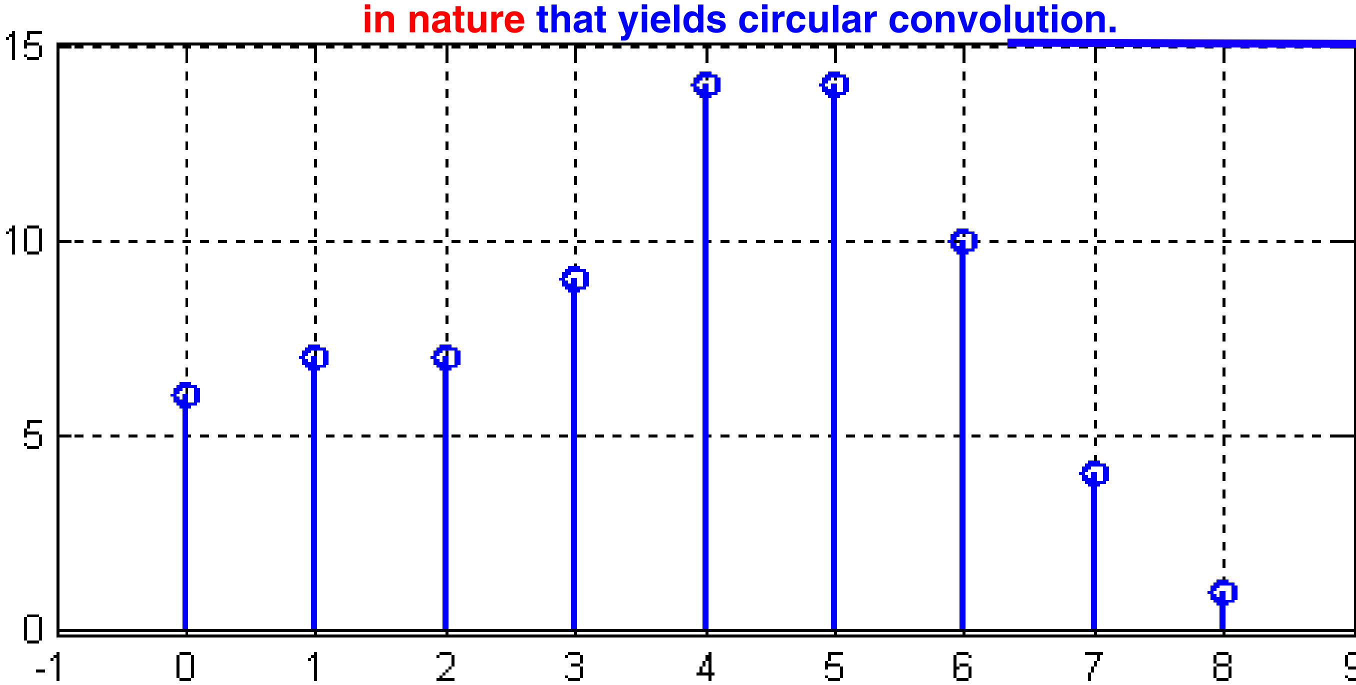
Unlike linear convolution, length is preserved.

Even though linear convolution and circular convolution look very different, watch how we can trick the channel into performing circular convolution.

How can we make the channel's linear convolution look like circular convolution? We trick it with a CP (longer than the channel delay spread) making the signal look periodic.

Add a CP to the original sampled sequence so that $x'(k)$ = [2, 1, 1, 2, 3, 2, 1], and perform linear convolution with the channel impulse response. The resulting sequence [6, 7, 7, 9, 14, 14, 10, 4, 1] is seen below.

We need a trick, because a signal $x(k)$ transmitted over a channel with impulse response $h(k)$ is received as the linear convolution $x(k) * h(k)$. There is NO mechanism in nature that yields circular convolution.



This signal [6, 7, 7, 9, 14, 14, 10, 4, 1] results from a *linear* convolution with channel $h(k)$. From the figure below, we see the similarity between the circularly convolved signal $x(k) \circledast h(k)$ and the linearly convolved signal $x'(k) * h(k)$.

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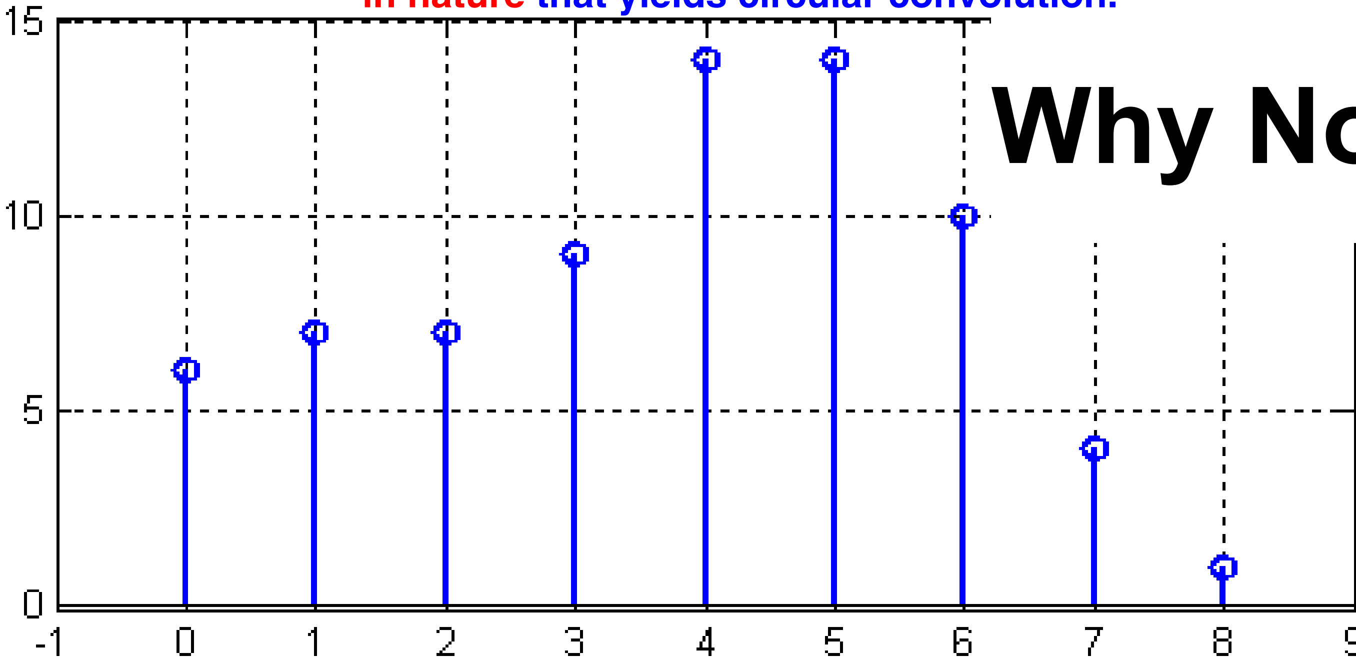

Why Not?

This signal [6, 7, 7, 9, 14, 14, 10, 4, 1] results from a *linear* convolution with channel $h(n)$. From the figure below, we see the similarity between the circularly convolved signal $x(k) \circledast h(k)$ and the linearly convolved signal $x'(k) * h(k)$.

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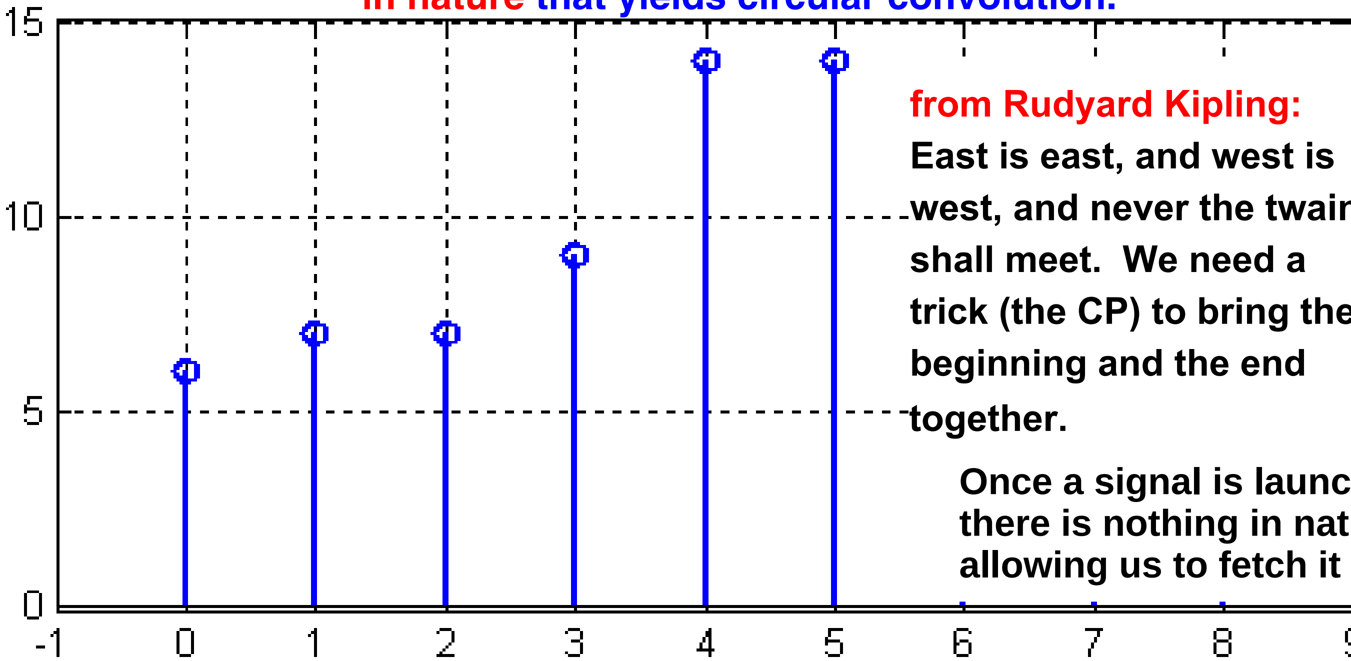
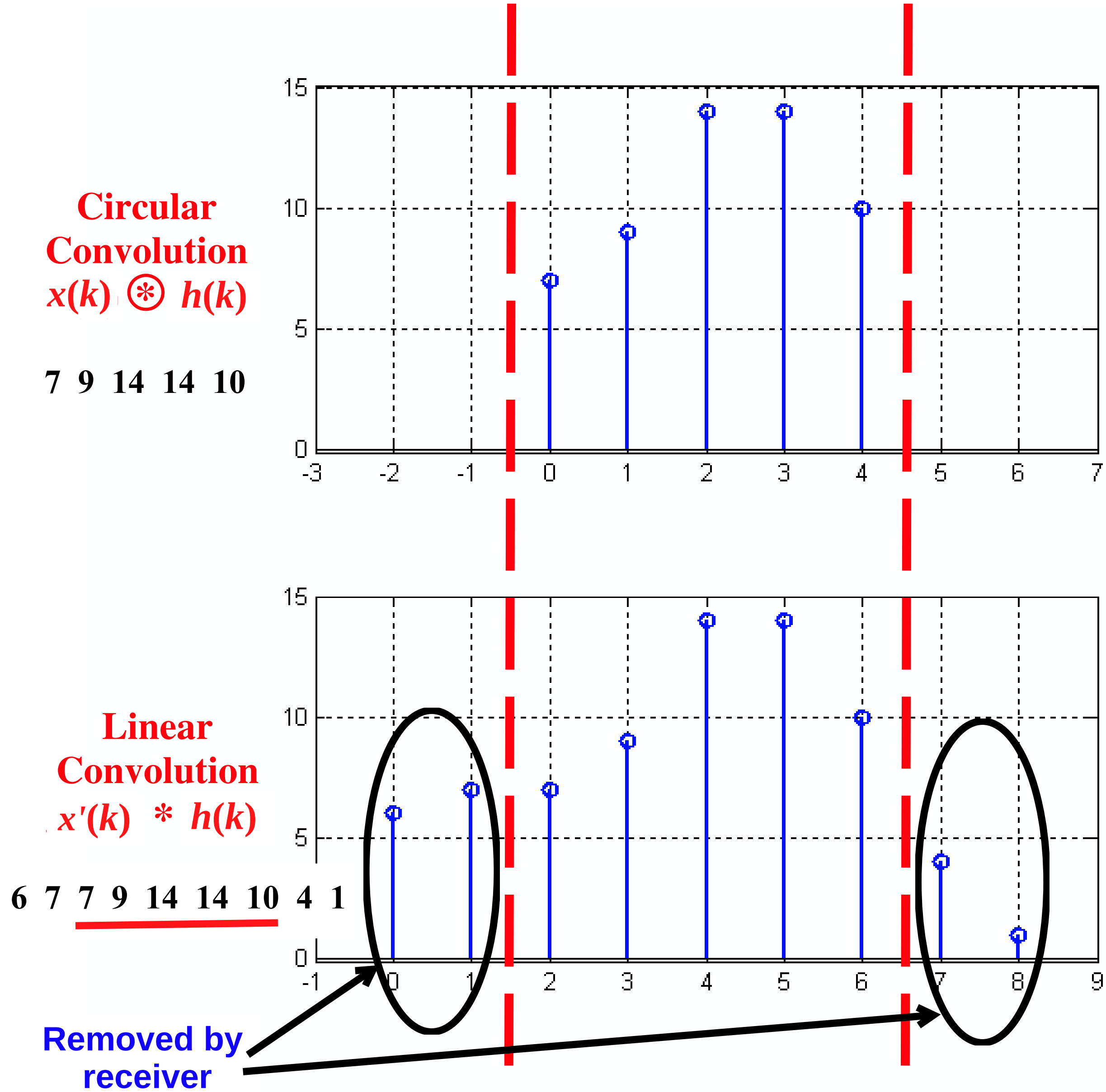


from Rudyard Kipling:
East is east, and west is west, and never the twain shall meet. We need a trick (the CP) to bring the beginning and the end together.

Once a signal is launched, there is nothing in nature allowing us to fetch it back.

This signal $\boxed{6, 7, 7, 9, 14, 14, 10, 4, 1}$ results from a *linear* convolution with channel $h(k)$. From the figure below, we see the similarity between the circularly convolved signal $x(k) \circledast h(k)$ and the linearly convolved signal $x'(k) * h(k)$.



Remember why we're doing this.

To help preserve orthogonality.

Isn't this clever?

The receiver removes the signal samples at both ends of $x'(k) * h(k)$, and feeds the result to the DFT unit. After DFT processing, we obtain the desired frequency-domain result $X(n)H(n)$. The CP has forced the linear convolution of $x'(k) * h(k)$ in the channel to yield the desired appearance of a circularly-convolved signal.

OFDM Modem Block Diagram

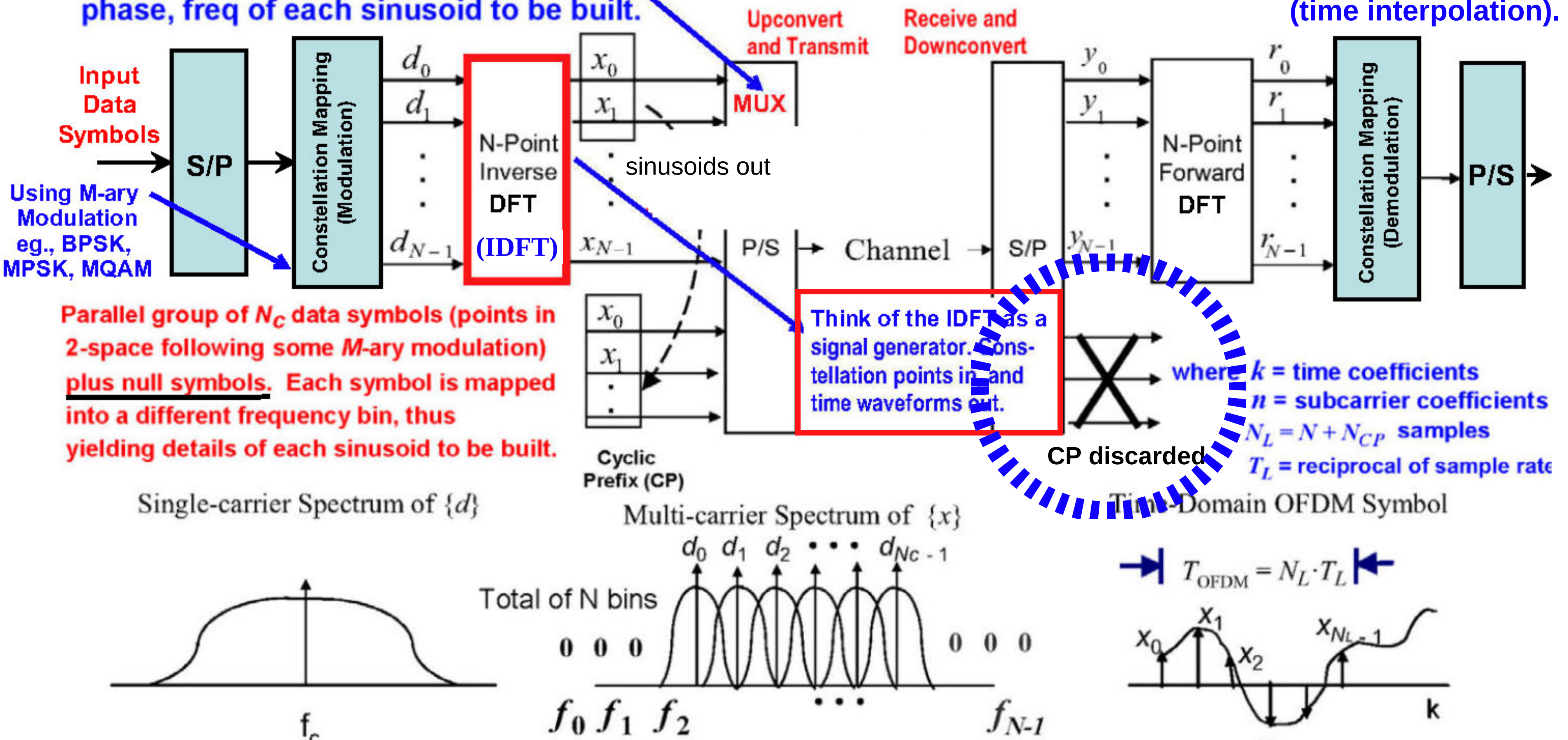
- An OFDM symbol is made up of a sum of N terms (N_C modulated orthogonal carriers plus null bins). Each k^{th} sample of a symbol can be represented as:

N is made larger than Nc by zero-padding Nc in the frequency domain, which raises the output sample rate (time interpolation).

IDFT operation starts by choosing coefficients of a sinusoidal basis set. These describe the magnitude, phase, freq of each sinusoid to be built.

$$x_k = \sum_{n=0}^{N-1} d_n e^{j\frac{2\pi}{N}nk} \qquad n,k = 0,1,2,\ldots,N-1$$

where some of the d_n values are zero



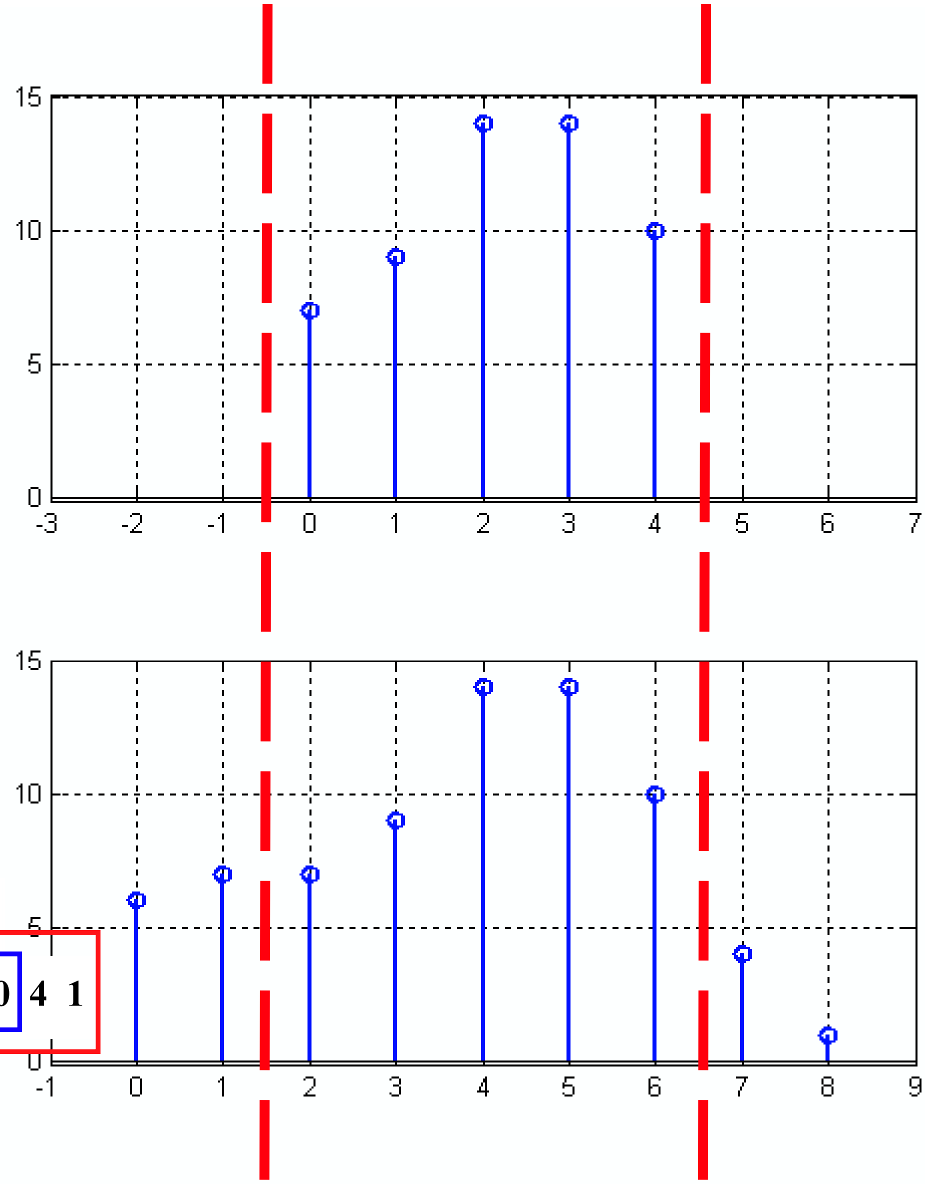
Don't confuse Nc with N. Nc represents the data (constellation points) or subcarriers, and N is the transform size. For building real analog filters, we use zero extensions (null bins) to form the transform such that $N > Nc$.

Circular Convolution
$x(k) \circledast h(k)$

7 9 14 14 10

Linear Convolution
$x'(k) * h(k)$

6 7 $\boxed{7\ 9\ 14\ 14\ 10}$ 4 1



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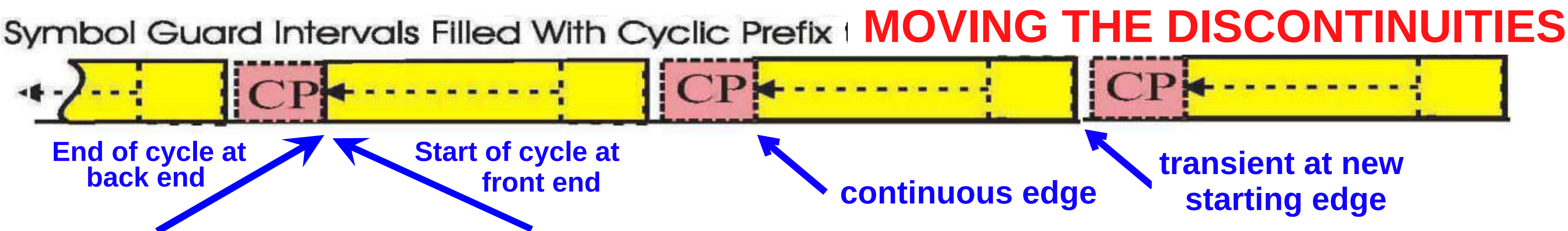
That's why we moved the discontinuities to the CP in the first place.

Removing the CP at the receiver, means that we've removed the discontinuities.



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- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

OFDM Applications

Standard 802.11a
Wireless Local Area Networks (WLAN)

OFDM Glossary (and Channel Parameters)

N_c
number of subcarriers

$N > N_c$
transform size (data-symbol samples)

$N_c = 0.6\,N$
typical subcarrier apportionment

N_{cp}
cyclic prefix samples

$N_L = N + N_{cp}$
total samples per OFDM symbol

T_L
sample time

$T_s = (N \times T_L)$
symbol time (data portion)

T_{cp}
cyclic prefix time

$T_{cp} = 0.25\,T_s$
typical CP apportionment

$T_{\mathrm{OFDM}} = (T_s + T_{cp}) = (N_L \times T_L)$
OFDM symbol time

$\Delta f = 1/T_s$
frequency difference between adjacent subcarriers

$f_s = (N \times \Delta f) = 1/T_L$
sample rate

$W_{\mathrm{signal}} = (N_c + 1)\,\Delta f$
OFDM modulation BW

Channel parameters:

T_m
max multipath delay

σ_τ
rms multipath delay

$f_0 \approx 1/T_m$
coherence BW

$f_0(50\%) \approx 1/5\sigma_\tau$
coherence BW over which the spectral correlation is at least 0.5

f_d
fading rate (Doppler spectral spreading)

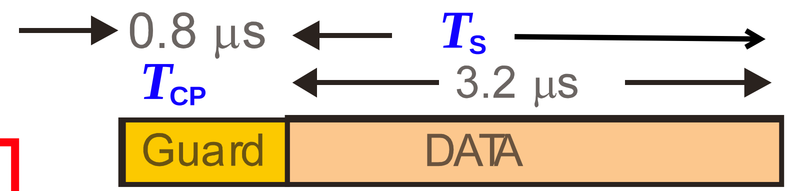
$T_0 \approx 1/f_d$
coherence time

An OFDM Application

OFDM 802.11a

Uplink and Downlink are Symmetrical

OFDM SYMBOL DURATION		4 µsec T_{OFDM}
Guard Interval		800 nsec
Orthogonal Tone Duration		3.2 µsec
Δf Tone Center Spacing	between FFT bins	312.5 kHz {1/3.2 µsec)
Number of Data Tones		48
Number of Pilot Tones		4
Total Number of Tones		$N_C = 52$
Total Bandwidth	$(N_C + 1)\,\Delta f$	16.56 MHz {(52+1)*312.5 kHz)}
Modulation		BPSK, QPSK, 16-QAM, 64-QAM
Coding Rate		1/2, 2/3, 3/4
Data Rates		6, 9, 12, 18, 24, 36, 48, 54 Mbps
Channel Spacing		20 MHz (func of reqd ACI)



4.0 µs

OFDM Symbol

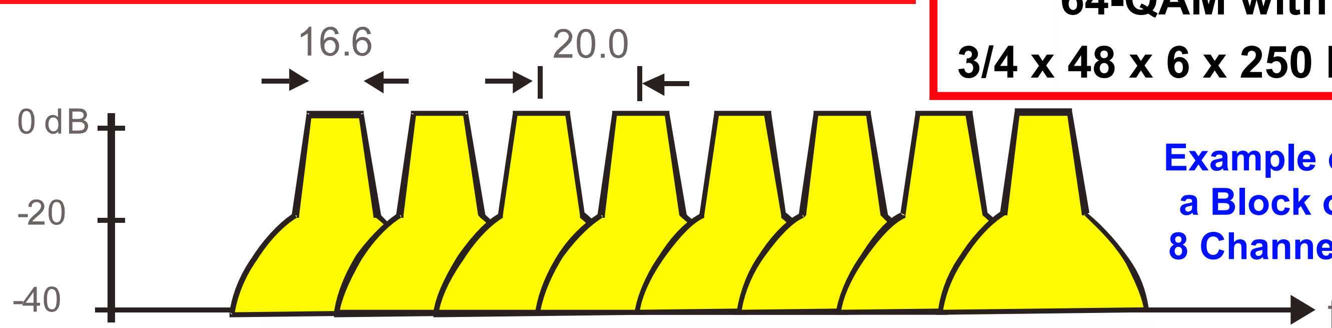
(250 OFDM ksymb/s)

BPSK with rate 1/2 code

1/2 x 48 x 250 ksymb/s = 6 Mbps

64-QAM with rate 3/4 code

3/4 x 48 x 6 x 250 ksymb/s = 54 Mbps



Example of: a Block of 8 Channels

Each channel shown with two-sided BW around its center frequency

N_C = 52 Subcarriers (48 data + 4 pilot) in each 20 MHz Channel

Typical *N*-point FFT used is *N* = 64 or *N* = 128

Efficient spectral spacing to reduce ACI. Thus, the spectral tails (guard bands) are about 4 MHz.

OFDM Parameters for 802.11 (Local Area Network)

Typical Example (various ways to express parameters)

N_c = no. subcarriers = 52   Modulation BW $W_{\text{signal}} \approx N_c \times \Delta f \approx 16\ \text{MHz}$

N = transform size $(N > N_c)$ = 64 (or 128) data samples
popular choices

N_{cp} = cyclic prefix samples = 16 (based on N = 64) $T_{\text{cp}} \gg \sigma_\tau$

$N_L = N + N_{\text{cp}}$ total samples per OFDM symbol = 80

T_L = sample time = $1/f_s$ = 1/(20 MHz) = 0.05 μs

$$f_s = \frac{1}{T_L} = \frac{N_L}{T_{\text{OFDM}}} = \frac{N_{\text{cp}}}{T_{\text{cp}}} = \frac{N}{T_s} = N \times \Delta f = 20\ \text{MHz} \quad \text{samp rate}$$

64 x 312.5 kHz

$T_s = 1/\Delta f \approx N_c / W_{\text{signal}} = N$ X T_L = 64 X 0.05 μs = 3.2 μs

$T_{\text{OFDM}} = T_s + T_{\text{cp}} = N_L$ X T_L = 80 X 0.05 μs = 4 μs

where Δf the spacing between subcarriers is (1/3.2) MHz = 312.5 kHz

OFDM Parameters for 802.11 (Local Area Network)

Typical Example (various ways to express parameters)

N_c = no. subcarriers = 52 Modulation BW $W_{\text{signal}} \approx N_c \times \Delta f \approx 16\ \text{MHz}$ two-sided BW

N = transform size $(N > N_c)$ = 64 (or 128) data samples popular choices

(where $N_{cp} = 0.25\ N$) N_{cp} = cyclic prefix samples = 16 (based on N = 64) $T_{cp} \gg \sigma_\tau$

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$$f_s = \frac{1}{T_L} = \frac{N_L}{T_{\text{OFDM}}} = \frac{N_{cp}}{T_{cp}} = \frac{N}{T_s} = N \times \Delta f = 20\ \text{MHz}$$ samp rate

(where $N_{cp} = 0.25\,N$, and $T_{cp} = 0.25\,T_s$)

Larger N dictates higher time resolution & increased sample rate.

$T_s = 1/\Delta f \approx N_c / W_{\text{signal}} = N \times T_L$ = 64 X 0.05 μs = 3.2 μs

$T_{\text{OFDM}} = T_s + T_{cp} = N_L \times T_L$ = 80 X 0.05 μs = 4 μs

where Δf the spacing between subcarriers is (1/3.2) MHz = 312.5 kHz

OFDM Transmission Bandwidth (802.11 Example)

- We define a time interval T_s over which the signaling is to be orthogonal. For 802.11, T_s = 3.2 μs

- Choose a quantity of subcarriers N_c (dependent on multipath channel). For 802.11, N_c = 52 (48 data plus 4 pilot)

- The reciprocal of T_s ($1/T_s = \Delta f$) gives the spacing between FFT bins (fequency-domain samples) = 312.5 kHz

- W_{signal} = OFDM Data BW = $(N_c + 1) \times \Delta f$ = 53 x 312.5 kHz = 16.56 MHz

- How does the transform size N enter the picture? In 802.11, typically N = 64, 128

- Increasing the transform size (FFT bins) extends the transform in the frequency domain. There will be more unoccupied bins, making the filtering less costly.

OFDM Transmission Bandwidth (802.11 Example)

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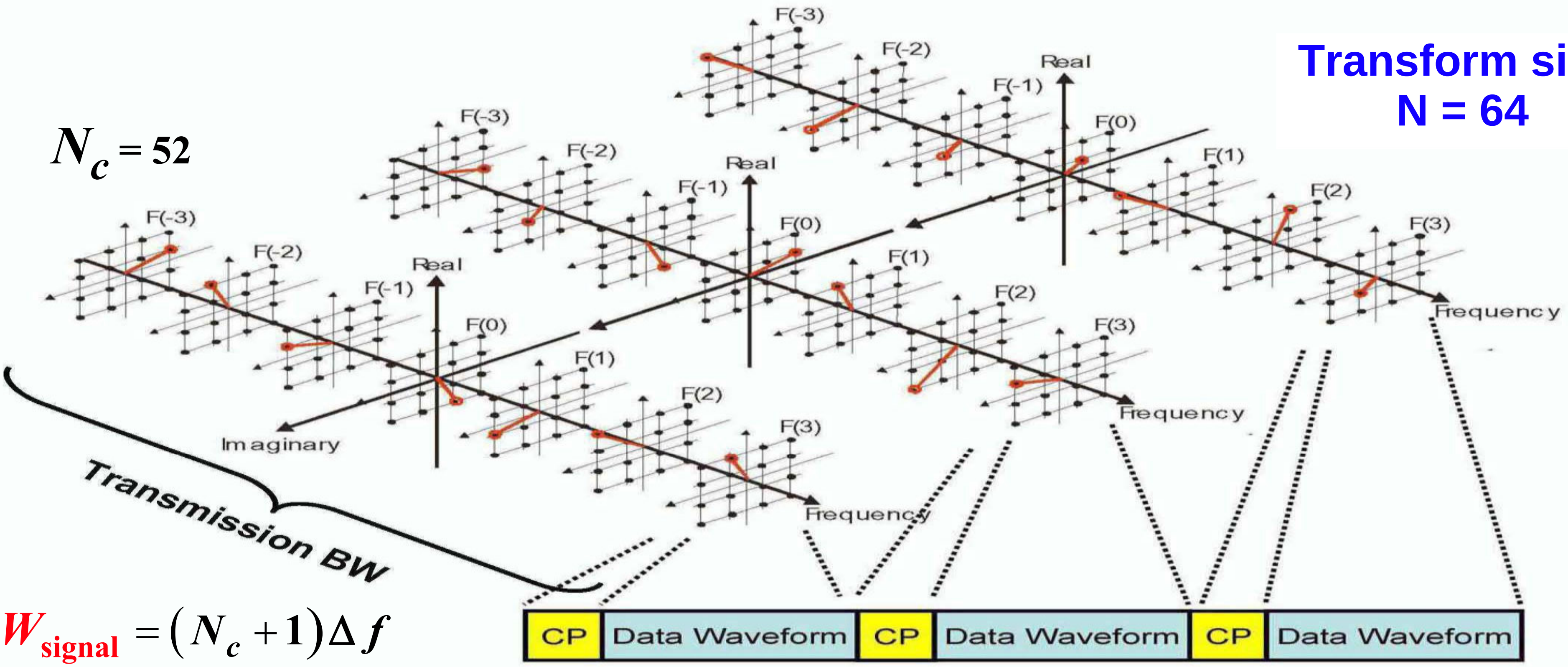
- **How does the transform size N enter the picture? In 802.11, typically N = 64, 128**

- **Increasing the transform size (FFT bins) extends the transform in the frequency domain. There will be more unoccupied bins, making the filtering less costly.** Time-resolution improves. Sampling rate increases (requiring interpolation). Spacing between spectral copies increases (eases analog filtering).

OFDM Parameters (802.11 Typical Example)

$N_c = 52$

Transform size N = 64



$W_{\text{signal}} = (N_c + 1)\Delta f$

$\Delta f = 1/T_s \quad = 1/3.2\ \mu\text{s}$

$= 312.5$ kHz

$W_{\text{signal}} = 53 \times 312.5$ kHz

$= 16.56$ MHz

Samples:

N_{cp}	N
16	64

$N_L = N_{cp} + N = 80$

Time duration:

T_{cp}	T_s
0.8 μs	3.2 μs

$T_{\text{OFDM}} = T_{cp} + T_s = 4\ \mu\text{s}$

Samp rate and time

$$f_s = \frac{N}{T_s} = \frac{64}{3.2\ \mu\text{s}} = 20\ \text{MHz} = \frac{1}{T_L}$$

$$T_{\text{OFDM}} = N_L \times T_L = 80 \times 0.05\ \mu\text{s} = 4\ \mu\text{s}$$

$$R_{\text{OFDM}} = 1/T_{\text{OFDM}} = 250\ \text{ksymb/s}$$

802.11 OFDM Exercise

Consider an 802.11 OFDM system, having the following parameter values: 64-point transform, with 48 message-occupied bins, OFDM symbol time = 4 μsec, CP time = 0.8 μsec, data modulation is 16-QAM, Error-correcting code rate = ¾. Find the following:

(a) Sampling rate
(b) Sample time
(c) Code-bits per subcarrier
(d) Code-bits per OFDM symbol
(e) Data-bits per OFDM symbol
(f) Data rate

(g) If the channel max delay spread = 20 samples, determine if the given CP time of 0.8 μsec will be long enough to mitigate the channel ISI.

Solution to 802.11 OFDM Exercise

(a) $\textbf{Tone spacing}: \ \Delta f = \frac{1}{T_S} = \frac{1}{T_{\text{OFDM}} - T_{\text{CP}}} = \frac{1}{3.2\,\mu\text{sec}} = 312.5\ \text{kHz}$

$\underline{\textbf{Sample rate}}: \ f_s = N \times \Delta f = 64 \times \Delta f = 64 \times 312.5\ \text{kHz} = 20\ \text{MHz}$

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(b) $$\underline{T_{\text{samp}}} = \frac{1}{f_s} = 50\ \text{nano-sec}$$

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(c) 16-QAM yields 4 <u>code-bits per subcarrier</u>.

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48 x 4 = 192 code-bits per OFDM symbol

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(e) Rate ¾ code: ¾ x 192 = 144 bits per OFDM symbol

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48 x 4 = 192 code-bits per OFDM symbol

(e) Rate ¾ code: ¾ x 192 = 144 bits per OFDM symbol

(f) Data rate: $\dfrac{144\text{ bits/symbol}}{4\,\mu\text{sec/symbol}} = 36 \ \text{Mbps}$

Solution to 802.11 OFDM Exercise

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(e) Rate ¾ code: ¾ x 192 = 144 bits per OFDM symbol

(f) Data rate: $\dfrac{144\text{ bits/symbol}}{4\,\mu\text{sec/symbol}} = 36\ \text{Mbps}$

(g) Max delay spread $20\text{ samples} \times T_{\text{samp}} = 20 \times 50\text{ nano-sec} = 1\,\mu\text{sec}$

$T_{\mathrm{CP}} = 0.8\,\mu\text{sec} < 1\,\mu\text{sec}$. **Therefore, we need a longer cyclic prefix.**

Sending an 84-bit data sequence as 21 16-QAM symbols sent as 3 OFDM symbols

84-bit data sequence:

10010100000111100011000001001011011111101001010000100001
0000001101111101000011000111

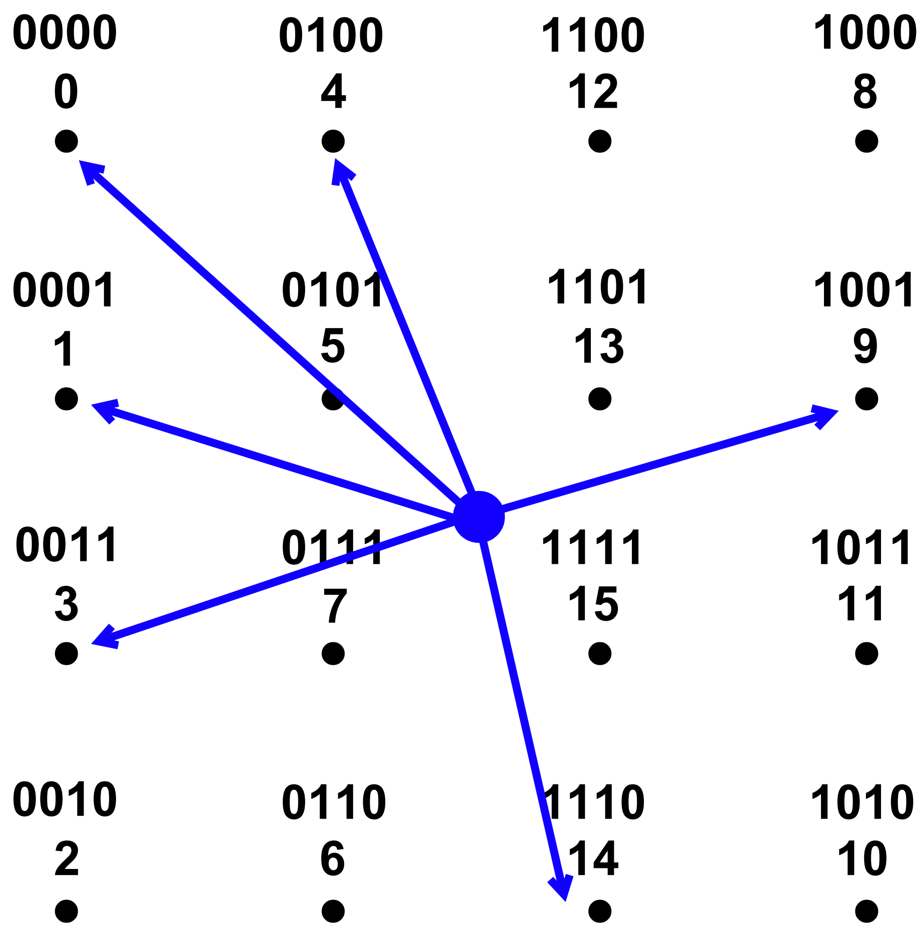
1001	0100	0001	1110	0011	0000	0100	OFDM
9	4	1	14	3	0	4	data symbol 1
1011	0111	1110	1001	0100	0010	0001	OFDM
11	7	14	9	4	2	1	data symbol 2
0000	0011	0111	1101	0000	1100	0111	OFDM
0	3	7	13	0	12	7	data symbol 3

Gray-Coded 16-QAM

0000 0 ●	0100 4 ●	1100 12 ●	1000 8 ●
0001 1 ●	0101 5 ●	1101 13 ●	1001 9 ●
0011 3 ●	0111 7 ●	1111 15 ●	1011 11 ●
0010 2 ●	0110 6 ●	1110 14 ●	1010 10 ●

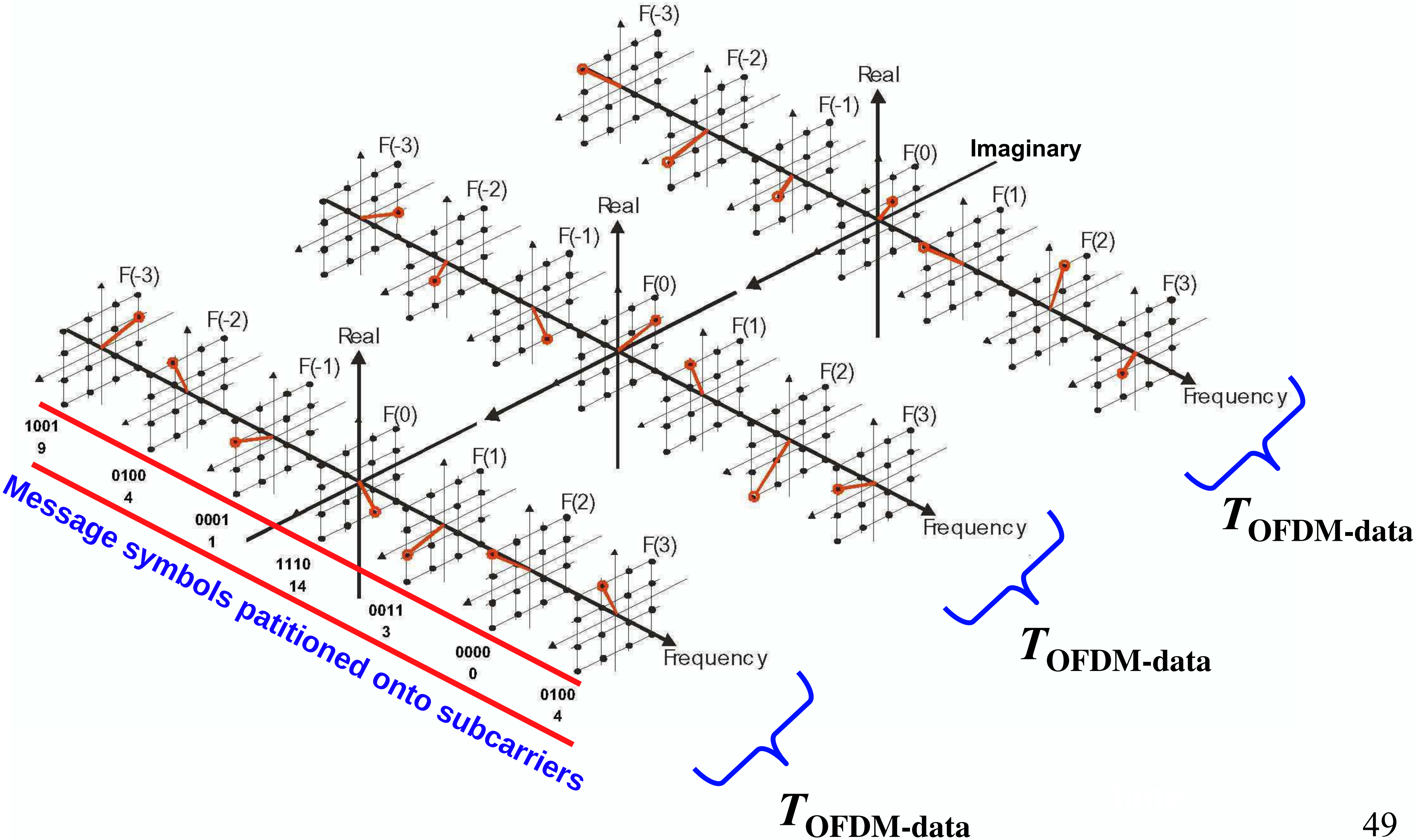
Shows the locations of the 4-bit data sequences in the 2-D constellation

Gray-Coded 16-QAM



message symbols: 9 4 1 14 3 0 4

Sending an 84-bit data sequence as 21 16-QAM symbols sent as 3 OFDM symbols



OFDM is Particularly Useful for High Data Rates, Such as Pictures or Video

- **"A picture is worth 1000 words." Is this age-old expression correct? Yes, in the behavioral sense. But what about the task (i.e., bandwidth requirements) of sending a picture versus 1000 words of text?**

- **A high-quality 8x10 photo is made up of about 3 Megapixels. A good quality color photo requires 8 bits per primary color per pixel, or 24 bits per pixel. Thus such an 8x10 photo can be represented as a sequence of 72 Megabits.**

- **English text on average has about 4.5 letters per word. Using ASCII with parity, each letter is made up of 8 bits. Thus on average, one English word, on average, can be represented as a sequence of 36 bits.**

- **How many such 36-bit words comprise one good quality 8x10 color picture?**

$$\frac{72 \times 10^{6} \text{ bits per picture}}{36 \text{ bits per word}} = 2 \text{ million words per } 8 \text{ x } 10 \text{ picture}$$

- **Therefore, sending one such picture via your cellphone is the equivalent of sending about six 500 page textbooks.**

- **1000 words = 36,000 bits. What size high-quality picture is worth a thousand words? Smaller than ¼ inch x ¼ inch.**

- **That's why source coding of images & BW efficiency is important.**

59. In the early "battle" for the best codes (convolutioal vs. Reed-Solomon), what are the arguments for each, and why did convolutional win out? (Sklar DIG notes, section 8)

60. In mobile channels, how does the terrain affect fading? How does the mobile-velocity affect it? (Sklar ADC notes, section 2)

61. What is the advantage of circular-convolution versus linear-convolution? How do we trick the channel into performing circular convolution? (Sklar ADC notes, section 3)

62. In OFDM, what is the mitigation technique for precluding ISI? For precluding ICI? (Sklar ADC notes, section 3)

63. Baseband OFDM symbols are typically made up of independent data at positive and negative spectral locations. How is this effected, and how is a real transmission-signal formed? (Sklar ADC notes, section 3)

64. For maintaining orthogonality among the subcarriers in OFDM, the tone spacing was chosen to be $1/T_S$. Why wasn't it chosen to be $1/T_{\mathrm{OFDM}}$?

65. How can SC-OFDM still be resistant to multipath when the data symbols are so short? (Sklar ADC notes, sec. 3) Hint: The time duration of a data pulse is longer than its main lobe.

66. Early skeptics about MIMO, claimed that it violated Shannon's capacity theorem. Why is that not the case? (Sklar ADC notes, section 4)

67. Why won't MIMO work in a multipath-free environment? (Sklar ADC notes, section 4)

68. Often, the signal-processing operations "DFT and IDFT" are called out as "FFT and IFFT," when one means the mathematical transformation. Why is this NOT precise?

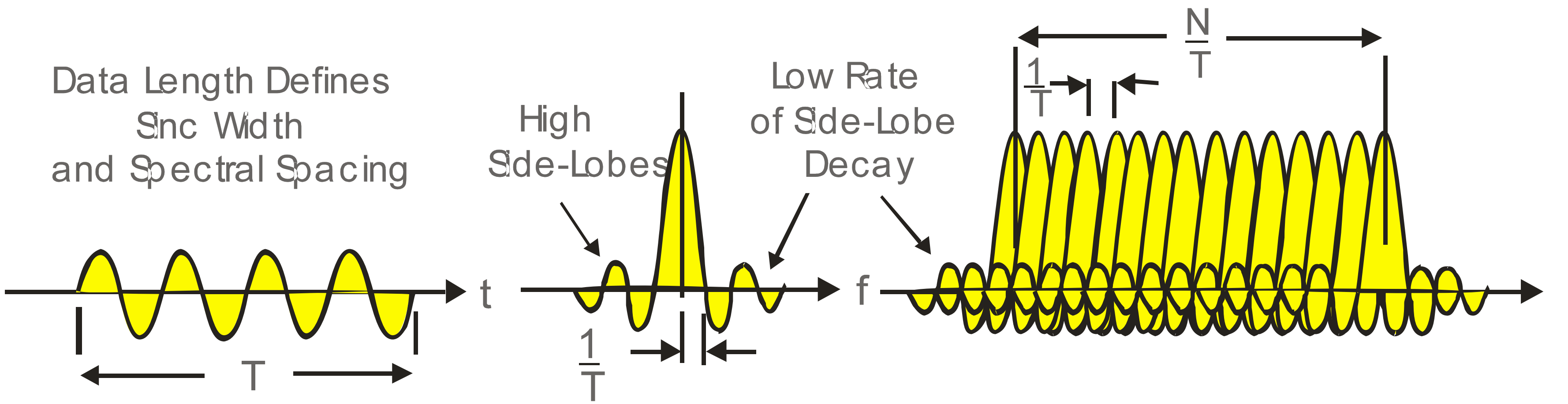
69. What are the Key Control Loops needed for system Synchronization? (fred harris, "Let's Assume the System is Synchronized," Sklar ADC notes, section 8)

70. How do you shape a time waveform to meet system spectral-confinement requirements? (Sklar DIG notes, sec 9 & ADC notes, sec 1) Hint: symbol rate, sample rate, filter length, transition BW, out-of-band attenuation.

Data Length Defines Sinc Width: Spectral Spacing Matches Width

Fourier Transform of an infinitely long sinusoid is an impulse function.

Fourier Transform of a gated-sinusoid is a sinc function.

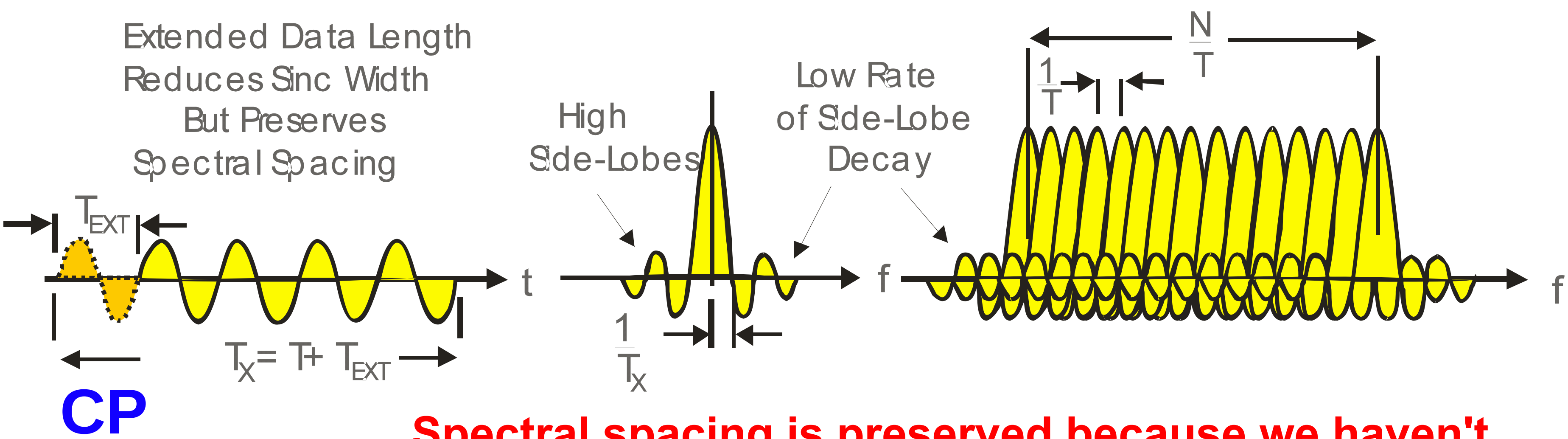


With the proper pulse spacing of $1/T$, the sequence is orthogonal, characterized by the peak of each pulse experiencing zero-value interference from neighboring pulses. Hence there is NO ISI.

Ref: fred harris, lecture on OFDM, San Diego State University

Extended Data Length Reduces Sinc Width: Spectral Spacing Preserved

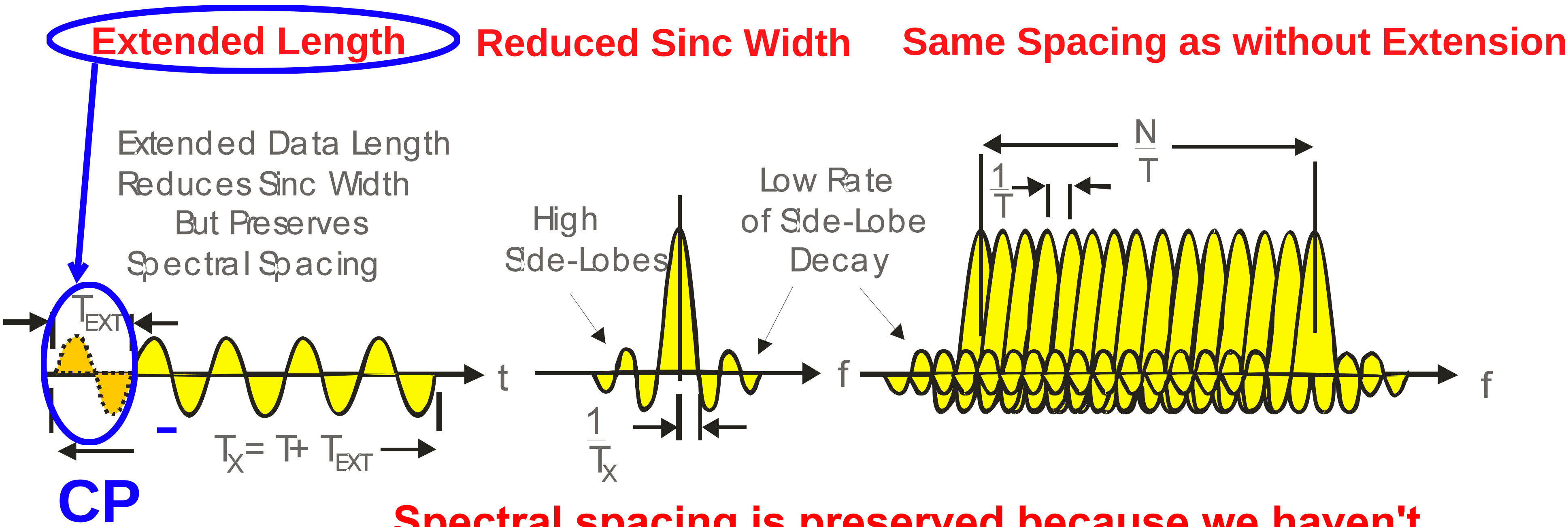
Extended Length   Reduced Sinc Width   Same Spacing as without Extension



Spectral spacing is preserved because we haven't changed the cycles per interval (frequency). We've just extended the length (as in the case of a cyclic prefix).

One might ask, with the extended length (reduced sinc width) why not reduce the sub-carrier spacing, thereby forming a family of extended orthogonal waveforms? Answer: Orthogonality in space is not what we're striving for. We want orthogonal waveforms at the receiver, after the extension is discarded.

Extended Data Length Reduces Sinc Width: Spectral Spacing Preserved



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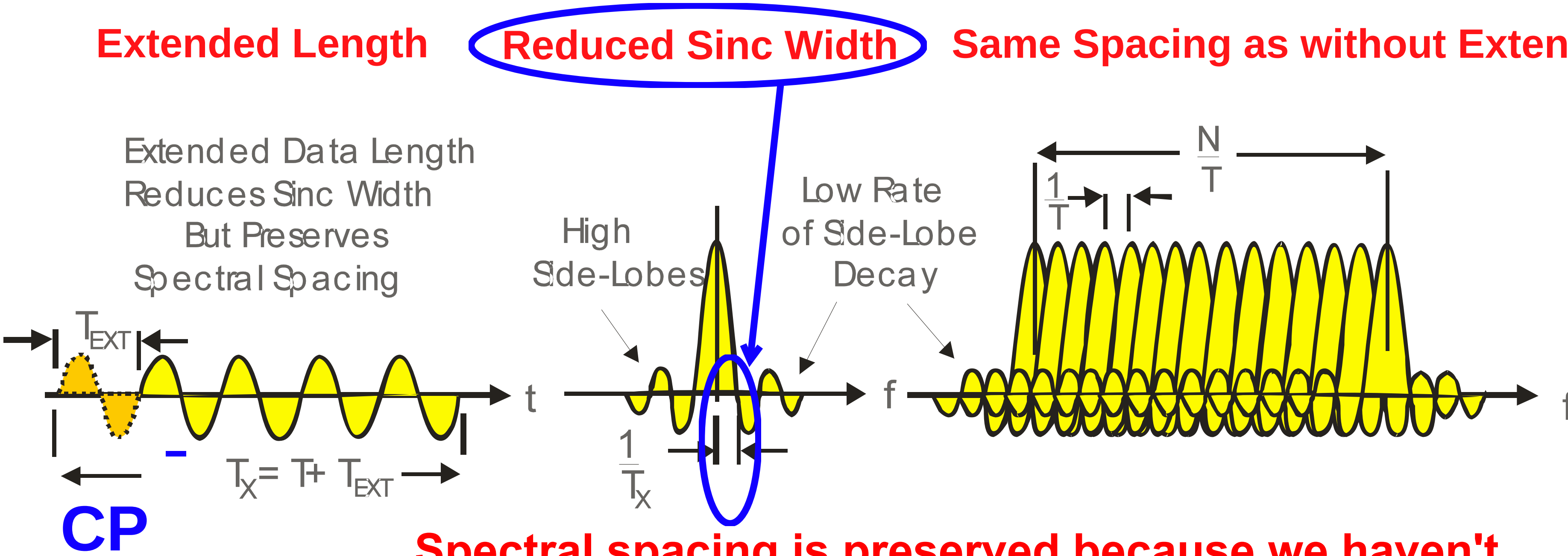

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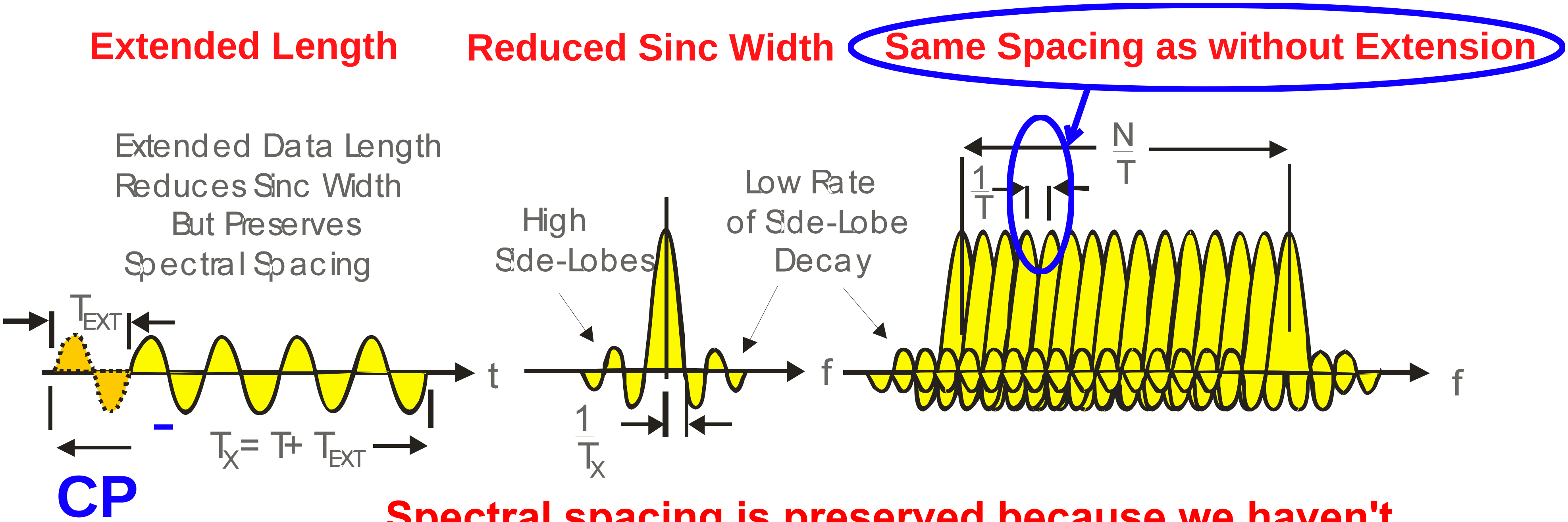

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OFDM Applications

Long Term Evolution (LTE)

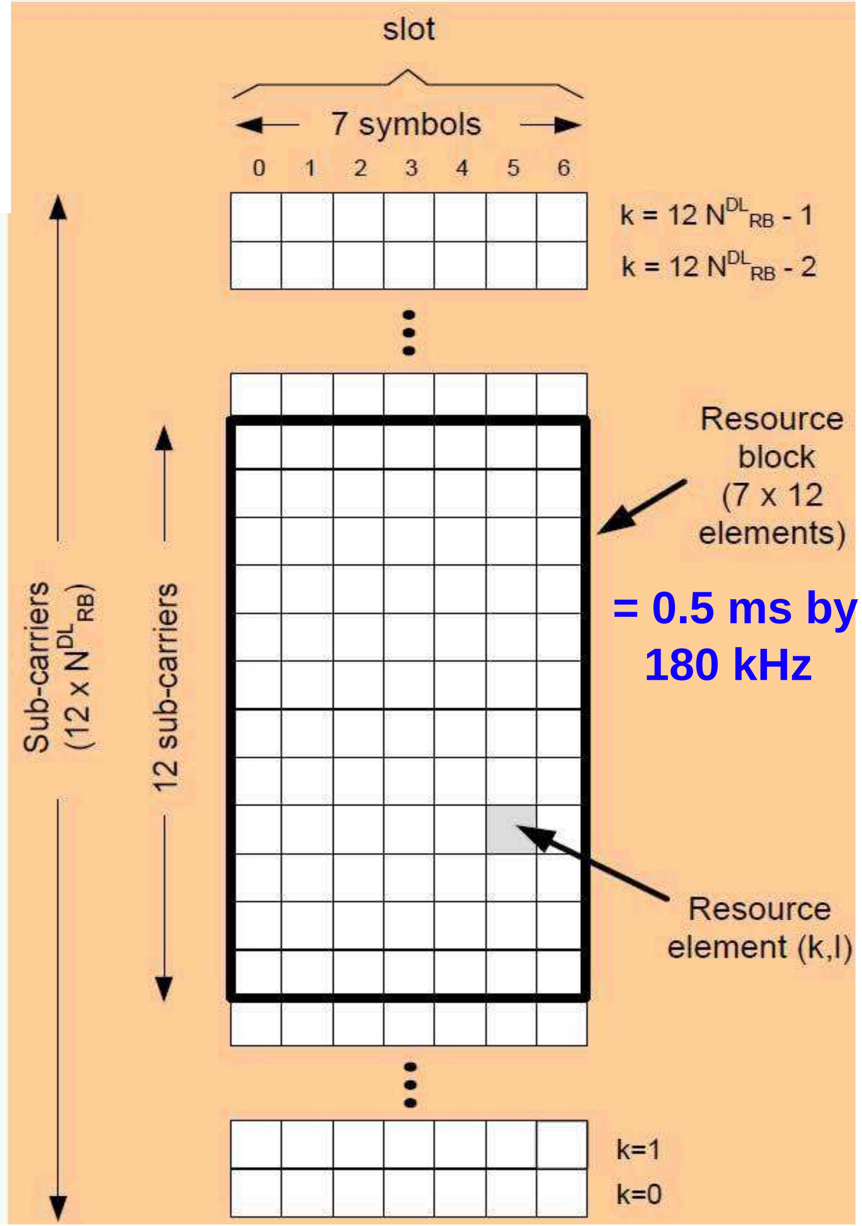
An OFDM Long Term Evolution (LTE) Application

LTE (Wide Area Network) Resources: Grid, Block, Element

Note: LTE standards involve Multiple Access

- Resource Grid
  - Several resource blocks (RB's).
  - Number of RB's adjusted to cover available BW.
- Resource Block (RB)
  - 7 by 12 elements.
  - Transmissions are allocated in discrete RB's.
- Resource element
  - one symbol width in time
  - one 15 kHz sub-carrier in frequency.

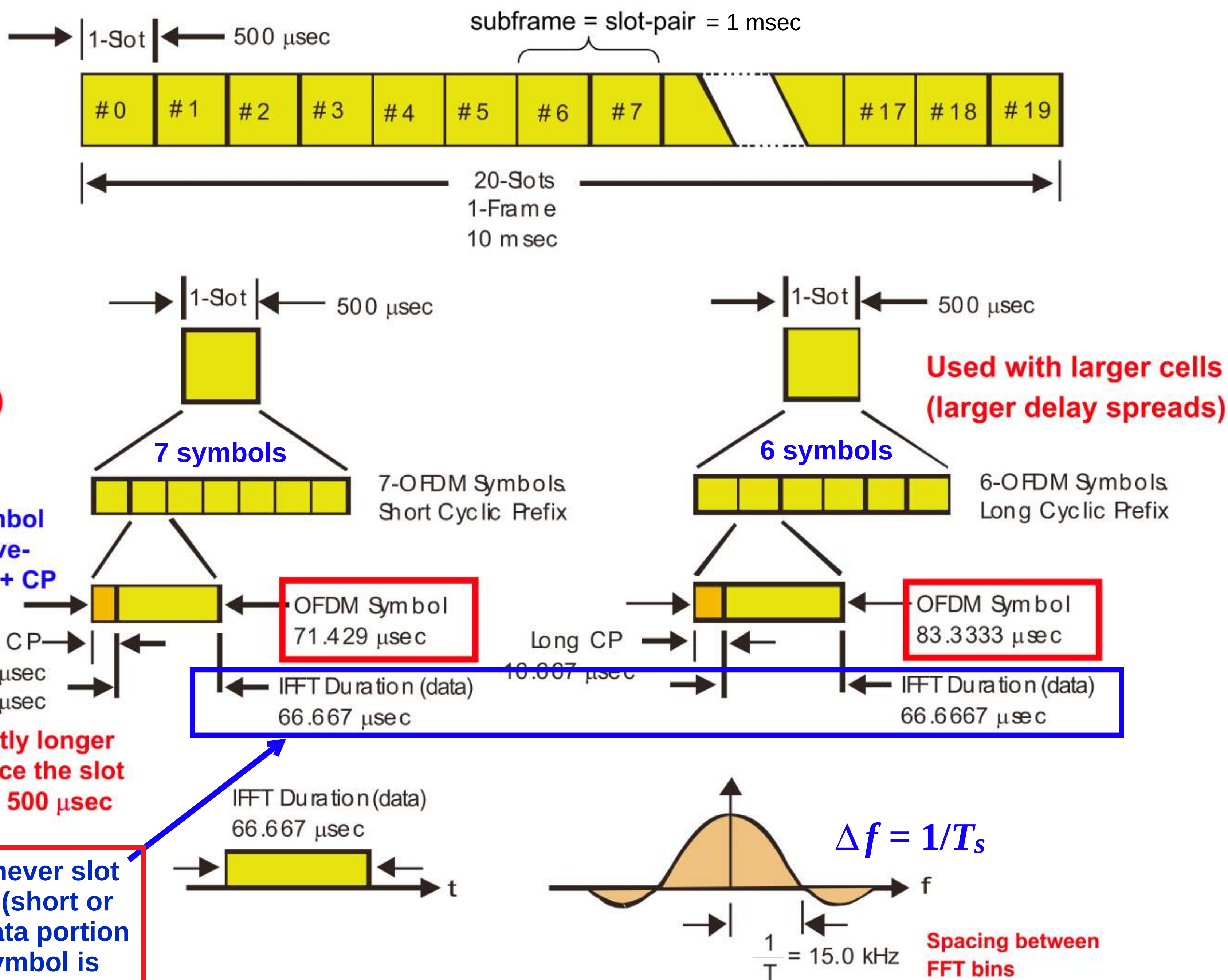
$\Delta f = 15$ kHz



The usual time-frequency grid needed for multiple-access assignments

Frames, slots, OFDM symbols, & data symbols

OFDM in LTE



A frame is made up of 20 slots. A slot is made up of 7 (or 6) symbols

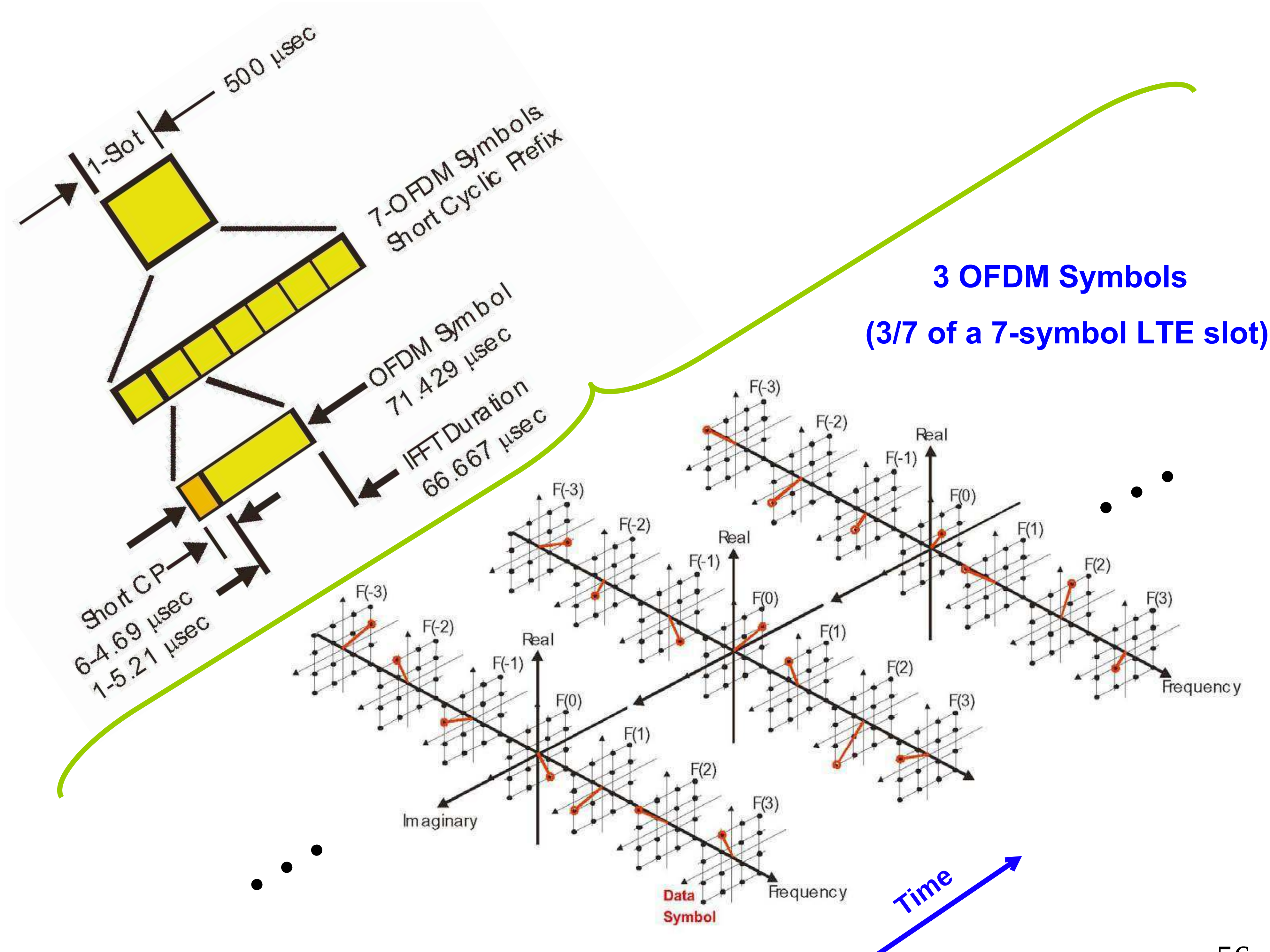
An OFDM symbol is an IFFT wave-shape output + CP

One slightly longer CP to force the slot size to be 500 μsec

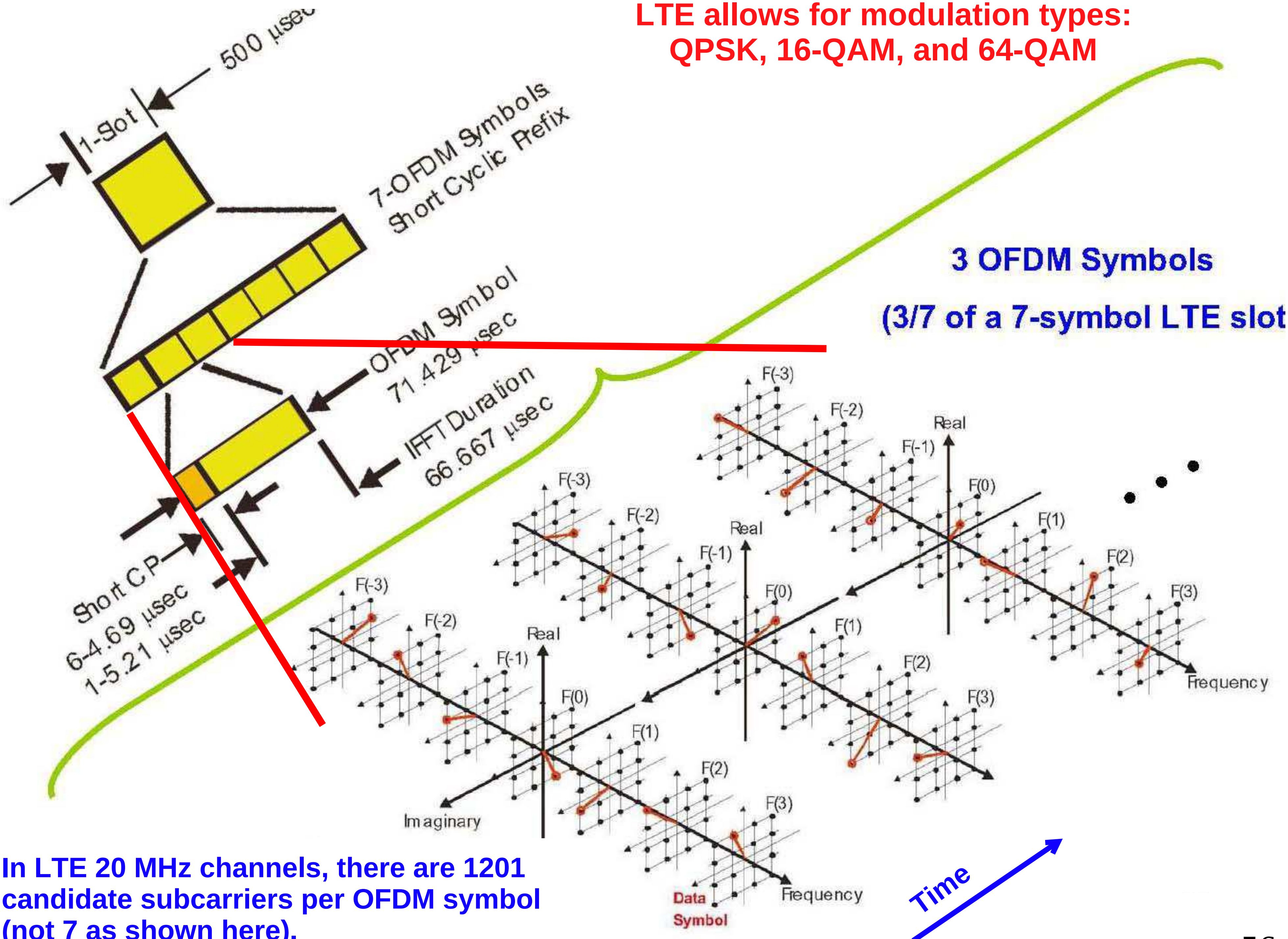
Note that whichever slot type is chosen (short or long CP) the data portion of the OFDM symbol is always the same size.

3 OFDM Symbols

(3/7 of a 7-symbol LTE slot)



LTE allows for modulation types: QPSK, 16-QAM, and 64-QAM



In LTE 20 MHz channels, there are 1201 candidate subcarriers per OFDM symbol (not 7 as shown here).

All BW values are 2-sided (MHz)

OFDM in LTE (six channel bandwidth options - Release 8)

Transition BW reqmt of filters dictate that the candidate subcarriers N_c be approx 60% of the FFT size N.

Required ACI dictates size of guard bands

Transmission BW	1.4	3.0	5.00	10.00	15	20
Modulation BW (MHz)	1.08	2.7	4.5	9.0	13.5	18.0
Slot Duration	500 μsec					
Sub Carrier Spacing Δf	FIXED 15 kH					
Sampling Frequency f_s	1.92 MHz	3.84 MHz	7.68 MHz	15.36 MHz	23.04 MHz	30.72 MHz
FFT Size N	128	256	512	1024	1536	2048
Number of Candidate Sub-Carriers (approx 60% of FFT size) N_c	73	181	301	601	901	1201
Number of OFDM Symbols Per Slot	7-Short Cyclic Prefix 6-Long Cyclic Prefix					
Short CP Length (in clock samples)	1-5.21 μsec 10 Samples 6-4.69 μsec 9 Samples	1-5.21 μsec 20 Samples 6-4.69 μsec 18 Samples	1-5.21 μsec 40 Samples 6-4.69 μsec 36 Samples	1-5.21 μsec 80 Samples 6-4.69 μsec 72 Samples	1-5.21 μsec 120 Samples 6-4.69 μsec 108 Samples	1-5.21 μsec 160 Samples 6-4.69 μsec 144 Samples
Long CP Length	16.67 μsec 32 Samples	16.67 μsec 64 Samples	16.67 μsec 128 Samples	16.67 μsec 256 Samples	16.67 μsec 384 Samples	16.67 μsec 512 Samples

channel spacing

signal BW

Sample Rate f_s = 15 kHz x Transform Size N

Modulation BW approx = 15 kHz x No. of occupied subcarriers

Includes DC subcarrier

In 802.11a, channel spacing is 20 MHz (func of reqd ACI). Also N = 64 or 128. Note the greater flexibility in LTE with 6 channel options.

LTE Channel Bandwidth Configurations

Slot time = 0.5 ms. Sub-frame time = 1 ms. Frame time = 10 ms

Transition BW Filter requirements dictate that $N_c \approx 0.6\,N$. BW of RB = 180 kHz

Guard-band BW equals 10% of Channel BW except for 1.4 MHz case (22.85%)

Subcarrier spacing $\boxed{\Delta f = 15\text{ kHz.}}$ Data portion of Symbol $T_s = 1/\Delta f =$ 66.667 μsec

Transmission BW (W_{xmt} MHz) 2-sided	1.4	3	5	10	15	20
Modulation Bandwidth (W_m MHz)	1.08	2.70	4.50	9.00	13.50	18.00
Guard Band for each side (kHz)	160	150	250	500	750	1000
Number of RBs $\#RBs = W_m / 180\text{ kHz}$	6	15	25	50	75	100
N Transform size	128	256	512	1024	1536	2048
N_c Occupied subcarriers	72	180	300	600	900	1200
Sample rate (MHz) DFT BW $f_s = N \times \Delta f$	1.92	3.84	7.68	15.36	23.04	30.72
Sample time (μsec) $T_L = 1/f_s$	0.5208	0.2604	0.1302	0.0651	0.0434	0.0326
Samples per slot	960	1920	3840	7680	11520	15360

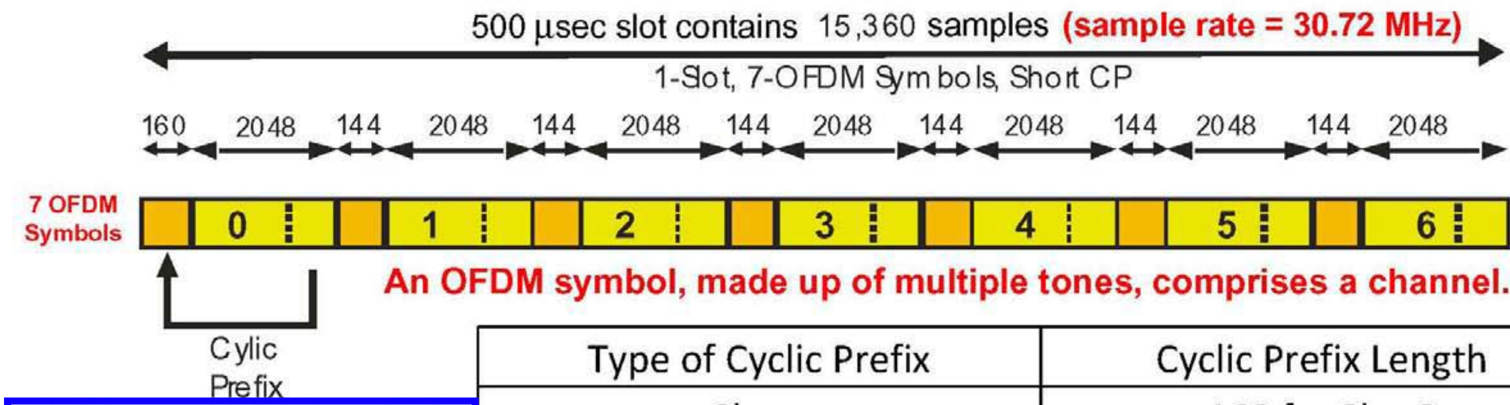
Sampling Requirements

- The 2-sided baseband spectrum of an OFDM symbol contains independent data on each of its two sides.
- Hence, OFDM baseband modulation doesn't have Hermitian symmetry properties, and is complex. When up-shifted onto a transmission carrier wave, it then becomes a real time-signal with even Hermitian symmetry.
- To meet the Nyquist sampling criterion, the required sampling rate has often been described as: exceeding twice the signal's single-sided baseband bandwidth.
- In the modern era, it is more appropriate to describe a signal's bandwidth in terms of its 2-sided spectrum. Thus we recast the Nyquist sampling criterion as: The sampling rate must exceed the 2-sided baseband bandwidth.
- The sampling rate f_S defines the distance between spectral copies. This distance must exceed the 2-sided bandwidth in order to avoid spectral-overlap of copies.
- Hence Nyquist sampling in OFDM (which will be complex) is best described as requiring a sampling rate that must exceed the 2-sided baseband bandwidth.
- We verify that it does indeed meet that rate for both 802.11 and LTE. In 802.11a, the 20 MHz channel has a 2-sided signal BW of approx 16 MHz and a sampling rate of 20 MHz. In LTE, the 20 MHz channel has a 2-sided signal BW of approx. 18 MHz and a sampling rate of 30.72 MHz.
- The chosen sampling rate is forced by the transform size to allow for the proper spacing between FFT bins to sustain orthogonality of gated complex sine waves.

$$f_s = N \times \Delta f$$

samples/slot = slot time x f_s

OFDM in LTE: Example of FFT size 2048



15,360 samples in slot comprised of seven symbols

In this example, each symbol is the sum of N_c = 1201 gated sinusoids, where each sinusoid has N = 2048 samples. Hence, the composite time waveform has N = 2048 samples/symbol.

Type of Cyclic Prefix	Cyclic Prefix Length
Short	160 for Slot 0 144 for Slots 1, 2, ... ,6
Long	512 for Slots 0, 1, ... ,6

- Duration for the IFFT part of the symbol (without CP) is a fixed 66.667 µsec. The reciprocal of this duration is 15 kHz, which is the spacing between the sin(x)/(x) channels or tones.
- The sample rate (BW spanned by the FFT) is this spacing times the number of channels. In this example sample rate equals 15 kHz X 2048 = 30.72 MHz. Thus, within one slot, the number of samples amount to sample rate times the slot time = 15,360 samples.

$$f_s = \frac{N}{T_s} = N \times \Delta f$$

- Since channel spacing is fixed, an increase in BW requires more tones, which means a larger size FFT, which in turn necessitates an increase in sample rate.
- Users need to be aligned in time, frequency, and signal strength by the base station.

Users are typically assigned several specific subcarriers (a segment of a channel) for a specific time interval.

Key OFDM Relationships

$f_s = N/T_s = N \times \Delta f$ $\Delta f = 1/T_s$ $T_{\mathrm{OFDM}} = T_s + T_{\mathrm{CP}}$

f_s and T_s are user independent parameters.
Once chosen, you've locked in N and Δf.

$N > N_c$ $W_{\mathrm{signal}} = (N_c + 1)\,\Delta f$

Describe the effects of increasing the number N of the N-point IDFT transform. Will it increase the sample rate? Will it increase the OFDM BW?

Describe the effects of decreasing Δf the frequency spacing between tones. And show the ways in which it can happen.

Key OFDM Relationships

$$f_s = N/T_s = N \times \Delta f \qquad \Delta f = 1/T_s \qquad T_{\mathrm{OFDM}} = T_s + T_{\mathrm{CP}}$$

f_s and T_s are user independent parameters.
Once chosen, you've locked in N and Δf.

$$N > N_c \qquad W_{\text{signal}} = (N_c + 1)\,\Delta f$$

Describe the effects of increasing the number N of the N-point IDFT transform. Will it increase the sample rate? Will it increase the OFDM BW?

An increase in N by itself, will increase the sample rate, but will NOT increase the BW because we are not sending samples; we are sending waveforms. The increased sample rate will result in greater time-domain resolution, and will spread the spectral periodic copies further apart. This makes the analog filtering easier and less costly.

Describe the effects of decreasing Δf the frequency spacing between tones. And show the ways in which it can happen.

The effect of decreased Δf will increase the length of the OFDM symbol T_s making it less vulnerable for a given channel ISI. Increased symbol length can come about by decreasing Δf, or increasing N (for fixed f_s), or by increasing N_c (for fixed W_{signal}).

Using the Table of LTE Channel BW Configurations Show that when using 64-QAM, the Max OFDM Uncoded Data Rate that can be supported for LTE ≈ 100 Mbps

Largest Channel BW		20 MHz
Largest Transform Size		2048
Candidate Subcarriers		1200
64-QAM		6 bits/subcarrier
Bits per OFDM Symbol		6 x 1200 = 7200 bits
Slot Time		0.5 ms
Symbol Time	0.5 ms / 7 =	71.429 μsec
Max Data Rate	7200 bits / 71.429 μsec ≈ 100 Mbps	
Overhead reduces this to ≈		86 Mbps

Another Way to Compute the Max OFDM Uncoded Data Rate using Resource Blocks

Max Channel BW	**20 MHz**
Max Occupied BW, W_m	**18 MHz**
#RBs = $W_m \div$ **180 kHz/RB =**	**100 RBs**
Each RB has 12 x 7 =	**84 REs**
100 RBs have	**8400 REs**
64-QAM yields	**6 bits per RE**
Bits per RB = 6 x 84 =	**504 bits**
Bits per 100 RBs =	**50400 bits**

Bit Rate = 50400 bits / 0.5 ms $\approx$ **100 Mbps**

Overhead reduces this to $\approx$ **86 Mbps**

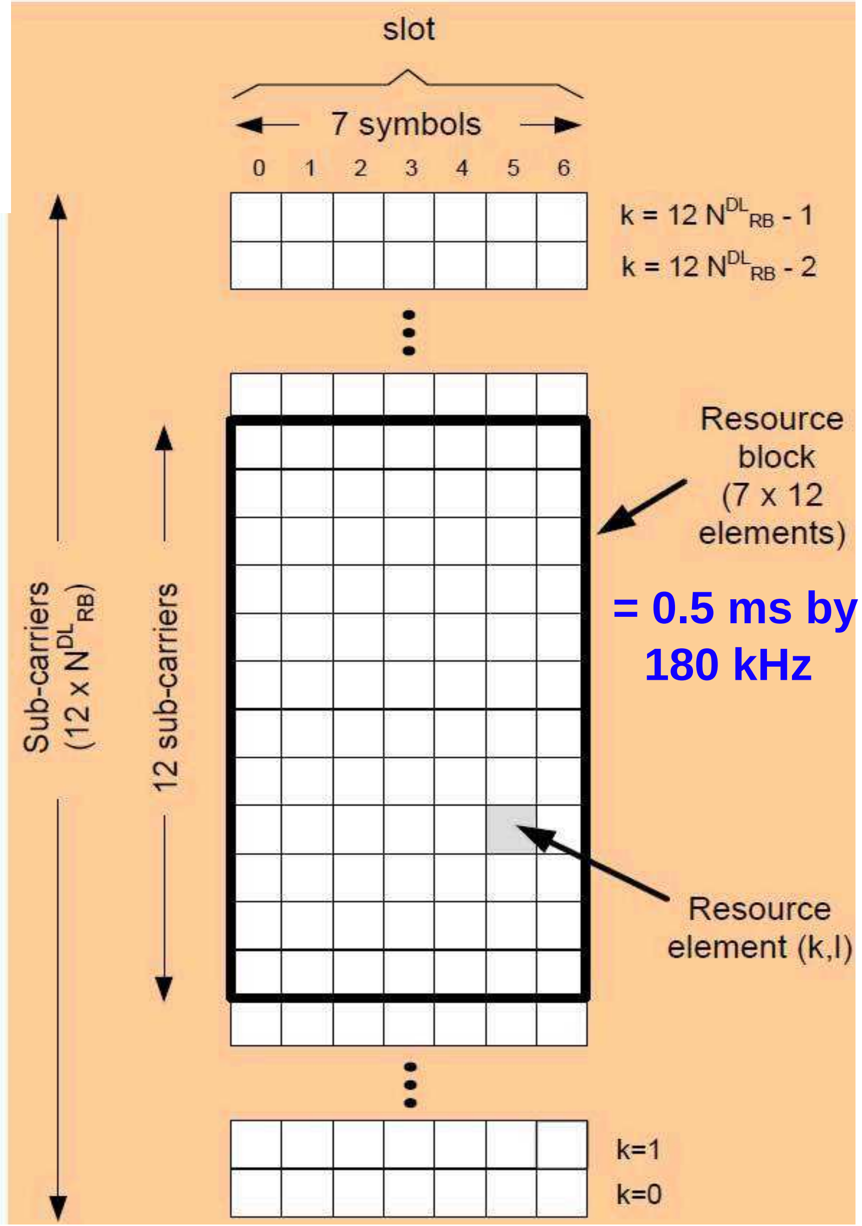
An OFDM Long Term Evolution (LTE) Application

LTE (Wide Area Network) Resources: Grid, Block, Element

Note: LTE standards involve Multiple Access

- Resource Grid
  - Several resource blocks (RB's).
  - Number of RB's adjusted to cover available BW.
- Resource Block (RB)
  - 7 by 12 elements.
  - Transmissions are allocated in discrete RB's.
- Resource element
  - one symbol width in time
  - one 15 kHz sub-carrier in frequency.

$\Delta f = 15$ kHz



The usual time-frequency grid needed for multiple-access assignments

Power vs Bandwidth OFDM Trade-Off for a 50 Mbps D/L Channel for an LTE System (neglecting overhead)

	Smaller BW - needs Larger SNR	Larger BW - needs Smaller SNR
Channel BW	10 MHz	15 MHz
Transform Size	1024	1536
Candidate Subcarriers	600	900
64-QAM bits/subcarrier:	6 bits	16-QAM: 4 bits
Bits/OFDM symb:	6 x 600 = 3600 bits	4 x 900 = 3600 bits
OFDM Symbol Time =	71.429 μsec	71.429 μsec
Data Rate =	$\frac{3600 \text{ bits}}{71.429\ \mu\text{sec}} \approx 50 \text{ Mbps}$	$\frac{3600 \text{ bits}}{71.429\ \mu\text{sec}} \approx 50 \text{ Mbps}$

A service provider typically responds to a wireless user's request for service (say a 50 Mbps channel) based upon the user's SNR. If the user's SNR is large, the provider will respond with a small BW channel (10 MHz here). If the SNR is small, the provider will offer a larger BW (15 MHz here) to support the requested bit rate.

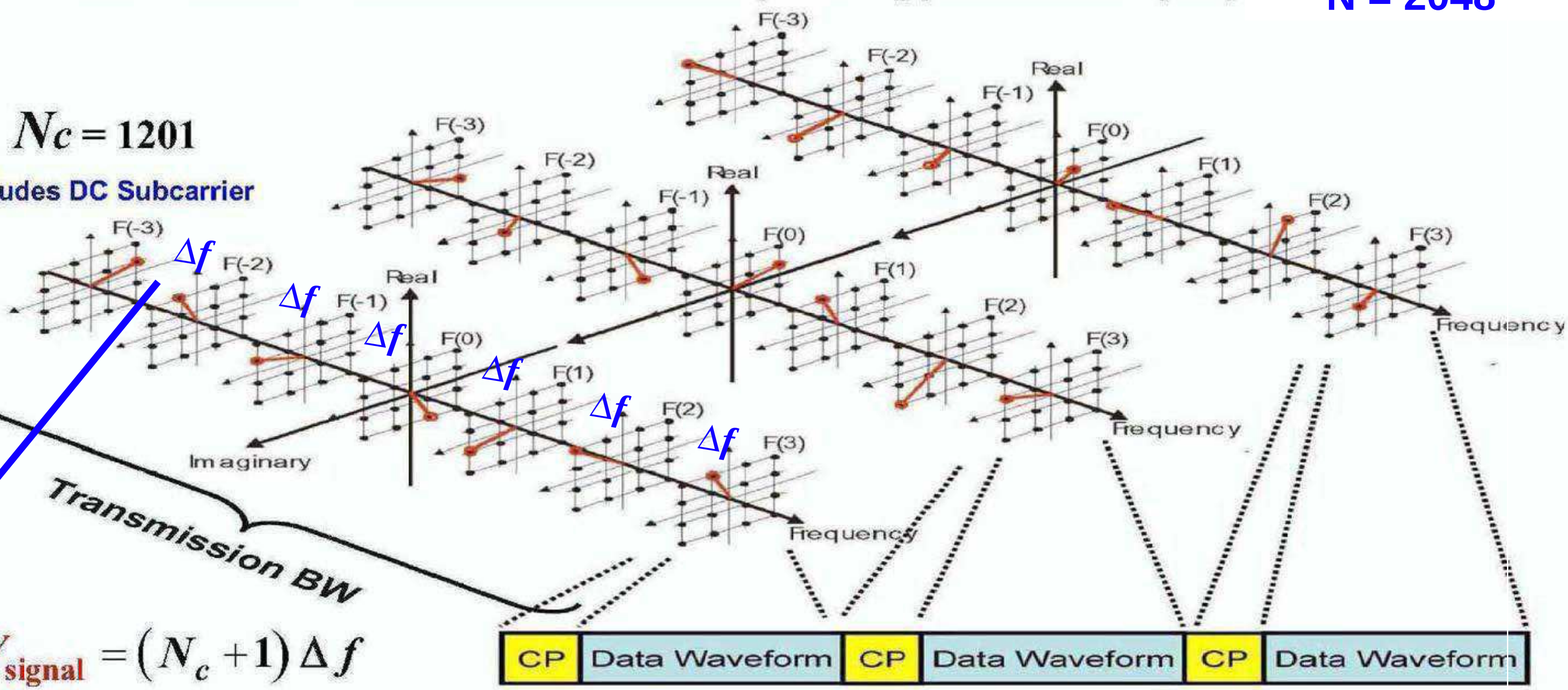
Shown above is a power versus bandwidth OFDM trade-off for a 50 Mbit/s downlink LTE channel (neglecting overhead). Either a 10 MHz channel and 64-QAM modulation or a 15 MHz channel and 16-QAM modulation can support the user's 50 Mbps request.

OFDM Parameters (LTE Typical Example)

Transform size
N = 2048

$N_c = 1201$

Includes DC Subcarrier



$W_{\text{signal}} = (N_c + 1)\,\Delta f$

$\Delta f = 1/T_s \quad = 1/66.67\ \mu s$

$= 15\ \text{kHz}$

$W_{\text{signal}} = 1202 \times 15\ \text{kHz}$

$\approx 18\ \text{MHz}$

Samples:

N_{cp}	N
144	2048

$N_L = N_{cp} + N = 2192$

Time duration:

T_{cp}	T_s
4.69 μs	66.67 μs

$T_{\text{OFDM}} = T_{cp} + T_s = 71.4\ \mu s$

Slot time $T_{\text{slot}} = 7 \times T_{\text{OFDM}} = 500\ \mu s$

Samp time $T_L = T_{\text{slot}} / \text{samp/slot}$

$= 500\ \mu s / 15{,}360 = 0.03255\ \mu s$

$T_{\text{OFDM}} = N_L \times T_L = 71.4\ \mu s$

Samp rate $f_s = \dfrac{N}{T_s} = \dfrac{1}{T_L} = 30.72\ \text{MHz}$

$R_{\text{OFDM}} = 1/T_{\text{OFDM}} = 14\ \text{ksymb/s}$

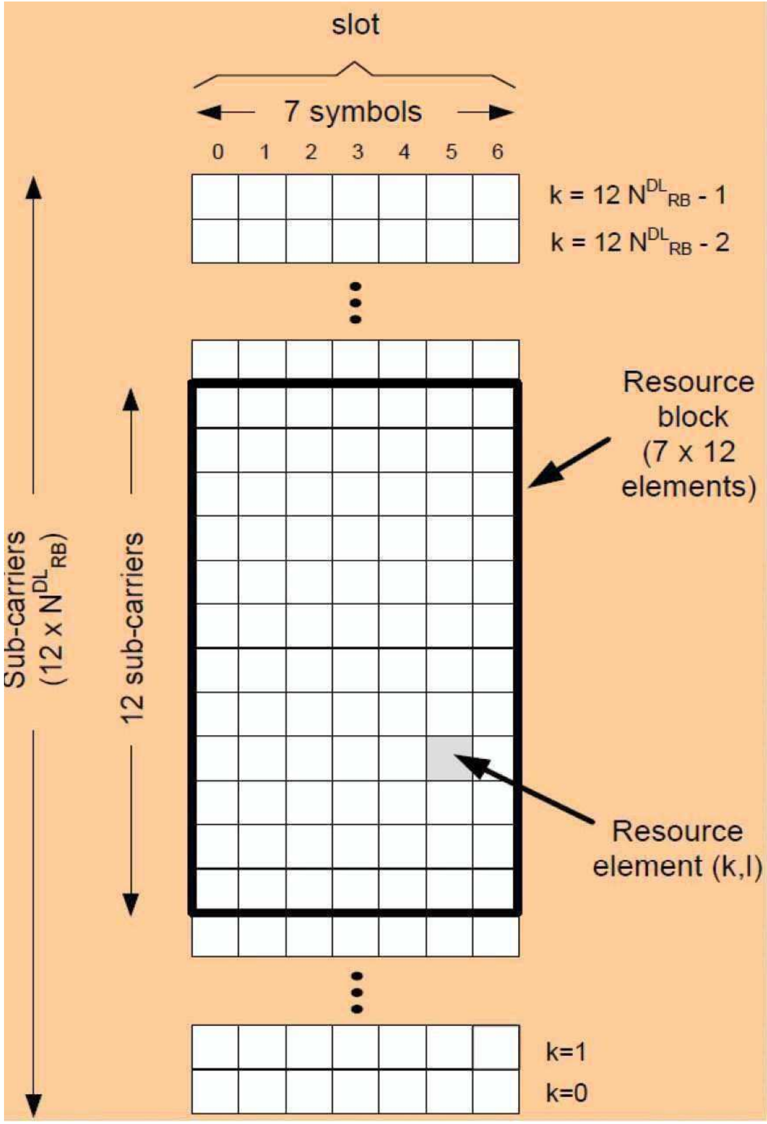
LTE Resources Scheduling Example

- Consider the slot and subcarrier architecture in the LTE resource blocks.

- Note that the slot time is 0.5 ms, that the subcarrier bandwidth (and separation between subcarriers) is 15 kHz. There are 12 x 7 = 84 REs/RB.

Find the theoretical peak downlink and uplink bit rates for a 20 MHz channel. The downlink uses 4 x 4 MIMO. The uplink is a single beam from UE to BS.

Assume that the modulation is 64-QAM, and that there is no error-correction coding. Consider that reference and control channels use about 25% of the resources.



LTE Throughput Solution

Downlink

20 MHz channel corresponds to 100 RBs, which contains 84 x 100 = 8,400 REs.

Theoretical Peak Data Rate (bits/ms) = number of REs per subframe x number of bits per modulation symbol (64-QAM for this example)

subframe = slot-pair = 1 ms

$$= \frac{(8400 \times 2)\ \text{REs}}{1\ \text{ms/subframe}} \times 6\ \text{bits/symbol}$$

$$\text{bits/sec} = 16.8 \times 10^6\ \text{REs/sec} \times 6\ \text{bits/symbol} = 100.8\ \text{Mbps}$$

In a MIMO 4 x 4 system, peak data rate = 4 x 100.8 Mbps = 403.2 Mbps

About 25% of resources are needed for reference and control signals. That leaves: ≈ 300 Mbps as the peak data rate.

Uplink

For a single beam from UE to BS, the peak data rate follows above computation, minus the MIMO, that is: 100.8 Mbps. After considering a 25% reduction for reference and control, the peak data rate is: ≈ 75 Mbps.

IEEE Virtual Presentation
The ABCs of OFDM
By Dr. Bernard Sklar

Part 1 March 18, 2021
Part 2 March 25, 2021

Abstract: The main benefit of OFDM is its ability to cope with Severe multipath channel conditions without needing Complex Equalization filters. How does it do this? In short, by "dividing and conquering." It partitions a High-data-rate signal into Smaller low-data-rate signals so that the data can be sent over many low-rate subchannels. We emphasize following:

- The Big Picture: Time/Frequency Relationships.
- Single-Carrier versus Multi-Carrier Systems.
- The 4 Key WSSUS Functions.
- OFDM Implementation Examples.
- Importance of the Cyclic Prefix (CP).
- Converting Linear Convolution to Circular Convolution.
- Periodic Outputs on a Unit Circle.
- OFDM Waveform Synthesis and Reception.
- Hermitian Symmetry.
- Our "Wish List."
- Testing for Orthogonality.
- Tricking the Channel.
- OFDM Applications (802.11a and LTE).
- Single-Carrier OFDM (SC-OFDM).

Single-Carrier OFDM
(SC-OFDM)

33. Why do binary and 4-ary orthogonal FSK manifest the same bandwidth-efficiency relationship? (Section 9.5.1)

34. Describe the subtle energy and rate transformations of received signals: from data-bits to channel-bits to symbols to chips. (Section 9.7.7)

35. Define the following terms: Baud, State, Communications Resource, Chip, Robust Signal. (Sections 1.1.3 and 7.2.2, Chapter 11, and Sections 12.3.2 and 12.4.2)

36. What are the two fading mechanisms that characterize small-scale fading? (Sec 15.2)

37. For a mobile fading channel, why is signal dispersion independent of fading rapidity? (Section 15.4.1.1)

38. What is the key difference between Rician fading and Rayleigh fading? (Sec 15.2.2)

39. In the context of a fading channel, define the terms: delay spread, coherence bandwidth, coherence time, Doppler spread. How are they related? (Sec 15.3 and 15.4)

40. What if any, are the differences in the terms: Doppler spread, spectrum spreading, fading bandwidth, fading rapidity, fading rate? (Section 15.4.2)

41. Why does signal distortion due to fading yield more serious degradation than a loss in SNR? (Section 15.5)

42. Why is OFDM useful for high-data-rate fading channels? (Sec. 15.5.1 & 15.20 Prob 15.20). What benefit of SC-OFDM makes it a natural for mobile systems?

43. To provide diversity between two fixed platforms, how large an interleaver span is needed? (Section 15.5.6)

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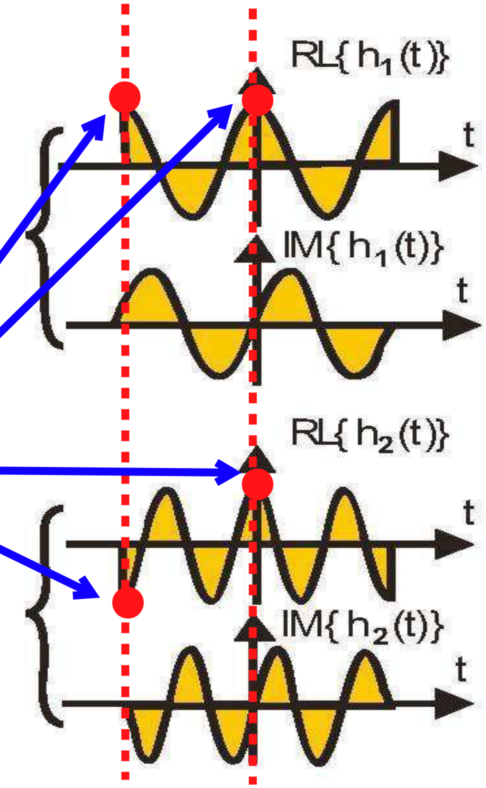
42. Why is OFDM useful for high-data-rate fading channels? (Sec. 15.5.1 & 15.20 Prob 15.20). What benefit of SC-OFDM makes it a natural for mobile systems?

SC-OFDM offers improved PAPR, which facilitates the efficient operation of power amplifiers.

SC-FDMA (DFT-Spread FDMA) versus OFDMA

• OFDMA systems achieve robustness in the presence of multipath by transmitting on M orthogonal frequency carriers, each operating at R/M bits/s. $M = Nc$ (earlier slides)



• OFDMA exhibits very pronounced envelope fluctuations (the output sum of gated sinusoids can yield a variety of amplitudes) resulting in high PAPR.

• This requires highly linear power amplifiers to avoid excessive IM distortion. Amplifiers have to operate with a large backoff from their peak power, resulting in lower power efficiency.

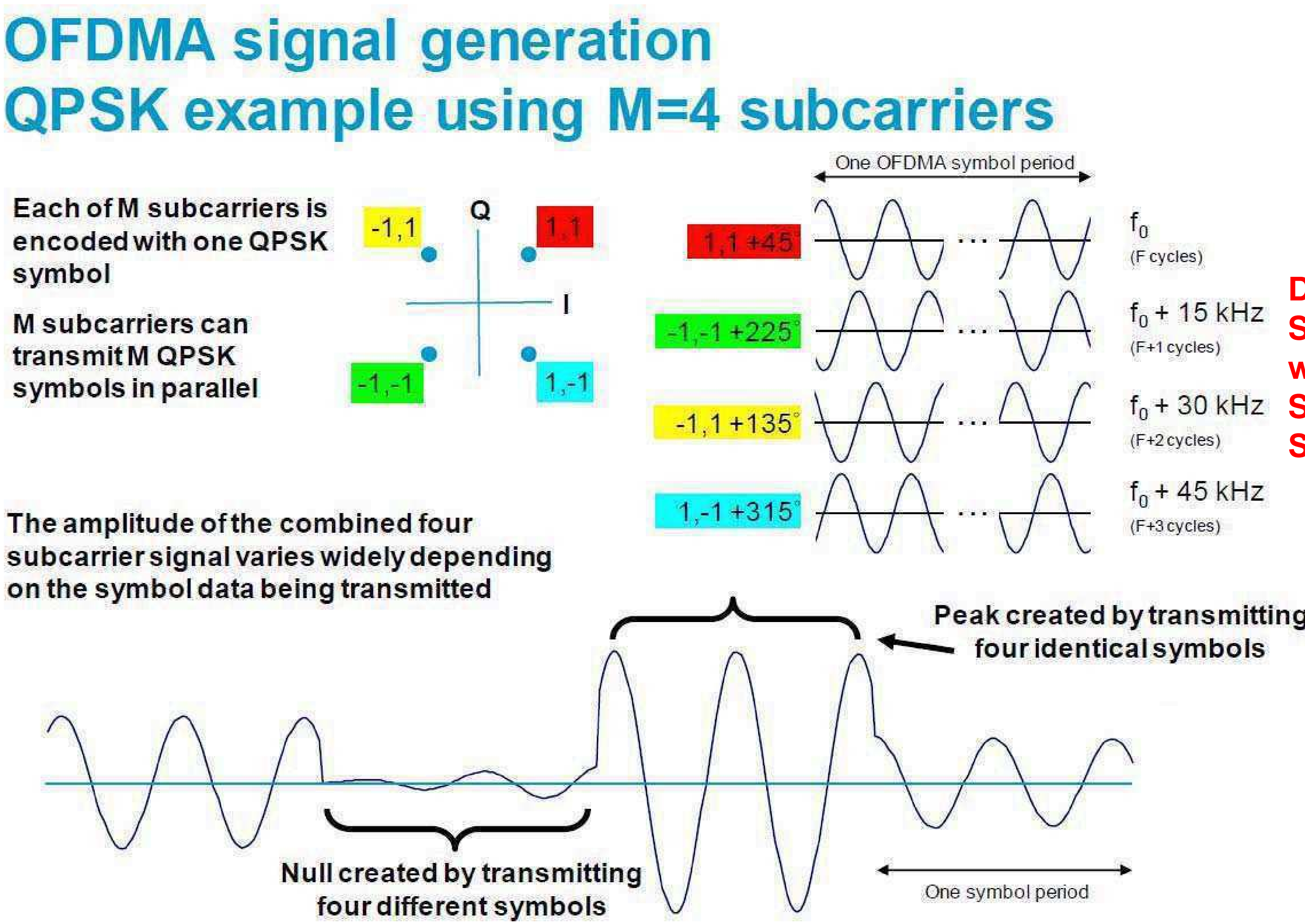
• SC-FDMA is a modified version of OFDMA for U/L transmission in the LTE of cellular systems. In the SC-FDMA output, individual (sin x)/x basis functions are transmitted sequentially (each peaks at a different time), thus reducing the PAPR.

• The Cyclic Prefix (CP) acts as a guard time. If the length of CP > T_m then there is no intersymbol interference (ISI). Since the CP is a copy of the last part of the symbol, it converts linear convolution (with the channel) into circular convolution, which in the frequency domain is a pointwise multiplication of DFT frequency samples.

• Equalization consists of dividing the DFT of the received signal by the DFT of the channel impulse response (a simple scaling – same for OFDM and SC-OFDM).

• For an ideal channel (no distortion), note that the receiver for SC-OFDM would only need to sample the received waveform at the appropriate times.

PAPR in OFDMA can be Large



Data Symbols with LTE Spectral Spacing

Ref: Moray Rumney, Agilent, March 20, 2008, "SC-FDMA The New LTE Uplink Explained"

On Trading Excess Bandwidth for Reduced Peak to Average Power Ratio in Single Carrier Shaped Dirichlet Kernel OFDM

fred harris, San Diego State University, fred.harris@sdsu.edu
Chris Dick, Xilinx Corporation, chris.dick@xilinx.com

Motivation for reducing Peak-to-Average Power Ratio (PAPR)

ABSTRACT
The waveform of choice for OFDM signaling is the sinusoid with an integer number of cycles per interval and with an appended cyclic prefix to obtain circular convolution with the channel. This combination makes the channel inversion particularly simple; performed as a ratio between the DFT of the received signal and the DFT of the channel. In fact, this relationship is valid for any periodic function formed as a sum of the basis sinusoids of the DFT. One particularly simple example of this class of signals is the Dirichlet kernel (the periodically extended sinc function). This kernel is used in single carrier OFDM [1]. An advantage of this kernel relative to the complex sinusoid kernel is a 3.4 dB reduction in peak to average power ratio (PAPR). We show here that a windowed version of this kernel exhibits a significantly lower, in fact up to a 10.0 dB reduction in PAPR. The cost to obtain the reduced PAPR is excess bandwidth but that may be a fair trade to obtain higher average transmitted power for a given peak power limited amplifier.

With average amplitude 1/4-th of peak amplitude, average power is 1/16-th of peak power. Power amplifiers are DC to AC converters and power pulled from the DC power supply, which is approximately constant, not delivered to the load is dissipated in the power amplifier. Amplifiers are very inefficient in their transduction process of turning DC power to signal power when they operate at small fractions of their peak power level. **Typical efficiencies for an amplifier operating with an IEEE 802.11a signal are on the order of 18 % [1]. Thus an amplifier required to supply 1 Watt would have a peak power capability of 16 watts and would be pulling 5.5 Watts from the power supply while squandering 4.5 Watts, raising the temperature of its heat sinks, while delivering 1 Watt to its external load.**

What are Basis Functions?

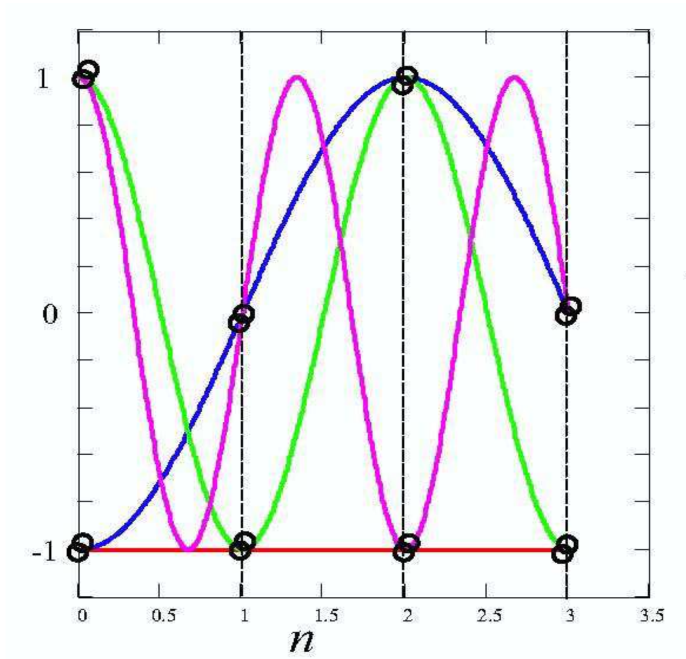
A common mathematical representation of a signal is a linear combination of elementary functions called basis functions.

In communication systems, examples of the most popular basis functions are gated sinusoids, square-root Nyquist pulses, and Nyquist pulses (also called (sin x)/x, sinc, raised cosine, and Dirichlet functions). Dirichlet is a periodically extended sinc function.

In OFDM, the basis functions in the time domain are gated sinusoids. In the frequency domain, they are sinc pulses.

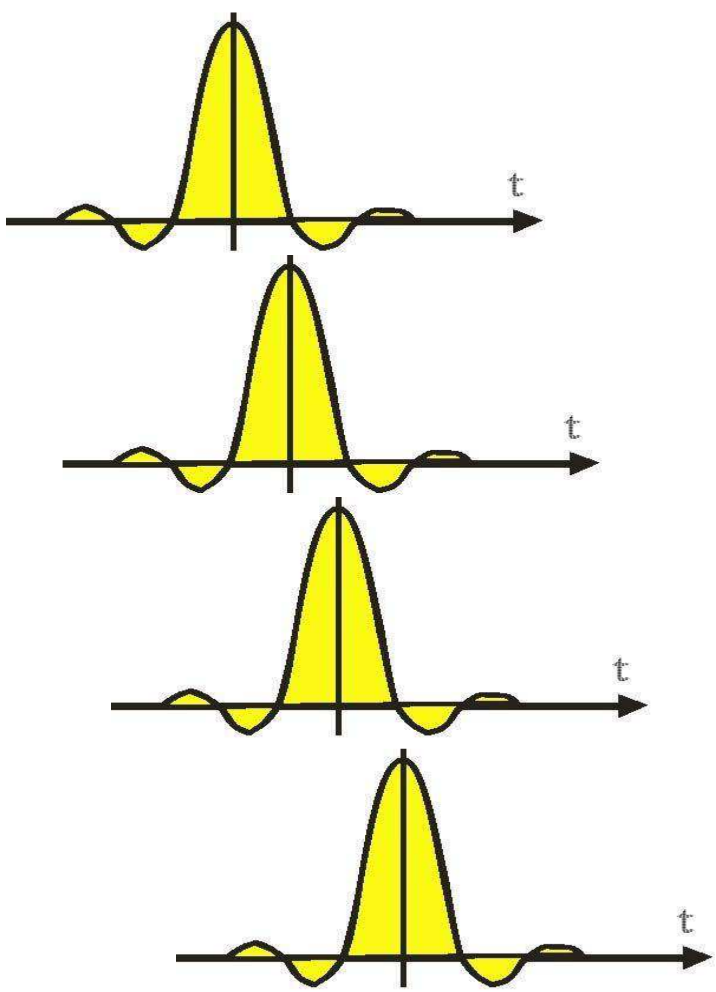
In SC-OFDM, the basis functions in the time domain are sinc pulses. In the frequency domain they are gated sinusoids.

Why SC-OFDM Offers Improved PAPR Performance Over Standard OFDM



Standard OFDM:

The output sum of gated sinusoids can yield a variety of amplitudes. Hence there is a large PAPR.



SC-OFDM:

Individual Dirichlet basis functions are transmitted sequentially, each peaking at a different time. Hence there is a reduced PAPR.

Generating OFDM : Sinc functions representing narrowband data phasors are inputted into the IFFT. Output time signal: Superposition of gated sinusoids.

Generating SC-OFDM: Sinc functions representing time waveforms are transformed (via the DFT) to wideband gated sinusoids. Such wideband spectra are inputted into the IFFT. Output time signal: Staggered sinc functions.

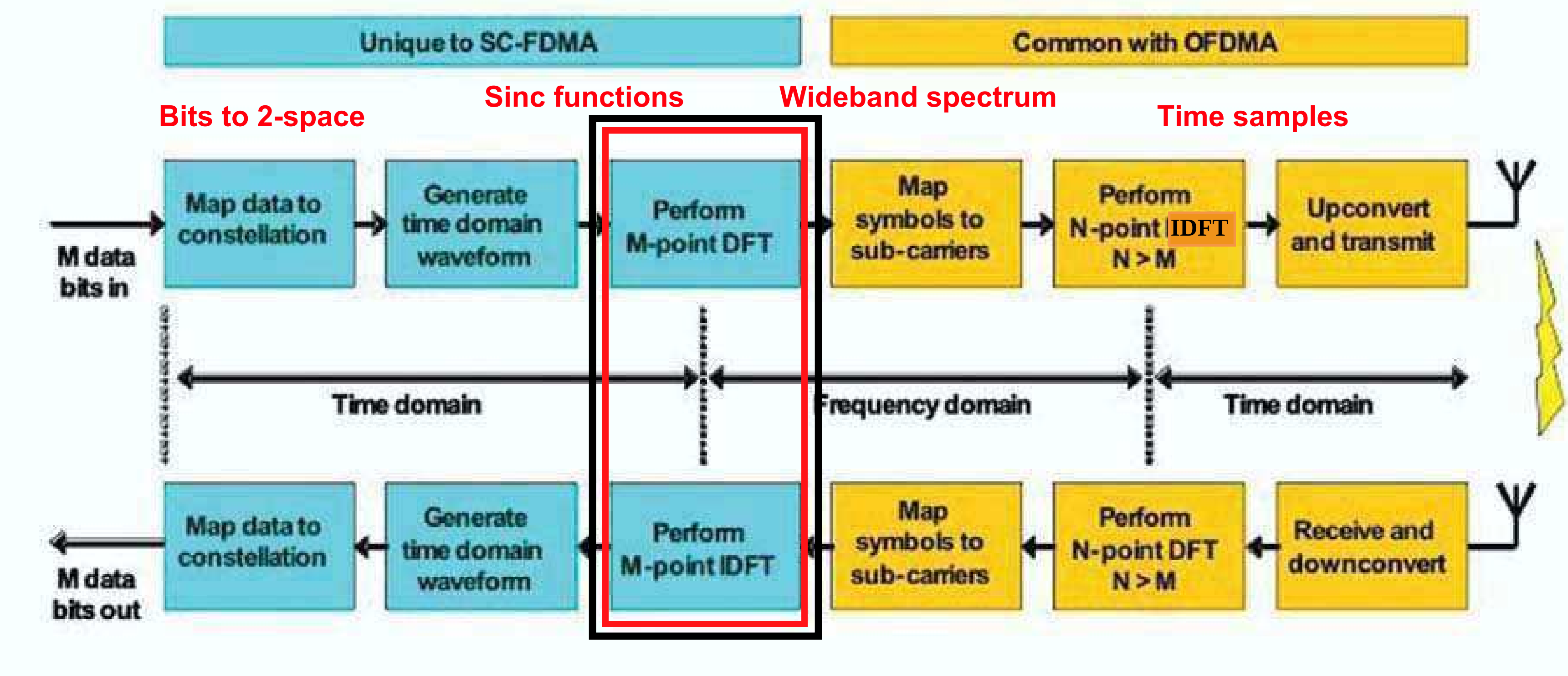
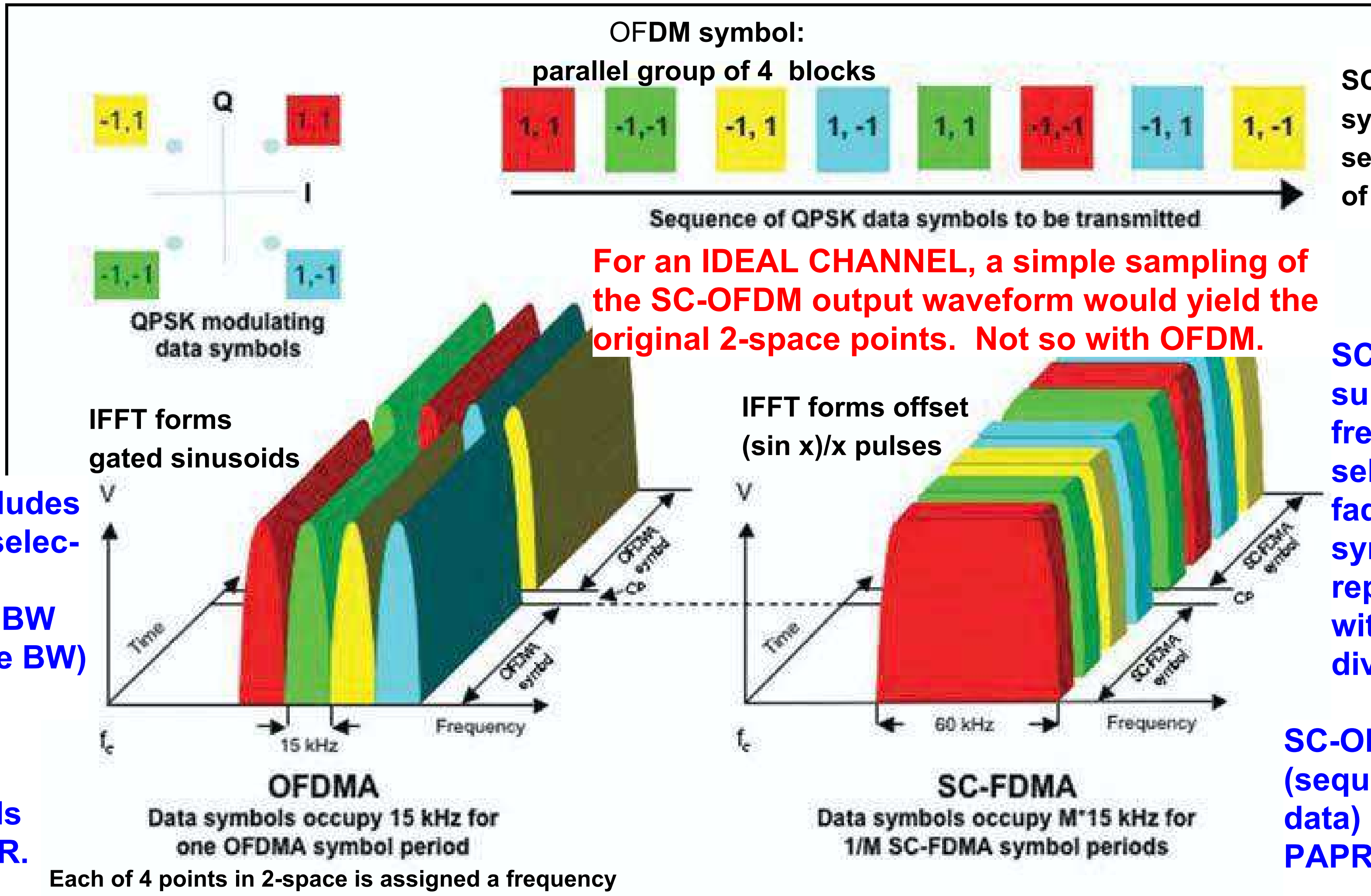


Figure 5 *Simplified model of SC-FDMA generation and reception*

Ref: White Paper, "De-mystifying SC-FDMA, The New LTE Uplink," Agilent Technologies, April 2008

SC-FDMA: Hybrid scheme, combining the low PAPR of single-carrier transmission systems with the long symbol time and flexible frequency allocation of OFDM. Note that: DFT is a a transform, and FFT is an algorithm. These names are often used inconsistently.

For SC-OFDM, the output waveform is made up of Dirichlet basis functions.



SC-OFDM symbol: serial group of 4 blocks

OFDM precludes frequency selec-tive fading (subcarrier BW < coherence BW)

SC-OFDM survives frequency selective fading (data symbol is represented with spectral diversity)

OFDM (parallel data) yields large PAPR.

SC-OFDM (sequential data) reduces PAPR.

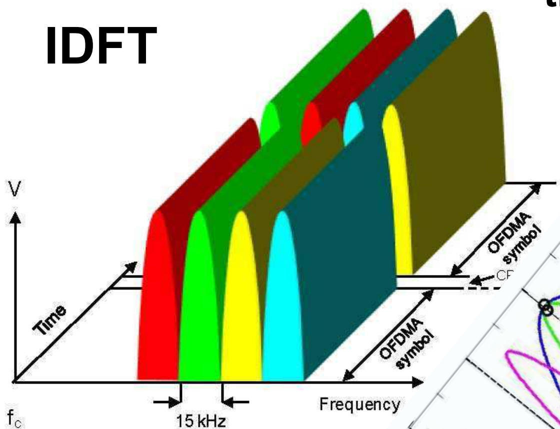
Figure 2 *Comparison of OFDMA and SC-FDMA transmitting a series of QPSK data symbols*

Ref: White Paper, "De-mystifying SC-FDMA, The New LTE Uplink," Agilent Technologies, April 2008

OFDM versus SC-OFDM Waveform Formation

OFDM

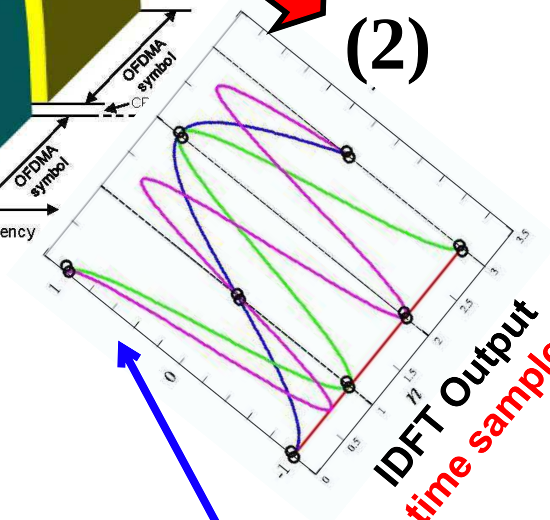
IDFT



Output waveform is the superposition of gated sinusoids

actually built

(2)

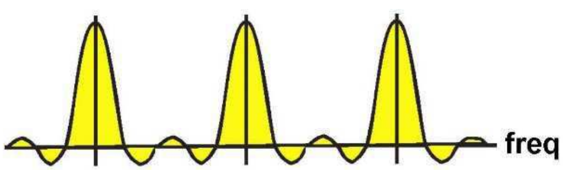


IDFT Output
time samples

The signal length is a multiple of its sinusoidal basis functions (there are an integer number of cycles per gated sinusoid).

(1)

IDFT Input
freq samples

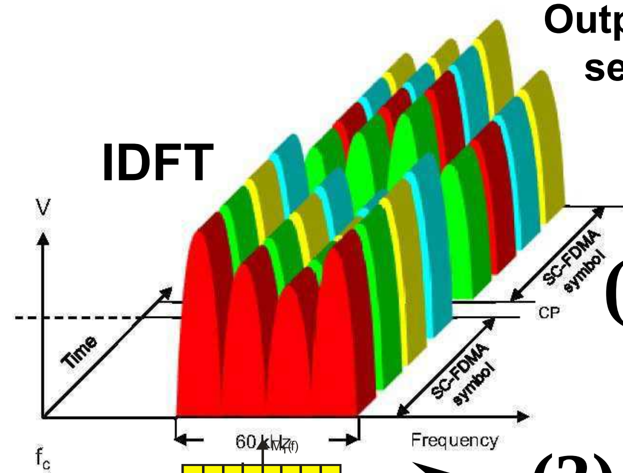


visulalization:
sinc functions
NOT actually built

OFDM

SC-OFDM

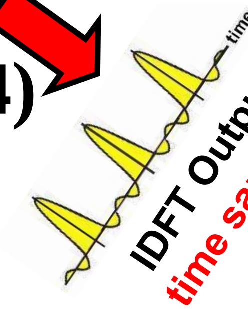
IDFT



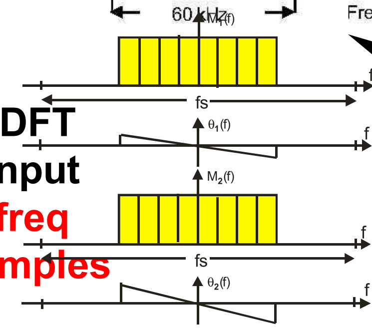
Output waveform is a sequence of sinc functions

actually built

(4)



IDFT Output
time samples



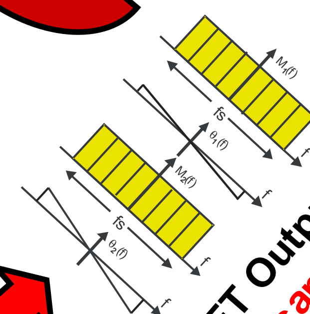
IDFT
Input
freq
samples

(3)

SC-OFDM

Zero padding of the IDFT input freq samples, raises the sample rate f_s

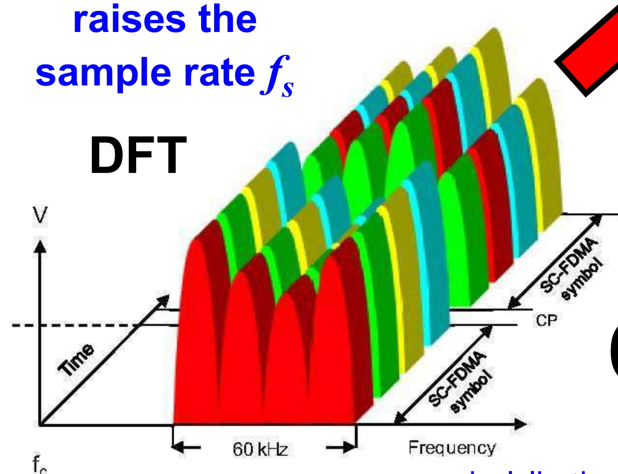
WB frequency DFT Output & IDFT Input



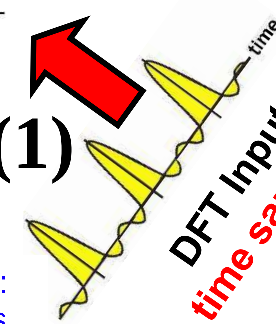
DFT Output
freq samples

(2)

DFT



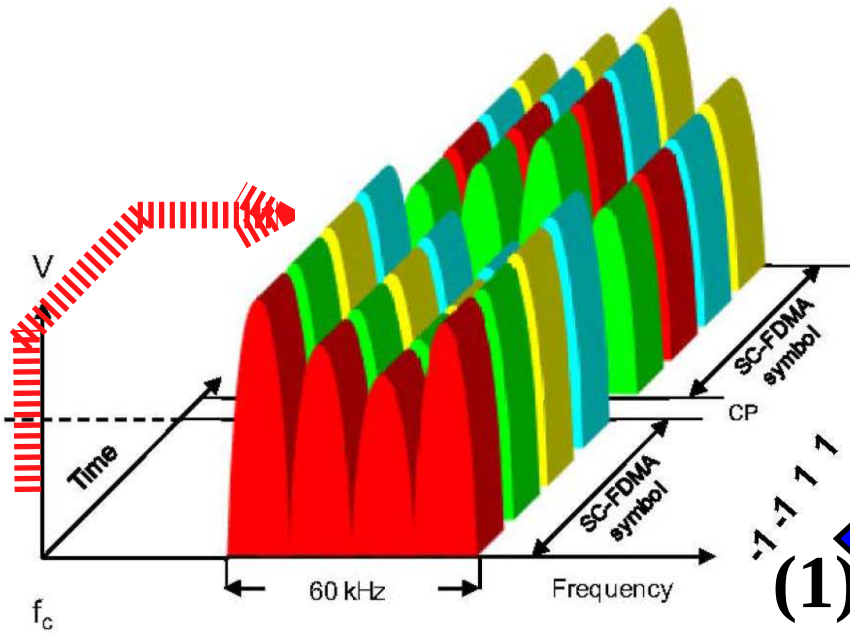
(1)



DFT Input
time samples

visulalization:
sinc functions
NOT actually built

SC-OFDM: Same Time-Frequency Plot for DFT and IDFT



0 -2+j2 0 -2-j2

(4) Finish Here

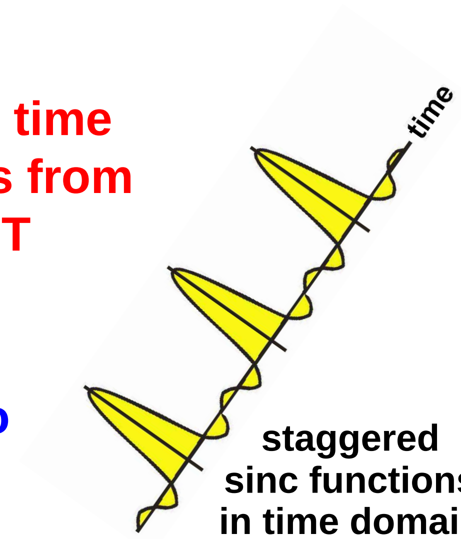
Output time samples from IDFT

Start Here

Input time samples to DFT

-1 -1 1 1

(1)



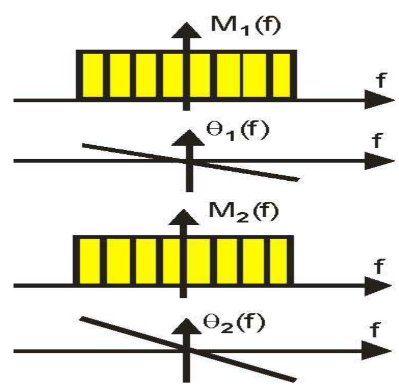
staggered sinc functions in time domain

(3)

(2)

Input frequency samples to IDFT

Output frequency samples from DFT

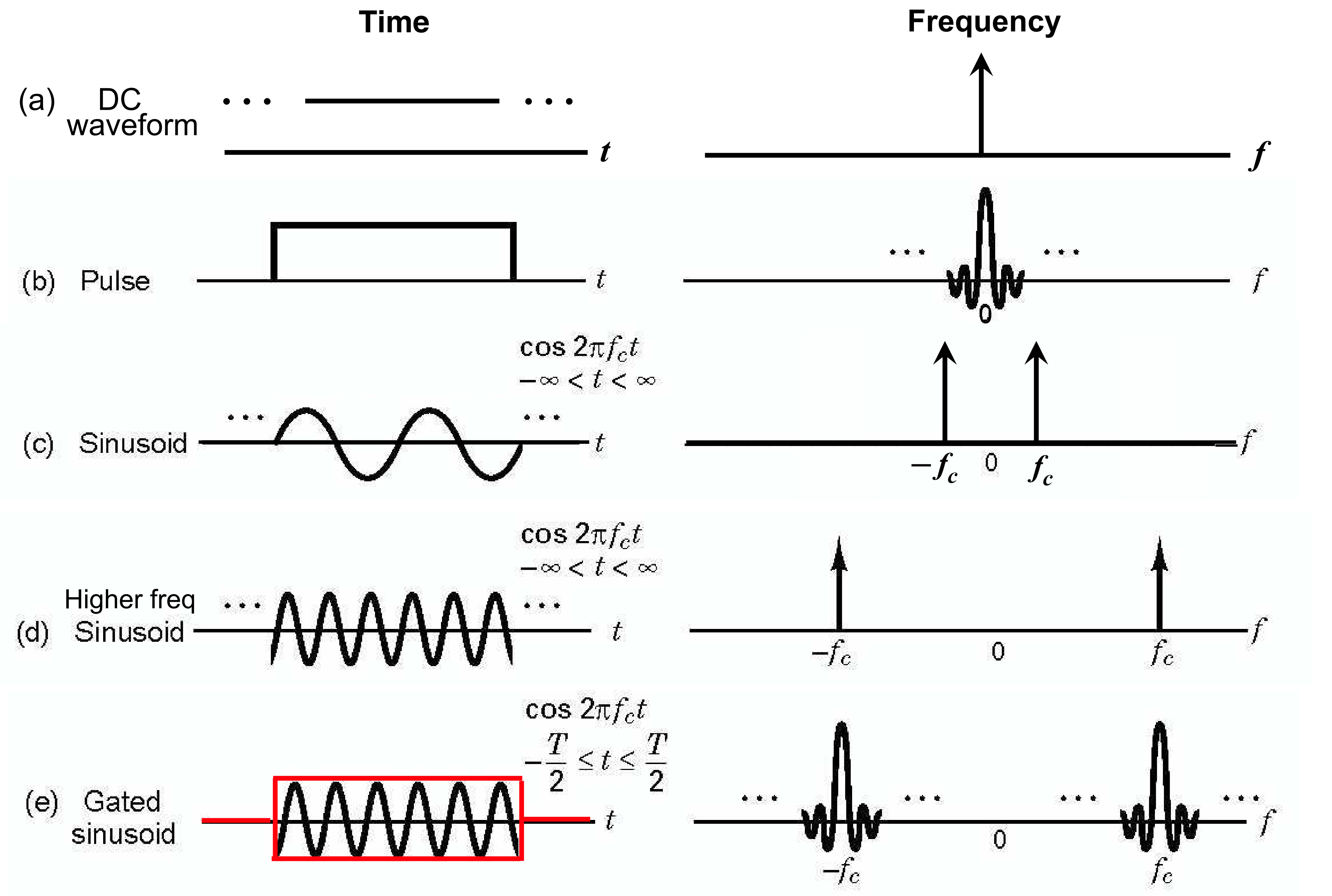


gated sinusoid functions in frequency domain

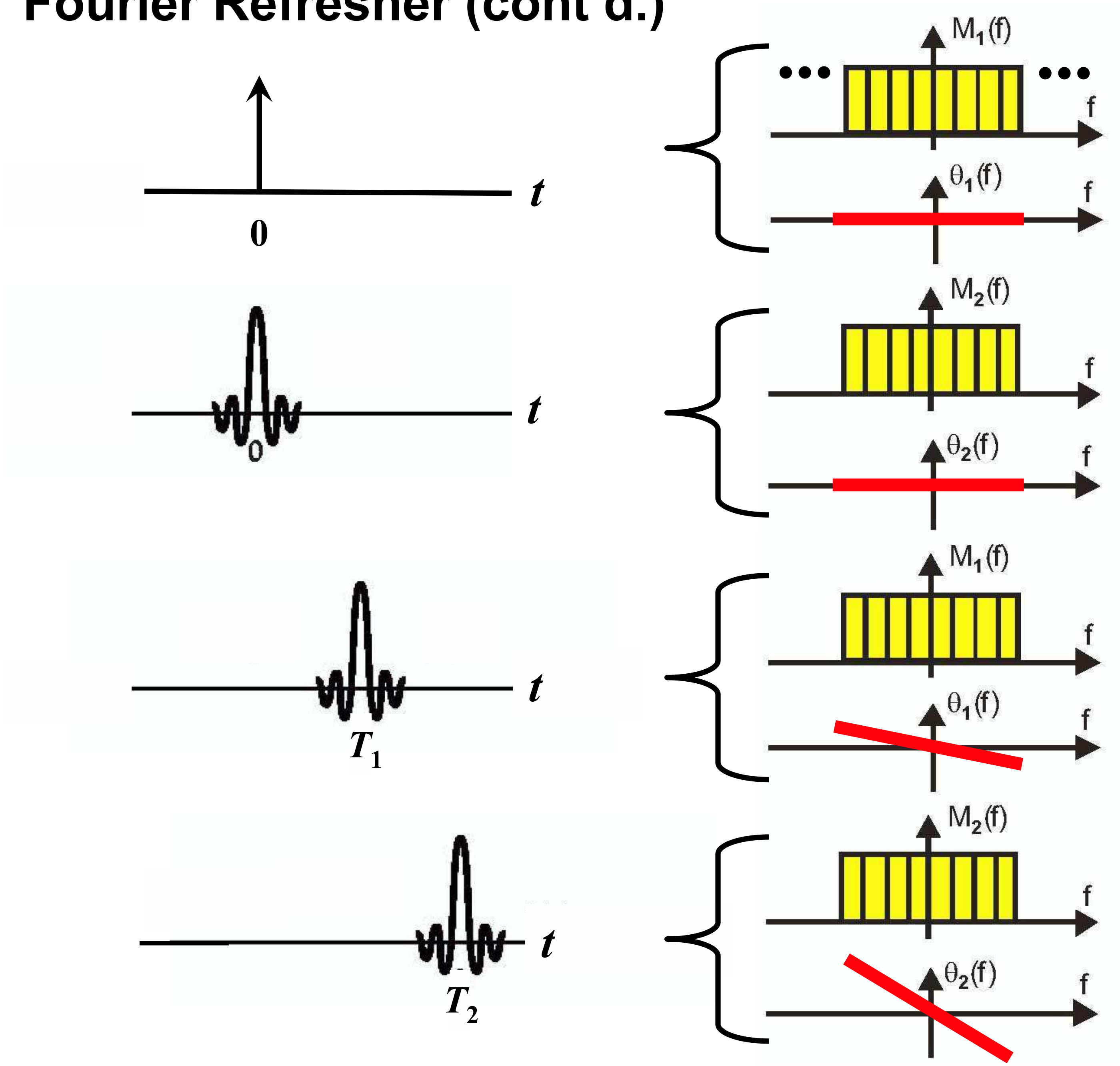
Each SC-OFDM time symbol is formed as a sum of constant envelope sinusoids cooperating over the same long OFDM symbol time. Each main lobe and tails extends over the same long-duration OFDM time interval.

In SC-OFDM, the narrow sinc-function main lobes are misleading, giving the impression that short symbols are sent. Just like OFDM sends the superposition of (long) gated sinusoids, here too SC-OFDM sends the superposition of (long) gated sinc functions.

Fourier Refresher



Fourier Refresher (cont'd.)



Infinite BW White (uncorrelated) Example: AWGN

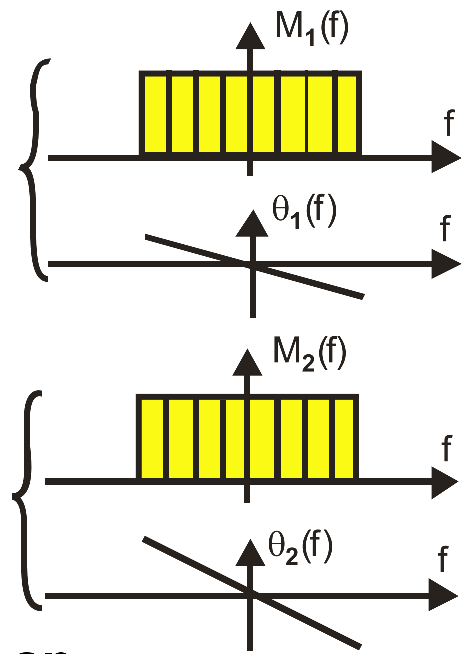
Wideband Spectrum of Gated Sinusoids Phase slope = 0

Wideband Spectrum of Gated Sinusoids Phase slope = -T_1

Wideband Spectrum of Gated Sinusoids Phase slope = -T_2

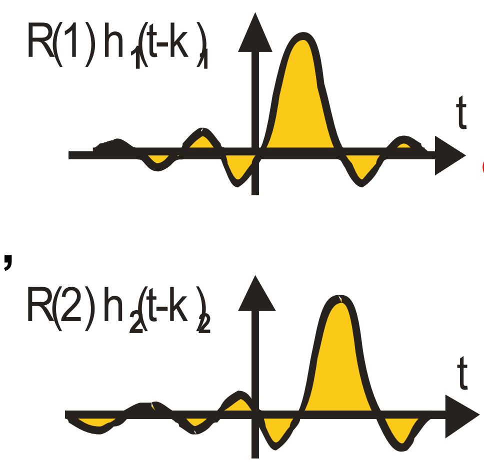
SC-OFDM

- A data sequence is transformed to a sequence of 2-space points $\{p_k\}$ following some *M*-ary modulation scheme. Next,
- The DFT transforms a time block of such points to a summation of wide BW spectral samples (complex). One data point (with zeros at all other times in the block) yields an $M_i(f)\angle\theta_i$
- The sample rate is raised (zero extension)
- The IFFT transforms each wide BW spectrum to an offset (sin x)/x pulse (a phase offset in the frequency domain yields a time offset in the time domain)
- Superposition of the offset (sin x)/x pulses yields the SC-OFDM output time waveform
- Such waveforms carry constellation amplitudes, which are retrieved by the MF at the receiver
- For an IDEAL CHANNEL, sampling the output would yield the original 2-space points



Visualization: output of DFT, Input to IFFT: wide BW spectrum (rectangular envelope)

Gated sinusoids spectrum



Actually built: output of IFFT:

Offset (sin x)/x pulses

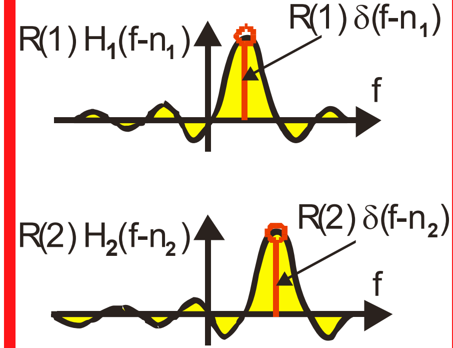
OFDM and SC-OFDM

OFDM

Data is characterized by a set of points in 2-space that are mapped into coefficients of a sinusoidal basis set that describe the details of each sinusoid to be built.

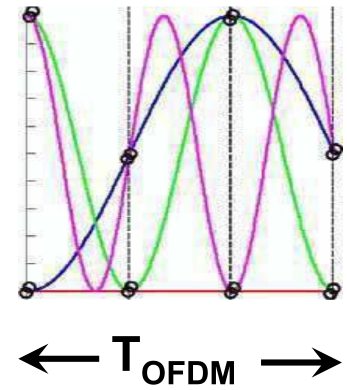
Each data symbol is mapped to a separate subcarrier (i.e., carried by a sinc function in frequency).

Staggered Sinc Functions



frequency coefficients in → **IDFT** → time samples out

Time Domain Gated sinusoids



Visualization: (sin x)/x (not actually built)

It is the transform of a gated sinusoid.

SC-OFDM

Data is characterized by a set of points that can be visualized as (sin x)/x functions, offset in time

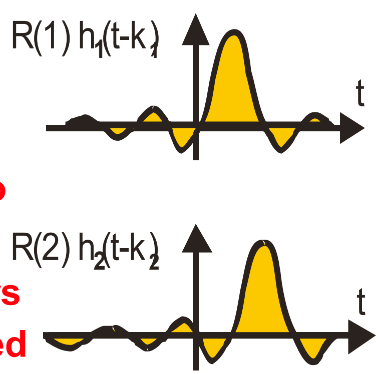
time samples in → **DFT** → frequency coefficients → **IDFT** → time samples out

Key added step for SC-OFDM

Wideband spectrum

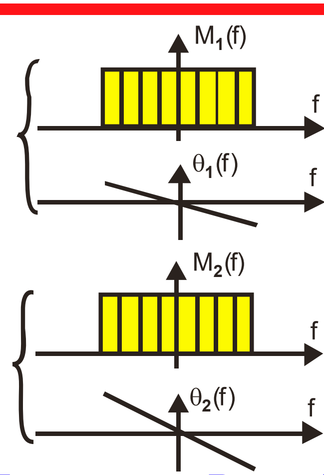
Actually built:
Time offset (sin x)/x pulses

Each data symbol is mapped to multiple subcarriers (i.e., carried by a sinc function in time).

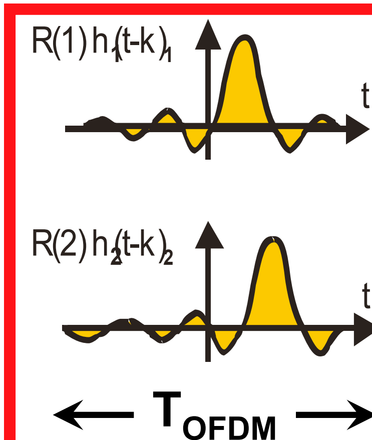


Each time point taken one at a time (with zero extension) yields a WB spectrum with a phase slope

Zero extensions provide frequency padding.



Frequency Domain Gated Sinusoids



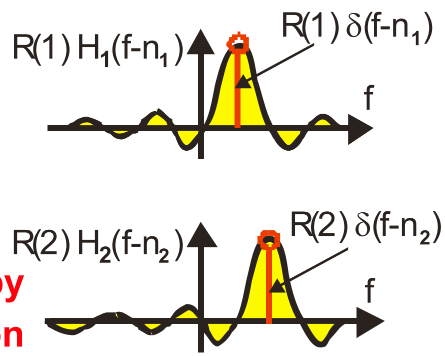
Staggered Sinc Functions

OFDM and SC-OFDM

OFDM

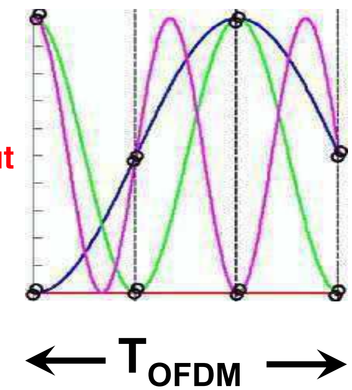
Data is characterized by a set of points in 2-space that are mapped into coefficients of a sinusoidal basis set that describe the details of each sinusoid to be built.

Each data symbol is mapped to a separate subcarrier (i.e., carried by a sinc function



frequency coefficients in → **IFFT** → **time samples out**

Time Domain Gated sinusoids

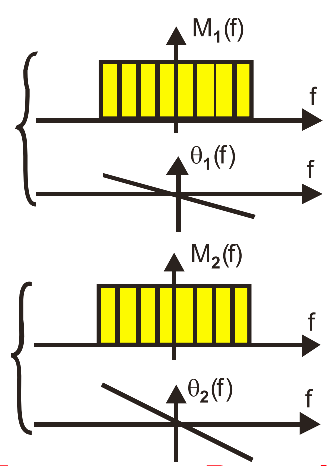


Visualization: (sin x)/x (not actually built)
It is the transform of a gated sinusoid.

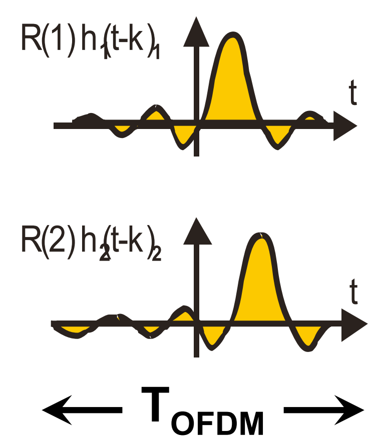
frequency coefficients → **IFFT** → **time samples out**

Wideband spectrum

Actually built: Time offset (sin x)/x pulses



Frequency Domain Gated Sinusoids



The IFFT performs a Dual-like function in SC-OFDM compared to ordinary OFDM.

Thank You

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